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Minimum Vertex Cover of a New Cluster Graph and its Application in Sensor Network

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Abstract

In this paper we have introduce a new cluster graph G(2n+6, 3n+m) and construct the graph as 4 regular graph. We have find minimum vertex cover of the graph G(2n+6, 3n+m). An algorithm has been developed to find the minimum vertex cover of any graph. In addition to this a theorem also developed. Finally an application of minimum vertex cover has been focused on sensor network.

Keywords: Minimum vertex cover, vertex color, sensor network, wireless network.

1. Introduction

Minimum vertex cover is a process that having the smallest possible number of vertices of a given graph and which covers all the edges. Coloring is a process that color the vertices but no adjacent vertices should be same color. Different authors have been working on Minimum vertex cover and coloring problem from different perception. Jin and Wei [1] have suggested that a graph G is k- closable if it has an L-coloring whenever L is a list assignment such that $|L(v)| \geq k$ for all $v \in V(G)$. For 3-choosability of plane graphs they have provided two sufficient conditions. Bello, Ali and Isah [2] have proposed that a new graph of groups, which is a commuting order product prime graph of finite groups as a graph having the elements of G as its vertices and two vertices are adjacent if and only if they commute and the product of their order is a prime power. They find some results and study some chromatic number.Kok, Kureethara [3] have proposed the notion of perfect Lucky k-colouring. Here Basic conditions for a perfect Lucky k-colourable graph are presented. They also find a theoretical prove thatthe chromatic number of these connected graphs is $\chi(G) = 3$ or 4. For $k = \max\{\chi(G) : 3 \text{ or } 4\} = 4$, it is possible to find Lucky 4-polynomials for all graphs on six vertices and ten edges. Kalita and dutta [4] have developed an algorithm to find the minimum vertex cover of different types of regular planar sub-graphs. They have also provided the application of the same on minimum vertex cover to reduce the power consumption of sensor network. Zhang, Liu and Li [5] have proved that planar graph G with maximum degree $\Delta \geq 12$ that the (2,1)-total labeling number $\lambda 2(G)$ is at most $\Delta + 2$. Ackerman , Keszegh and Vizer [6] have proved that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges. Graphs that admit such drawings are related to quasi-planar graphs and to maximal 1-planar and fan-planar graphs.. Lu and Wang [7] have obtained a sharp result that for any even $n \geq 34$, every { Dn , Dn +1} regular graph of order n contains (n/4) disjoint perfect matchings, where Dn = 2[n/4]-

1. As a consequence, for any integer D≥Dn, every {D, D +1} regular graph of order n contains (D-[n/4]+1) disjoint perfect matching's. Islam, Haloi [8] have proposed a peculiar structure of 4-regular planar graph for G(2m+2,4m+4) where m≥2. Based on the experimental results they proposed a theorem and also prove it. Interestingly they also provide an application of the peculiar structure graph on region based map coloring and GSM network coloring. Islam, Choudhury, Kalita [9] have proposed Various properties to find out the Minimum Vertex Cover of different types of circulant graphs of even and odd values of m≥2 have been studied. An algorithm an application has been developed to avoid deadlock in OS using Minimum Vertex Cover .Complements of line graphs (of graphs) enjoy nice coloring properties that for all graphs. They show that in this class the local and usual chromatic numbers are equal. The topological method is especially suitable for the study of coloring properties of complements of line graphs of hyper graphs have been proposed by Daneshpajouh, Meunier and Mizrahi[10]. Gusev[11] has been proposed the methods of the cooperative game theory. The author studied the vertex cover properties and introduce vertex cover game. Since surveillance cameras are to cover all areas of the network, the characteristic function depends on the vertex covers of the graph. The Shapley–Shubik index is used as the measure of centrality. The Shapley–Shubik index is shown to be efficient in a vertex cover game for the allocation of cameras in a transport network. Proceeding from the Shapley–Shubik indices calculated in this study, recommendations were given for the allocation of surveillance cameras in a specific transport network in a district of the City of Petrozavodsk, Russia. In most of the works, it is seen that almost all the works have been done on regular planar graphs and planar graphs. On 3-regular planar graph and 4-regular planar graphs, very few works are seen to be done. But in the research paper hardly any work is seen on even region and odd region of 4-regular planar graph. But the regions are used in different Application in coloring and biological diversity. Therefore in the present work we propose two new theorems on the odd region of 4 – regular planar graphs. Section 1 includes the introduction which contains the works of other researcher. Section 2 includes the definition. Section 3 contains Theoritical studies and one theorem which are stated and proved. Section 4 includes an algorithm to find out Minimum Vertex Cover and Section 5 includes the Application and Section 6 includes the conclusion.

