

Relevance of Boolean Logic to Modern Society

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Abstract

The aim of this paper is to critically and historically situate the relevance of Boolean logic to the structure and dynamism of the contemporary society. In his groundbreaking work; *An Investigation of the Laws of Thought* (1854), Boole traces the transformation of algebraic logic from a philosophical project into the formal backbone of modern computational systems. The modern society relevance of this study becomes obvious hence, situating Boolean logic within a broader intellectual lineage that includes Claude Shannon's formalization of switching circuits, Alan Turing's theory of computation, and subsequent developments in artificial intelligence, information retrieval, and cryptography. Adopting an expository and analytical method of research, the paper demonstrates that Boolean logic functions not merely as a historical artifact but as a foundational grammar in the modern society underlying digital infrastructures, from microprocessor design to database querying and algorithmic decision-making. At the same time, it critically examines the limitations of strict bivalent reasoning in contexts characterized by uncertainty, ambiguity, and ethical complexity. By engaging with extensions such as fuzzy logic, probabilistic reasoning, and contemporary critiques in algorithmic ethics, the study argues that Boolean logic persists as both a necessary foundation and an evolving framework.

Keywords: Logic, Boole, digital computing, artificial intelligence, information retrieval, and algorithmic ethics.

Introduction

During the mid-nineteenth century the English mathematician George Boole undertook an intellectual project that would in a direction that he could hardly have envisaged transform the material and conceptual foundations of human civilization. Investigation of the laws of thought, on which are founded the mathematical theories of logic and probabilities, in 1854, has been his major work, which advanced a form of algebraic notation with sufficient power to represent the basic procedures of the human mind.¹ The ambition of Boole did not start and finish with the technical, but he was determined to show that the laws of thought itself could be expressed in the language of exact mathematics, in this way to bring logic and algebra into concord, a congruity which had been sought in vain by philosophers since Aristotle. His system (consisting of an algebra of classes which acted upon binary values of one and zero, under the laws of conjunction, disjunction, and negation) was the present-day universal idea of Boolean algebra.²

The value of the contribution made by Boole goes much farther than the philosophical background on which it was created. Although Boole himself was more focused on the formalisation of logical inference and calculus of probabilities, it is his binary structure that would become the exact language of mathematics that would be needed in the twentieth century with the electronic revolution.³ Not the history question, as to how the conceptions of Boole came to be applied in technology, but the philosophical and the evaluative question is the following: in what ways, and in what ways, do the conceptions of Boolean logic continue to apply to the epistemic, technological, and ethical aspects of contemporary society, and what are the boundaries of the applicability of this concept to an age of growing computational complexity? The paper has followed the approach of exposition and critical literature analysis. The expository aspect would require a meticulous reconsideration of the logical system formulated by Boole based on primary documents, and the way the conceptual framework of his algebra was appropriated by other mathematicians or computer scientist including Claude Shannon and Alan Turing. The analytical side entails critical interaction with secondary scholarship that has questionably assessed, elaborated and in certain instances doubted the sufficiency of any fantastic system of Booleans to accommodate the pressures of current computing, artificial intelligence and information guard. The citation system is

¹George Boole, *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities* (London: Walton and Maberly, 1854), 1–20.

²Ivor Grattan-Guinness, *The Search for Mathematical Roots, 1870–1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel* (Princeton: Princeton University Press, 2000), 45–67.

³George Boole, *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities* (London: Walton and Maberly, 1854), 37–42.

the notes-bibliography system recommended by the seventh edition of The Chicago Manual of Style, and uses endnotes in reference to documents.

