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Analytical And Numerical Expression For The Infection Model Of Coffee Rust Disease

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ABSTRACT.

Coffee rust disease is one of the major threats to the economy faced by many countries in which coffee production is their main occupation. The effective way to control the growth of coffee rust spore causing fungi Hemileia vastatrix is to use the bacteria Bacillus thuringiensis which maintains the soil health rather than using copper fungicides which harms humans as well as soil health. In this paper Mathematical Modeling of the Consumer population C , the Resource population R , the size of the population of coffee rust spores Hemileia vastatrix $H_{\{i, j\}}$ and the size of the population of bacteria Bacillus thuringiensis $B_{\{i, j\}}$,where {i,j} denotes the position of the coffee tree in i^{th} row and j^{th} column of a plantation area, are considered. The analytical solution of the system of differential equations has been derived using New Homotopy Perturbation Method (NHPM) & Homotopy Perturbation Method (HPM) and also compared with the numerical simulation satisfactorily. The solution thus obtained gives us a better understanding of the effect of various parameters over the coffee rust disease model.

Keywords: Infection model, Coffee rust, Bacillus thuringiensis, New Homotopy Perturbation Method, Homtopy Perturbation Method.

1. INTRODUCTION

Coffee first originated in the mountains of Ethiopia which were observed by Arneson [1]. C.Gupta [2] described that coffee comes under the genus Coffea and it belongs to the order Rubiales, family Rubiaceae, sub-family Ixoroideae and tribe Coffeeae. The three natural geographic areas where wild coffee exists are Africa, Madagascar and Mascarenes. There are 124 different species of coffee out of which only two species are cultivated commercially and they are C. canephora (Robusta coffee) and C. arabica (Arabica coffee). Another species of coffee C. liberica (Liberica, Liberan or Excelsa coffee) is grown mainly for local consumption and has a very poor quality. Dinesh et al. [3] Arabica and Robusta are the two types of coffee species that are cultivated on a commercial scale in India which constitutes 4.54% of world production. Fernando Haddad et al. [4] describes that Brazil is the largest coffee producing country and has an increasing demand for coffees that are produced organically. Patricia Esquivel et al. [5] have explained that usually coffee drink is prepared from roasted coffee beans and by mixing hot water.

Coffee plants are affected by several diseases. Here coffee rust disease caused by the fungus Hemileia vastatrix on Coffea arabica is considered. Maria do Céu Silva et al. [6] have said that other than coffee rust disease, coffee plants are also affected by diseases like the coffee wilt disease, the brown eye spot or berry blotch, the American leaf spot of coffee and the halo blight of coffee. The list of coffee diseases that the coffee plant faces are coffee leaf rust caused rust which is caused by the fungus Hemileia vastatrix and the symptoms include yellow-orange powdery spots on the underside of leaves. The coffee berry disease is caused by the fungus Colletotrichum kahawae which is characterized by the symptoms like leaves dark brown-black, slightly sunken lesions on green berries. The coffee wilt disease is caused by the fungus Gibberella xylarioides and the symptoms are leaves become yellow, dry, fall from tree and berries ripen prematurely. The coffee berry borer disease is caused by the insect Hypothenemus hampei and they cause small, round holes of 1 mm diameter near the tip of the berry. The white stem borer disease is caused by the insect Monochamus leuconotus which causes holes of about 1cm diameter in the stem of the tree. The leaf miner disease is due to the insect Leucoptera species and the symptoms are irregular brown spots or patches on the upper surface of the leaves. The brown eye spot disease is because of the fungus Cercospora coffeicola which causes small pale yellow-white circular or angular spots to develop between leaf veins and on the upper leaf surface and these spots enlarge to become reddish-brown with greyish centre [7].

Arneson [1] demonstrates that Coffee rust is the most economically important coffee disease in the world where coffee is the most important agricultural product in international trade. The coffee leaf gets infected through the stomata of the leaf. After infection, it takes 10 to 14 days for the new uridinia to develop and the lesions enlarge for a period of 2 to 3 weeks. This disease was first reported by an English explorer on wild Coffea species in the Lake Victoria region of East Africa in 1861. In 1869, H. J. Berkeley and his assistant, Mr. Broome, named the devastating fungus as Hemileia vastatrix. Laercio Zambelin [8] explains that the urediniospores of H.vastatrix are the main reason for coffee rust which land, germinate and form a spore on the adaxial side of the leaves. Pedro Talhinhas et al. [9] have given the information that the germination of urediniospores is possible at 24^0C and it requires water.