2. Definition of Vertex Cover:

A vertex cover of an undirected graph is a subset of its vertices such that for every edge (u, v) of the graph, either u or v is in vertex cover and the set covers all the edges of the given graph.

The Minimum Vertex Cover is a vertex cover having the smallest possible number of vertices of a given graph and which covers all the edges. For example the graph Figure-1 has Minimum Vertex cover is 3 having {v_1, v_3, v_5} which covers all the edges.

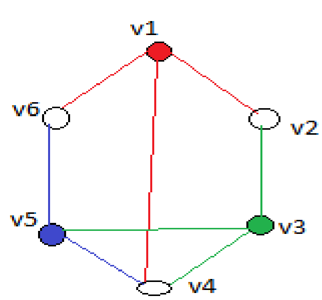


Figure 1 Minimum Vertex Cover is=3 .

3. Our Work

3.1 Theoretical Investigation

For the G(V,E)=(2n+6,3n+m), where V=2n+6 is the set of vertices and E=3n+m is the set of edges and n and m is the arbitrary values.

Case1

In the graph G(V,E)=(2n+6,3n+m), if n=1 and m=10, then the graph contain 8 vertices and 13 edges. In the structure of the graph degree of two vertices should keep 4 and others should keep 3 then the graph have minimum vertex cover is n+4and the minimum vertex cover set is {v_4, v_2,v_8,v_5,v_6 } is shown in Figure-2

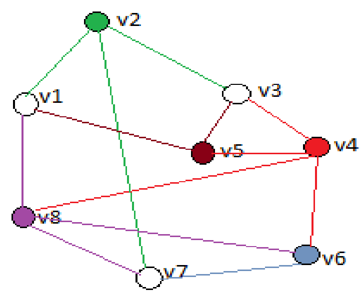


Fig-2- Minimum Vertex Cover=5

Again, in the graph G(V,E)=(2n+6,3n+m), if we keep n value is same but we increment the m value that is if n=1 and m=11 then the graph contain 8 vertices and 14 edges. In the structure of the graph, degree of four vertices should keep 4 and others should keep 3 then also the graph have minimum vertex cover is n+4, and the minimum vertex cover set is remains same i.e {v_4, v_2,v_8,v_5,v_6 } which is shown in the Fig-3

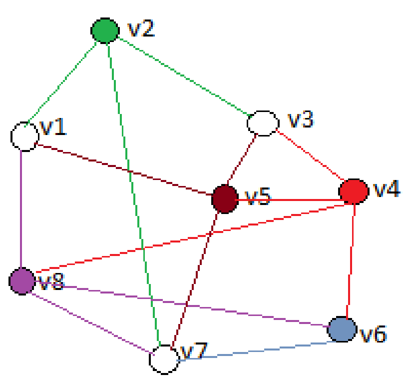


Fig-3- Minimum Vertex Cover=5

Again if n value remains same that is n=1 and m value is incremented by 1 that is m=12 then graph contain 8 vertices and 15 edges. The graph have degree of 6 vertices keep 5 others should keep 3 then the graph have also minimum vertex cover is n+4and the minimum vertex cover set is {v_4, v_2,v_8,v_5,v_6 } which is shown in the Fig-4.

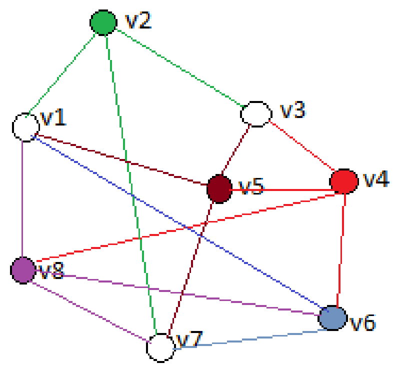


Fig-4- Minimum Vertex Cover=5

After that if we again n value keeps remain same and increase the value of m then the graph structure contains same degree of all the vertices and which is a 4 -Regular graph. Therefore in case 1 we can increase m value up to 12. We consider case 1 as cluster graph-1 as the properties and vertices of the graph are same. The experimental results are given bellow-