Understanding Boole's Logical System

In order to value the applicability of the Boolean logic to the contemporary society, we must find out what the internal structure and philosophical inspirations of the system Boole described. Before Boole, the most common school of thought in formal logic was Aristotelian syllogistic, a system defining the different valid kinds of deductive inference by the structure of a set of categorical propositions. Although the syllogistic continued to be the mainstay of rational training during the course of more than two millennia, it was severely inadequate in breadth and expression. John Corcoran has also pointed out persuasively that Boole did not accomplish the rejection of Aristotelian logic but rather the radical generalisation of that logic, the translation of the qualitative classifications of the syllogistic into a general quantitative algebraic system with much more combinatorial capacity..⁴

Boole's system rests upon several foundational principles. First, he introduced the concept of classes or sets, represented by symbols such as x , y , and z , which denote collections of objects sharing a common property. Second, he defined operations upon these classes that correspond to logical connectives: multiplication (xy) represents the intersection of classes, addition ($x + y$) represents their union, and the complement of a class x is given by $(1 - x)$. Third, and most consequentially, Boole restricted the values that his variables could assume to the binary pair 0 and 1, where 1 represents the universal class (the totality of objects under consideration) and 0 represents the empty class (nothing). This restriction yields what Boole called the "fundamental law of thought," expressed algebraically as $x^2 = x$, or equivalently $x(1 - x) = 0$, which asserts that every object either belongs to a class or does not, with no intermediate possibility.⁵

The philosophical importance of this law can be exaggerated as it captures in an algebraic identity the classical logical principles of non-contradiction and excluded middle. The idempotent law $x^2 = X$ has been proven by Theodore Hailperin not only to be such a convenient property but the defining property that separates Boolean algebra, as opposed to ordinary numerical algebra, and it is this property that makes the system relevant to binary digital systems.⁶ Stanley Burris has also demonstrated that the equational-foundation of the logic of Boole, in its proper formalisation, is isomorphic to a complete lattice structure that has the ability to model any finite propositional logic.⁷

From Boole to the Digital Age: Shannon and the Electronic Circuit

The turning point in transferring the translation of Boolean logic as a philosophical interest to a technological basis came in 1938 when Claude Shannon, at then a graduate student at the Massachusetts Institute of Technology, wrote his master thesis entitled, "A Symbolic Analysis of Relay and Switching Circuits".⁸ The idea offered by Shannon was both intuitive and ground breaking, and it was as follows: he understood that the two binary states of electrical relay switches open and closed, on and off, could be directly mapped onto the truth values of the propositional logic of Boolean truisms: 1 as a true proposition (or a closed circuit) and 0 as a false proposition (or an open circuit).⁹ With this isomorphism, Shannon showed that any logical function that could be represented in Boolean algebra could be realised physically as a system of electrical switches, and vice versa, that any switching network could be designed and analysed using the concept of Boolean algebra.

The effects of the discovery made by Shannon were enormous. It gave the conceptual foundations to the methodology of digital circuit design, the practise of electrical engineering leaving an empirically developed field, but giving it a mathematical base. All the logic gates of a modern microprocessor, such as AND, OR, NOT, NAND, NOR, and XOR, exist physically as a realisation of a Boolean operation. The Boolean algebraic techniques are used in designing, checking, and optimization of the combinational and sequential circuits that form the arithmetic logic units, memory controllers, and the most recent instruction decoders of modern computers..¹⁰ Relying on the Boolean abstraction over binary switching, the whole trail of digital design, starting with more or less basic elements like transistors and switches and up to open

⁴John Corcoran, "Aristotle's Prior Analytics and Boole's Laws of Thought," *History and Philosophy of Logic* 24, no. 4 (2003): 261–88.

⁵Claude Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Transactions of the American Institute of Electrical Engineers* 57, no. 12 (1938): 713–23.

⁶Claude Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Transactions of the American Institute of Electrical Engineers* 57, no. 12 (1938): 715.

⁷Raul Rojas, "The Design Principles of Konrad Zuse's Mechanical Computers," in *The First Computers: History and Architectures*, ed. Raúl Rojas and Ulf Hashagen (Cambridge, MA: MIT Press, 2000), 95–108

⁸Andrew Hodges, *Alan Turing: The Enigma* (London: Burnett Books, 1983), 96–130.

⁹Martin Davis, *The Universal Computer: The Road from Leibniz to Turing* (New York: W. W. Norton, 2000), 147–65.