Copper containing fungicides are effective in controlling the coffee rust. But there are two disadvantages in using these copper fungicides. The first disadvantage is they must be applied on leaves before the infection occurs which is a difficult task. The second disadvantage is they are toxic to the other organisms in the environment [1]. Raoul A. Muller et al. have studied that a long time ago Bordeaux Mixture was commonly used which contains copper sulphate compounds and this acts as an effective anti-sporulant but still this pollutes the richness of the soil [10]. Oscar Luaces et al. [11] conveyed that in earlier days chemical fungicides are used which contaminates the environment and causes a lack of taste in coffee. Laercio Zambelin [8] have investigated that several researchers suggested alternative control methods like biological control of coffee rust by making use of bacteria like Bacillus thuringiensis and Bacillus subtilis. Here the coffee rust disease model is a system of non-linear differential equations as proposed by Jorge Arroyo-Esquivel et al. [12] which consists of the consumer population C, the resource population R, the size of the population of coffee rust spores Hemileia vastatrix $H_{\{i,j\}}$ and the size of the population of bacteria Bacillus thuringiensis $B_{\{i,j\}}$, where {i,j} denotes the position of the coffee tree in i^{th} row and j^{th} column of a plantation area. The analytical solution of this system is obtained by using New Homotopy Perturbation Method (NHPM) and Homotopy Perturbation Method (HPM). Maple and Matlab software are used to compare the analytical and numerical values of the system of differential equations effectively.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

A non-linear differential equation model that describes the spread of coffee rust through a coffee plantation and its interaction with the bacteria B. thuringiensis is introduced by Jorge Arroyo-Esquivel et al. [12]. Let C be the consumer population with carrying capacity K, which consumes the resource population with an effective rate a and has a mortality rate m. Let R be the resource population, which grows at a rate r and is consumed by the consumer population with a conversion rate c and $H_{\{i,j\}}$ and $B_{\{i,j\}}$ be the size of the populations of coffee rust spores H. vastatrix and the bacteria B. thuringiensis B157 on tree {i,j},respectively. Consider a coffee plantation of n×m coffee trees, distributed in a rectangular array, where the tree in the i^{th} row and j^{th} column occupies the position{i,j}. Let $H_{\{i,j\}}$ and $B_{\{i,j\}}$ be the size of the populations of coffee rust spores H. vastatrix and the bacteria B. thuringiensis B157 on tree {i,j},respectively.

For each $i \in \{1,2,\ldots n\}, j \in \{1,2,\ldots m\}$ both $H_{\{i,j\}}$ and $B_{\{i,j\}}$ are assumed to grow logistically, where b and h are the growth rates and K_B and K_H are the carrying capacities of the bacteria and coffee rust, respectively. Given that coffee rust appears primarily during the rainy season, we will consider the carrying capacities constant. $B_{\{i,j\}}$ has a natural death rate d and the bacteria limits the growth of coffee rust with a constant conversion rate γ. The principal method of dispersal is through rainfall, which arrives at the neighbor trees at rate α and is removed from their original tree by rainfall at rate β. We suppose that the bacteria are being irrigated into the plantation at rate μ{i, j}. The $I_{\{i,j\}}$ function is defined as a random variable dependent on the coffee tree's neighbors, which correspond to the trees in the cardinal and intermediate directions of the {i, j}th tree.

The model is then described by the system of non-linear differential equations:

(2.1) $$\frac{dC}{dt}=a\left(\frac{K-C}{K}\right)CR-mC$$

(2.2) $$\frac{dR}{dt}=rR(1-R)-a\left(\frac{K-C}{K}\right)CR$$

(2.3) $$\frac{dB_{\{i,j\}}}{dt}=bB_{\{i,j\}}\left(1-\frac{B_{\{i,j\}}}{K_B}\right)+\mu_{\{i,j\}}(t)-dB_{\{i,j\}}$$

(2.4) $$\frac{dH_{\{i,j\}}}{dt}=hH_{\{i,j\}}\left(1-\frac{H_{\{i,j\}}}{K_H}\right)+\alpha I_{\{i,j\}}-\left(\gamma B_{\{i,j\}}+\beta\right)H_{\{i,j\}}$$