Experimental result of case-1

Table-1

Graph G(2n+6,3n+m) where n=1,m=10,11,12						
Value of n	**Value of m**	**Total No. of Edges**	**Total No. of Vertices**	**Set of 4 degree vertices**	**Set of three degree vertices**	**Minimum vertex cover**
1	10	13	8	V_4,V_8	V_1,V_2,V_3,V_5,V_6,V_7,	5
1	11	14	8	V_4,V_5,V_7, $V_{8,}$	V_1,V_2,V_3, V_6	5
1	12	15	8	V_1,V_4, V_5, V_6,V_7, $V_{8,}$	V_2,V_3	5

Case 2

In the graph G(V,E)=(2n+6,3n+m), if n=2 and m=10, then the graph contain 10 vertices and 16 edges. In the structure of the graph degree of two vertices should keep 4 and others should keep 3 then the graph have minimum vertex cover is n+4 that is 6 and the minimum vertex cover set is {v_1,v_4,v_6,v_8,v_9,v_5} which is shown in Fig-5.

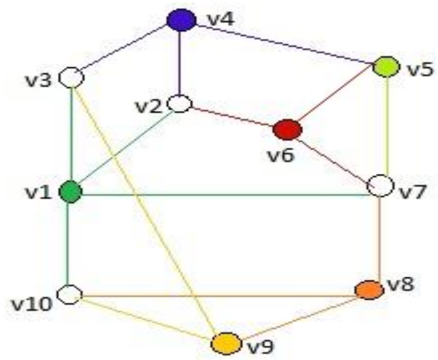


Fig-5- Minimum Vertex Cover=6

Again, in the graph G(V,E)=(2n+6,3n+m), if we keep n value is same but we increment the m value that is if n=2 and m=11 then the graph contain 10 vertices and 17 edges. In the structure of the graph, degree of four vertices should keep 4 and others should keep 3 then also the graph have minimum vertex cover is n+4 which is shown in the Fig-6.

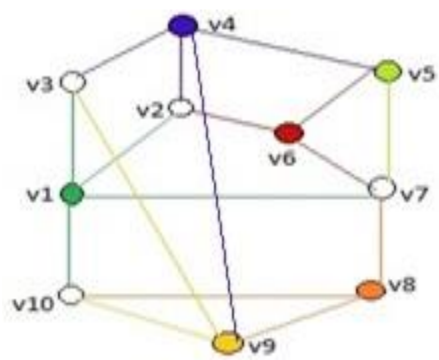


Fig-6- Minimum Vertex Cover=6

Again, in the graph G(V,E)=(2n+6,3n+m), if we keep n value is same but we increment the m value that is if n=2 and m=12 then the graph contain 10 vertices and 18 edges. In the structure of the graph, degree of six vertices should keep 4 and others should keep 3 then also the graph have minimum vertex cover is n+4 which is shown in the Fig-7.

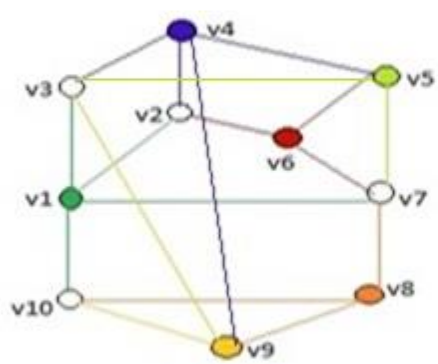


Fig-7- Minimum Vertex Cover=6

In this way if we keep n value 2 and increment m value as 13 and 14. In this structure of the graph, the total edges will be 19 and 20 and new edge will be connected by v_1 and v_8 and v_6 connected with v_{16}then also Minimum Vertex Cover is 6. In this stage the all vertices degree will be 4. In this paper our objective is that the graph has all vertices degree 4. In case 2 when n=2 and m value is 13, 14 then all the degree of the vertices are 4 so the graph is Four regular graph and also there have no parallel edge. Here we consider the graph structure as cluster graph-2 as the properties and vertices are same. The experimental results are given bellow-

Experimental results for Case-2:

Table-2

Graph G(2n+6,3n+m) where n=2,m=10,11,12,13,14						
Value of n	**Value of m**	**Total No. of Edges**	**Total No. of Vertices**	**Set of 4 degree vertices**	**Set of three degree vertices**	**Minimum vertex cover**
2	10	13	10	V_1,V_7	$V_2,V_3,V_4,V_5,V_6,V_8,V_9$, V_{10}	6
2	11	14	10	V_1,V_7V_4,V_9	V_2,V_3,V_5, V_6,V_8 , V_{10}	6
2	12	15	10	$V_1,V_7V_4,V_9,V_3,$ V_5	$V_2,V_{6,}V_8$, V_{10}	6
2	13	16	10	$V_1,V_7V_4,V_9,V_3,$ V_5, V_2,V_{10}	$V_{6,}V_8,$	6
2	14	17	10	$V_1,V_7V_4,V_9,V_3,$ V_5, $V_2,V_{10},$ $V_{6,}V_8,$	NIL	6