¹⁰George Boole, *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities* (London: Walton and Maberly, 1854), 24-30

architectures of complex systems, is based on this concept, as described by Jan Rabaey, Anantha Chandrakasan, and Borivoje Nikolić in their standard treatment of digital integrated circuits.¹¹

It should be mentioned that Shannon did not work alone. Such similar parallels between mechanical and electromechanical computing as demonstrated by Raul Rojas led to the parallel development of such principles by Konrad Zuse in Germany.¹² Similarly, the formal description of the universal computing machine, found theoretically, independently of Boolean algebra, by Alan Turing assumed that information was represented using a binary code that the system of Boole had rendered mathematically explicit.¹³ In discussing the intellectual lineage between the dream of a calculus ratiocinator by Leibniz and the actual creation of programmable digital computers, Martin Davis has traced that history as running through the works of Boolean logic and algebra by an intellectual lineage that he considers to be one of the most significant threads in the history of thought.¹⁴

Boolean Logic in Information Retrieval and the Digital Economy

In addition to its seminal contribution to the hardware design, Boolean logic is ubiquitous in the software systems and information architecture of the present-day digital life. The lateral and relatively immediate result is the application in information retrieval, the science involved in storage, organisation, and access of text and multimedia information in large bodies of data. In their thorough coverage of the topic, Christopher Manning, Prabhakar Raghavan and Hinrich Schütze cite the Boolean retrieval model as the first and most natural formal model of document collection search.¹⁵ Within this model, query is in the form of the Boolean expression of search terms and the operators of AND, OR, and NOT which a user has typed and documents are returned on the conditions of meeting the logical conditions of the query.¹⁶

Although modern searching engines (like Google) are far beyond the plain Boolean model, including probabilistic ranking algorithms, natural language processing, and machine learning methods, the logic architecture of such a system is of a strictly Boolean nature. In their canonical account of the Google search architecture, Sergey Brin and Lawrence Page admitted that indexing and matching operations that form the core of web search are based on the use of Boolean set operations on the inverted index structures.¹⁷ Whenever a user types in a query to a search engine, the system carries out intersections, unions, complements of sets of documents, which are exactly the conjunction, disjunction and negation of the algebra introduced by Boole. The fact that they are now implemented in billions of documents within fractions of a second does not change the logical nature thereof, only testifies to the remarkable scalability of the Boolean formalism, as applied to relatively modern hardware.

The extent to which Boolean logic has been adopted in information retrieval is very interesting to the organisation of the digital economy. The business model of the advertising activities of the large technology corporations is conditional upon the capability to connect the user request with the correspondent material and advert, and this business model is based on Boolean matching and filtering operations. SQL Database query language uses Boolean logic queries in a WHERE clause to define under what conditions records are returned, updated, or deleted. Enterprise resource planning systems, customer relationship management systems and financial trading systems, themselves, in the formal, logical expression of their functionality, are extended uses of Boolean algebra. Frank Vahid has shown that design of digital systems (using hardware description languages) to software-level Boolean expressions is a continuous Boolean reason of the design.¹⁸

Boolean Logic, Cryptography, and Digital Security

The applicability of Boolean logic to the contemporary society goes severely further into the area of cryptography and information security. Majority of the operations of symmetric and asymmetric encryption algorithm are executed using binary data and Boolean operations. The most used operation is that of exclusive-or (XOR) which is also a Boolean operation, and it results in a false when it has the same inputs, this is the most common operation in the cryptographic design. The XOR operation is also found to be the most predominant mixing operation in stream cyphers, block cyphers,

¹¹Theodore Hailperin, *Boole's Logic and Probability: A Critical Exposition from the Standpoint of Contemporary Algebra, Logic and Probability Theory*, 2nd ed. (Amsterdam: North-Holland, 1986), 58–73.

¹²Stanley Burris, "The Laws of Boole's Thought," *History and Philosophy of Logic* 21, no. 1 (2000): 39–54.

¹³Claude Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Transactions of the American Institute of Electrical Engineers* 57, no. 12 (1938): 718–20.

¹⁴Jan Rabaey, Anantha Chandrakasan, and Borivoje Nikolić, *Digital Integrated Circuits: A Design Perspective*, 2nd ed. (Upper Saddle River, NJ: Pearson, 2003), 11–45.