Case -3

In the graph G(V,E)=(2n+6,3n+m), if n=3 and m=10, then the graph contain 12 vertices and 19 edges. In the structure of the graph degree of two vertices should keep 4 and others should keep 3 then the graph have minimum vertex cover is n+4 that is 7 and the minimum vertex cover set is $\{v_1,v_4,v_6,v_{12},v_9,v_7,v_{10}\}$ which is shown in Fig-8.

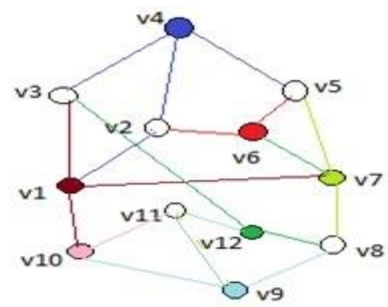


Fig-8- Minimum Vertex Cover=6

In the graph G(V,E)=(2n+6,3n+m), if n=3 and m=11, then the graph also contain 12 vertices and 20 edges. In the structure of the graph degree of 4 vertices should keep 4 and others should keep 3 then the graph have minimum vertex cover is n+4 and the minimum vertex cover set is $\{v_1,v_4,v_6,v_{12},v_9,v_7,v_{10}\}$ which is shown in Fig-9.

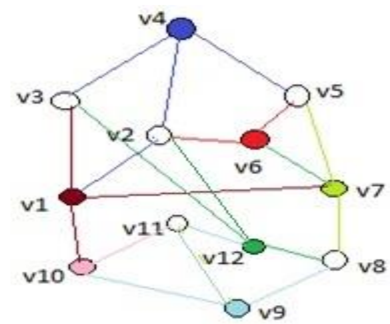


Fig-9- Minimum Vertex Cover=7

In the graph G(V,E)=(2n+6,3n+m), if n=3 and m=12, then the graph also contain 12 vertices and 21 edges. In the structure of the graph degree of 6 vertices should keep 4 and others should keep 3 then the minimum vertex cover of the graph is n+4 and the minimum vertex cover set is $\{v_1,v_4,v_6,v_{12},v_9,v_7,v_{10}\}$ which is shown in Fig-10.

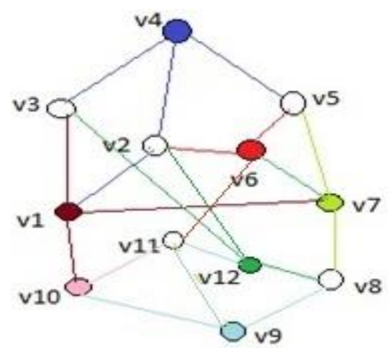


Fig-10- Minimum Vertex Cover=7

In this way if we keep n value is 3 and increment of m value as 13,14, 15 then also the total vertices are 12 and the total edge will be 22,23 and 24,and then new edges will be connected by v_4 tov_8 and v_3 to v_{10} and v_5 to $v_9$then also Minimum Vertex Cover is n+4 that is 7. In this stage the all vertices degree will be 4 so the graph is 4 regular graph. In case 3 when n=3 and m value is 15 then all the degree of the vertices are 4 and also there have no parallel edge and the graph is 4 regular graph. Here we consider the graph structure as cluster graph-3 as the properties and vertices are same. The experimental results are given bellow-

Experimental results for case-3

Table-3 Graph G(2n+6,3n+m) where n=1,m=10,11,12						
Value of n	**Value of m**	**Total No. of Edges**	**Total No. of Vertices**	**Set of 4 degree vertices**	**Set of three degree vertices**	**Minimum vertex cover**
3	10	13	10	V_1,V_7	V_2,V_3,V_4,V_5,V_6,V_8,V_9 , V_{10}	6
3	11	14	10	V_1,$V_7$$V_4$,$V_9$	V_2,V_3,V_5, V_6,V_8 , V_{10}	6
3	12	15	10	V_1,$V_7$$V_4$,$V_9$,$V_3$, V_5	V_2,$V_{6,}$$V_8$, V_{10}	6
3	13	16	10	V_1,$V_7$$V_4$,$V_9$,$V_3$, V_5, V_2,V_{10}	$V_{6,}$$V_8$,	6
3	14	17	10	V_1,$V_7$$V_4$,$V_9$,$V_3$, V_5, V_2,V_{10}, $V_{6,}$$V_8$,	NIL	6

In the above Experimental results (Table-1, Table-2, Table-3)we may increase the value of n, then increase the minimum vertex cover. From the above experimental results we find a theorem.