¹⁵Frank Vahid, *Digital Design with RTL Design, VHDL, and Verilog*, 2nd ed. (Hoboken, NJ: John Wiley, 2011), 35–70.

¹⁶Christof Paar and Jan Pelzl, *Understanding Cryptography: A Textbook for Students and Practitioners* (Berlin: Springer, 2010), 29–51.

¹⁷Douglas Stinson, *Cryptography: Theory and Practice*, 4th ed. (Boca Raton, FL: CRC Press, 2018), 60–85.

¹⁸Christopher Manning, Prabhakar Raghavan, and Hinrich Schütze, *Introduction to Information Retrieval* (Cambridge: Cambridge University Press, 2008), 1–18.

and hash functions because it has certain algebraic properties: it is its own inverse, it leaves the distribution of random inputs uniform and it is computationally cheap to execute in hardware.¹⁹

In an authoritative itself treatment of cryptographic theory and practise, Douglas Stinson has shown that the security of current encryption standards, such as the Advanced encryption standard (AES), relies upon the artful transformation of the Boolean functions of substitution, permutation and XOR operations ordered into a sequence of encryptions to provide the so-called properties of confusion and diffusion that Claude Shannon himself identified as needed to create secure cypher constructions.²⁰ It is curious that Shannon, the initial creating force behind the relationship between Boolean algebra and electrical circuits, was the founder of theoretical cryptography, and the Boolean operations that he advocated in both fields are still the bits of cryptographic computation. In a world where electronic security forms the basis of monetary transactions, governmental correspondence, and health records and personal privacy, the Boolean principles of cryptographic systems have become not only a research concern but a subject of significant societal effects..

Boolean Logic and Artificial Intelligence: Foundations and Tensions

The connexion between artificial intelligence and Boolean logic is both substantive and constructive as well as productive tension. The main representational formalism of knowledge and the reasoning during the initial decades of the artificial intelligence research was Boolean propositional logic and first-order propositional logic. All expert systems, theorem provers and rule-based inference engines ran in a broad sense of a Boolean environment, and represented knowledge as a collection of logical propositions based on which conclusions were drawn by the mechanical execution of modus ponens and other rules of inference. In their popular textbook, Stuart Russell and Peter Norvig have described the history of logical treatments of artificial intelligence beginning with the propositional calculus on through first-order predicate logic up to the emergence of knowledge representation languages..²¹

Nevertheless, the shortcomings of entirely Boolean reasoning in artificial intelligence came to the forefront as researchers had to deal with uncertainty and partial information, and continuous variation. The classical Boolean picture recognizes two true values (true and false) only and does not inherently include a way of expressing levels of belief, probabilistic instances, and predicates of vagueness. Russell and Norvig have reported the motivation behind these limitations, which led to the creation of probabilistic logic, Bayesian networks, and statistical learning algorithms filling in or in a few cases replacing Boolean logic..²²

The emergence of machine learning and deep neural networks has only intensified the problem with the posing connexion between Boolean logic and artificial intelligence. Instead of operating on discrete Boolean values, neural networks work on continuous valued activation functions, and instead of the logical rules, neural networks train by increasing or decreasing numerical weights using gradient descent. This is introduced, and the main point of their definitive treatment of deep learning, by Ian Goodfellow, Yoshua Bengio and Aaron Courville, characterises architectures where the crisp binary operations of Boolean logic are replaced by the differentiable operations of calculus..²³ Pedro Domingos has proposed that the search of a single framework that could have integrated the power of logical thinking, statistical learning, and neural calculation is one of the key problems of the modern research on artificial intelligence..²⁴ However, it would be wrong to assume that the developmental changes have rendered the Boolean logic useless. On hardware, each action of a neural network is eventually performed by performing Boolean operations on binary encoding of floating-point numbers. The conditional branching; the evaluation of stopping criteria; the selection of the training examples, form some of the control flow of machine learning pipelines, and is based on Boolean conditions at the architectural level. Theoretically, the Boolean satisfiability problem (SAT) is one of the most researched problems in computational complexity theory, SAT solvers are becoming more and more popular as subroutines in verification, planning and constraint satisfaction systems. The connexion between Boolean logic and artificial intelligence is thus not that of replacement but rather one of manifested complement, whereby the Boolean operations play the role of being the substrate over which more complicated non-Boolean computations can be implemented..