3.2 Theorem

The minimum vertex cover of the graph G (2n+6, 3n+m) is n+4 when n≥1, m≥10 and case value i.e c ≥1.

Proof: We have proceed to prove that the minimum vertex cover of the graph graph G (2n+6, 3n+m) isn+4 when n≥1, m≥10 and c≥1. The result is true for n=1,m=10 and c=1 fig3(a) which gives the graph G1(8,13) and minimum vertex cover is 5. When we keep n value remains same i.e 1 and increase the value of m ie.11and c=1 then increase the number of edges but the minimum vertex cover is 5. When c=2 and n=2,m=10 then minimum vertex cover is 6 .After increasing of m value the edges are increases then also the minimum vertex cover in the graph is 6 i.e when c=2 and n=2 and m=11.

Now Let the graph G (2k+6, 3k+m) has the minimum vertex cover k+4 when n=k and m>=10. We now show that the graph G (2n+6, 3n+m) has minimum vertex cover k+4+1. Now when we put n=k+1, then the form of the given graph G (2k+6, 3k+m) is G(2(k+1)+4,3(k+1)+m-3)=G(2k+6,3k+m). But our theorem states for the values of n≥1.Hence n=k+1≥1 =>k>0 which is true for the graph G(2k+6,3k+m) for k. Hence the theorem has been found.

4. Algorithm

Input: G (2k+6, 3k+m) Output: To find out the minimum vertex cover of the Graph.

Find out Minimum Vertex Cover

- Start
- While e ∈ E = NULL do
- Select V_i with max(degree(V_i)), ∀ i,j = 1,2,3 .... n and discard the adjacent edges i.e E(V_i,V_j)
- If degree(V_i) = degree(V_j) then
- Select V_i or $V_j$with max(degree(V_i or V_j)) and discard the adjacent edges i.e E(V_i,V_j)
- When select verities then always choose those V_i or V_j which edges are not connected
- with previous selected vertices i.e V_i or V_j
- End if
- End While
- Exit

5. Application

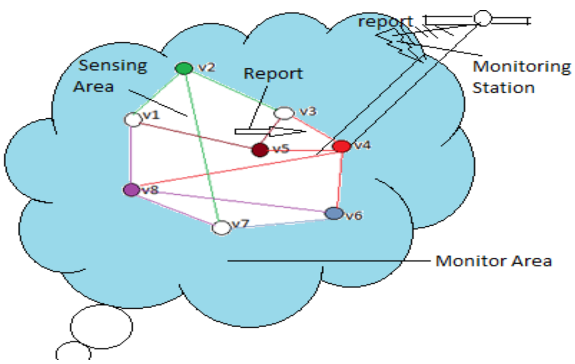
Application of Minimum Vertex Cover in Energy Saving Area Monitoring For Sensor Network or Wireless Sensor Networks

Network life time in Wireless sensor network (WSN) is a key and unsolved NP complete problem. The vertex cover assist in organizing to determine the cluster head to reduce the power loss and ensure that all sensors in the network are accessible by the cluster head. WSN are a rising communication technology that offers a possibilities to improve interaction model with the environment. In data processing and communication capabilities are generally used in Sensors networks. Data are collected by sensors and send the data to a base station by either directly or through another network.WSN supports nodes mobility and sensor have limited capabilities. In military and civilian field WSN offer a wide range of possible application in both the field. A sensor network contains of numerous power nodes. Therefore, such type of networks needs high power. Using Minimum Vertex Cover we can minimize the high power consumption. Maximizing the network life time is an important issue in sensor network due to its scarce resources. So using Minimum Vertex Cover algorithm minimizes the power consumption and extends the network life time. Sensor networks are characterized by dynamic topology, multihop communication, and limited resources.

In a symmetric and asymmetric way and data flow using wireless communications they constitute a sensor network when a number of devices work together. For collection the sensor nodes generally send their data to a specific sink node or monitoring station. In ad hoc networks Sensor networks are a unique case.