Beyond Bivalence: Fuzzy Logic and Probabilistic Extensions

¹⁹Christopher Manning, Prabhakar Raghavan, and Hinrich Schütze, *Introduction to Information Retrieval* (Cambridge: Cambridge University Press, 2008), 13-16.

²⁰Sergey Brin and Lawrence Page, "The Anatomy of a Large-Scale Hypertextual Web Search Engine," *Computer Networks and ISDN Systems* 30 (1998): 107-17.

²¹Stuart Russell and Peter Norvig, *Artificial Intelligence: A Modern Approach*, 4th ed. (Hoboken, NJ: Pearson, 2021), 226-55.

²²Stuart Russell and Peter Norvig, *Artificial Intelligence: A Modern Approach*, 4th ed. (Hoboken, NJ: Pearson, 2021), 240-48.

²³Ian Goodfellow, Yoshua Bengio, and Aaron Courville, *Deep Learning* (Cambridge, MA: MIT Press, 2016), 164-80.

²⁴Pedro Domingos, *The Master Algorithm: How the Quest for the Ultimate Learning Machine Will Remake Our World* (New York: Basic Books, 2015), 76-98.

A major intellectual innovation that occurred under the constraints of Boolean bivalence is the theory of fuzzy sets and fuzzy logic, which was proposed by Lotfi Zadeh in 1965.²⁵ Zadeh proposed that the classical Boolean membership function, which assigns to each element of a universe of discourse a value of either 0 (non-membership) or 1 (membership), be generalized to a continuous-valued membership function that can assume any real value in the interval [0, 1].²⁶ This generalization permits the formal representation of vague or imprecise concepts—such as “tall,” “warm,” or “approximately five”—that resist crisp Boolean classification.

The theoretical background and practical use of the concept of fuzzy logic has been fully explicated through the works of George Klir and Bo Yuan which show that this idea can be used in control systems, pattern recognition, decision analysis, and natural language processing.²⁷ Consumer electronics, industrial automation and motor engineering Fuzzy logic controllers that operate on continuous-valued truth tests to produce smooth control efforts are used. The philosophical aspect of it is that fuzzy logic does not reject the use of Boolean logic, it is rather a generalisation of it: classical Boolean logic is the special case where all membership values are constrained to the extremes of 0 and 1. The Boolean frame is therefore the example of a more general theory, much as Newtonian mechanics is obtained as the limit case of relativistic mechanics.

The same extension is paralleled by probability logic and Bayesian analysis. The seminal paper by Judea Pearl on probabilistic reasoning in intelligent systems established that the certainties of Boolean deduction can be augmented with the calculus of uncertainty, where propositions get a priority, and inferences are made based on the laws of probability theory.²⁸ The Bayesian networks developed by Pearl are structured to give both the mechanical understandability of directed acyclic graphs and the operationalism of conditional probability distributions to make rigorous decisions under uncertainty, in fields including medical diagnosis and natural language analysis. These probabilistic models do not replace Boolean logic, but instead extend the structural observations of Boolean logic, the nodes of the probabilistic graphical models being Boolean variables.

Algorithmic Ethics and the Social Critique of Boolean Reasoning

The ubiquity of the digital systems, which are dictated by Boolean logic, emerges in contemporary life as a source of the major ethical and social dilemma, which has gained the growing academic interest. In his recent work on the ethics of artificial intelligence, Luciano Floridi has contended that binary determination structures in algorithmic systems such as approve or deny, include or exclude, flag or ignore are products of the Boolean architecture on which those systems are implemented and which can have morally objectionable results when applied to situations of complexity (that are not easily classified in binary terms).²⁹ Whenever an algorithmic credit score system cases down to an approve or deny judgement based on the financial history of an individual, or when a predictive policing algorithm designates a neighbourhood as a high-risk area or a low-risk area, the Boolean manipulation operating on these judgments splits the socially complex world into sharp binary categories.