Increase the Life Time of Sensor Network by Using Minimum Vertex Cover

The monitored area is the union of all individual node of monitoring areas. Solving the problem to finding the Minimum Vertex Cover that is, the smallest subset of sensor nodes that covers the monitored area. Nodes does not belonging to this set and do not participate in the monitoring area. Here vertices are considered as nodes. The vertex cover set changes periodically both as a function of activity scheduling and to extend the network's monitoring capability. We have developed an algorithmic approach (previously discussed) to reduce the life time of sensor network problem as long as the given sensor nodes cover the area taking, the graphs as discussed above theorems. A Minimum Vertex Cover set is a subset of network nodes. A Minimum Vertex Cover set is connected if any two nodes or vertices in the set can communicate, possibly through other nodes via multihop broadcasting. The broadcasting task is to send a message from one node or one vertex to all network nodes using only nodes in a connected Minimum Vertex Cover set. Moreover, each node makes decisions without communications between nodes beyond the message exchanges that nodes use to discover each other and establish neighborhood information. The local information must observed for a node to decide whether or not it is in a connected Minimum Vertex Cover set otherwise, the increased communication overhead could balance the energy savings. The following figure 11(a), 11(b) and 11(c) - have been considered from the graph G(V,E)=(2n+6,3n+m)which give the minimum vertex cover.



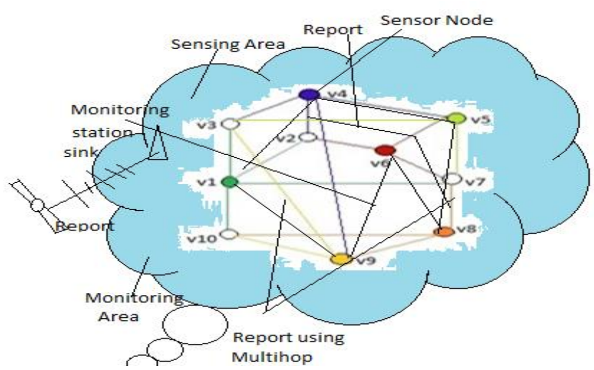


Fig-11(a)-Minimum Vertex Cover=5(Case-1) Fig-11(b)-Minimum Vertex Cover=6(Case-2)

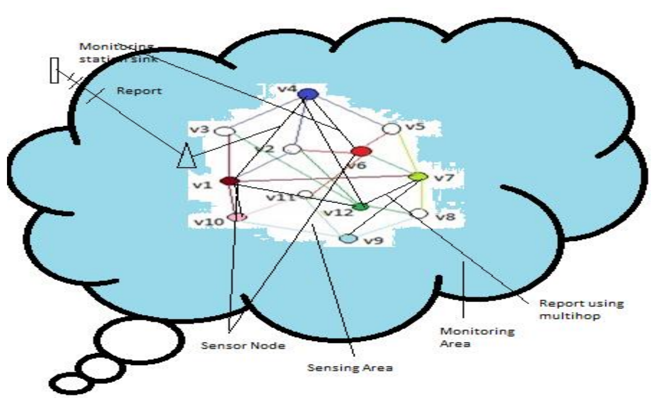


Fig-11(c)-Minimum Vertex Cover=7(Case-3)

In the application we consider one graph from each case. In the graph G(2n+6, 3n+m) consider vertices as a nodes or sensors and edges are consider as connection between two sensors. In case -1, case-2 and case-3 the graph G (2n+6, 3n+m) has total number of vertices is 8, 10, 12 respectively. After implantation of Minimum vertex cover algorithm in case-1, case-2 and case-3 the minimum vertex cover of the graph is 5,6,7 respectively and which covers all the vertices and edges.In a sensor network instead of 8,10,12 sensor if one can use 5,6, 7 sensors respectively and covers the area monitoring network then the network consumes less power which is very much useful in village area where load shading problem of light or power supply problem. From the above experimental results (Table -1, Table-2) we observe that after increment of edges that is connection of the nodes in the graph than also minimum vertex cover is same. Hence one can use the minimum vertex cover algorithm for the special types of networks G(2n+6, 3n+m) for different values of $n \geq 1$, $m \geq 10$ and $c \geq 1$, it will increase the life time of sensor network through less power consumption. So we can say that one can use our Minimum vertex cover algorithm and reduce the power consumption in sensor network.

6. Conclusion

The above theoretical and experimental results are discussed and justified our claims that the graph G (2n+6, 3n+m) has minimum vertex cover is n+4 when and n>=1, m>=10 and c>=1. When we increase the edges of the graph then we find 4 regular graph in case -1 case- and case3.The above discussion is useful for the researchers of different fields like: Minimum vertex cover, construction of new graph and reducing the power consumption of sensor network.

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