Cathy O’Neil wrote several articles recordings many instances, where the Boolean and quasi-Boolean decision rules within the algorithmic structures have yielded discriminatory results which disproportionately impacted the marginalised groups.³⁰ Safiya Umoja Noble has applied this criticism to the search engine field, claiming that the Boolean matching logic of information retrieval systems may strengthen and multiply already present social biases where the underlying data are historical patterns of discrimination.³¹ These criticisms do not argue with the essence of Boolean logic as a formal system, which itself, as a formal system, is ethically neutral, but only highlights the implications of using the Boolean decision-making structures in social situations where the dichotomous framework may be too unsophisticated to represent the ethical dimension of the instances it is applied to decide..

Floridi has said that it is important to resolve these ethical issues by considering not the renunciation of Boolean logic, but the enrichment of it with more realistic representations, such as multi-valued logics, probabilistic models, and deliberative processes that add human control over automated decision processes.³² The ethical intervention into algorithmic Boolean thinking therefore highlights a more general philosophical observation, which is that formal systems

²⁵Lotfi Zadeh, “Fuzzy Sets,” *Information and Control* 8, no. 3 (1965): 338–53.

²⁶Lotfi Zadeh, “Fuzzy Sets,” *Information and Control* 8, no. 3 (1965): 340–45.

²⁷George Klir and Bo Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications* (Upper Saddle River, NJ: Prentice Hall, 1995), 1–35.

²⁸Judea Pearl, *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference* (San Mateo, CA: Morgan Kaufmann, 1988), 29–75.

²⁹Luciano Floridi, *The Ethics of Artificial Intelligence: An Introduction* (Oxford: Oxford University Press, 2023), 80–110.

³⁰Cathy O’Neil, *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy* (New York: Crown, 2016), 3–31.

³¹Safiya Umoja Noble, *Algorithms of Oppression: How Search Engines Reinforce Racism* (New York: NYU Press, 2018), 15–40.

³²Luciano Floridi, *The Ethics of Artificial Intelligence: An Introduction* (Oxford: Oxford University Press, 2023), 95–105.

are only given the social relevance they have with respect to their usage, and that formal application of Boolean logic to consequential decision-making is only in a critical awareness of both their advantages and their drawbacks.

Conclusion

The above discussion has shown that Boolean logic is not a fossil of the nineteenth-century mathematical philosophy but it represents an active, and essential, part of the intellectual and material infrastructure of the modern society. The architecture of search engines and databases, the principles behind the design of microprocessors and digital circuits, the principles behind the design of cryptographic security, the logic of the artificial intelligence system, the formal language in which the operations of the digital world are described, analysed, and optimised is all found in Boolean algebra. The binary formalism which George Boole arrived at writing about in 1854 and which was inspired by the philosophical ambition of putting the laws of thought down into an algebraic notation is exactly the mathematical framework that has been proved to be specific to the needs of the electronic and computational technologies that characterise the modern day..³³

Simultaneously, the review of the literature has shown that the applicability of the Boolean logic to the contemporary society is not unconditionally high. The two-sided nature of the classical model of Boolean reasoning, which is quite useful in the design of deterministic digital systems, is severely challenged by the requirements in the uncertain, ambiguous, or morally hard areas. The wifings of Boolean logic into fuzzy logic, probabilistic reasoning, and multi-valued logics are evidence of creative fruitfulness of the original structure by Boole, and are concessive to its limitations. The moral argument against algorithmic decision-making also helps to keep in mind that the application of Boolean frameworks to social life follows some moral implications that are not directly related to the formalities of the logical system.

Conclusively, the application of the Boolean logic in society to date is more apt to be seen as a living and changing thing than just a fact. The algebra of Boole offers the grammar of the digital world, and to realise its potential, and to make its limitations responsible, the further interaction of philosophers, computer scientists, engineers, and ethicists with the project of explaining and designing the logical fabric of the present-day world is necessary..³⁴

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³³Ivor Grattan-Guinness, *The Search for Mathematical Roots, 1870–1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel* (Princeton: Princeton University Press, 2000), 50–55.

³⁴Martin Davis, *The Universal Computer: The Road from Leibniz to Turing* (New York: W. W. Norton, 2000), 155–60.

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