The initial conditions at time $t=0$ are $C=C_0$, $R=R_0$, $B_{\{i,j\}}=B_0$ and $H_{\{i,j\}}=H_0$

The description of the parameters are represented in Table 1

TABLE 1. Nomenclature

Symbol	Meaning
C	Consumer population
R	Resource population
$B_{\{i,j\}}$	Size of population of Bacillus thuringiensis
$H_{\{i,j\}}$	Size of population of Hemileia vastatrix
K	Carrying capacity
a	Effective rate
m	Mortality rate
r	Growth rate
b	Natural growth rate of Bacteria
K_B	Carrying capacity of Bacteria
$\mu_{\{i,j\}}$	Rate of irrigation of Bacteria into plantation
d	Natural death rate of Bacteria
h	Natural growth rate of rust
K_H	Carrying capacity of coffee rust
$I_{\{i,j\}}$	Number of spores
α	Immigration rate of coffee rust
β	Emigration rate of coffee rust
γ	Conversion rate of Bacteria to coffee rust

3.SOLUTION OF THE MODEL USING NEW HOMOTOPY PERTURBATION AND HOMOTOPY PERTURBATION METHOD

3.1 SOLUTION USING NEW HOMOTOPY PERTURBATION METHOD

In various fields of sciences and engineering linear and non-linear differential
equations gain much more importance. Both Homotopy Perturbation Method and New Homotopy Perturbation Method uses the imbedding parameter up as a small parameter, and only a few iterations are needed to search for an asymptotic solution [13].

To find the solution of equation (2.1), (2.2), (2.3), (2.4) we construct the homotopy as follows:

(3.1) $$(1-p)\left[\frac{dC}{dt}+C[-a.R(t=0)+\frac{a}{K}.C.R(t=0)+m]\right]+p\left[\frac{dC}{dt}+C(-a.R+\frac{a}{K}.C.R+m)\right]=0$$

(3.2) $$(1-p)\left[\frac{dR}{dt}+R[(r.R-r+a.C-\frac{a}{K}C^2)](t=0)\right]+p\left[\frac{dR}{dt}+R(r.R-r+a.C-\frac{a}{K}C^2)\right]=0$$

(3.3) $$(1-p)\left[\frac{dB_{\{i,j\}}}{dt}+B_{\{i,j\}}(-b+\frac{bB_{\{i,j\}}}{K_B}+d)(t=0)-\mu_{\{i,j\}}(t)(t=0)\right]+p\left[\frac{dB_{\{i,j\}}}{dt}+B_{\{i,j\}}(-b+\frac{bB_{\{i,j\}}}{K_B}+d)-\mu_{\{i,j\}}(t)\right]=0$$

(3.4) $$(1-p)\left[\frac{dH_{\{i,j\}}}{dt}+H_{\{i,j\}}(-h+\frac{hH_{\{i,j\}}}{K_H}+\gamma B_{\{i,j\}}+\beta)(t=0)-\alpha I_{\{i,j\}}\right]+p\left[\frac{dH_{\{i,j\}}}{dt}+H_{\{i,j\}}(-h+\frac{hH_{\{i,j\}}}{K_H}+\gamma B_{\{i,j\}}+\beta)-\alpha I_{\{i,j\}}\right]=0$$

The solution of equations (2.1), (2.2), (2.3) and (2.4) are written as a power series as follows:

(3.5) $$C=C_0+pC_1+p^2C_2+\dots$$

(3.6) $$R=R_0+pR_1+p^2R_2+\dots$$

(3.7) $$B_{\{i,j\}}=B_0+pB_1+p^2B_2+\dots$$

(3.8) $$H_{\{i,j\}}=H_0+pH_1+p^2H_2+\dots$$

Substituting the equations (3.5), (3.6), (3.7), (3.8) in (3.1), (3.2), (3.3) and (3.4) respectively

(3.9) $$(1-p)\left[\frac{d(C_0+pC_1+p^2C_2+\dots)}{dt}+(C_0+pC_1+p^2C_2+\dots)[-a.R(t=0)+\frac{a}{K}.C.R(t=0)+m]\right]$$
$$+p\left[\begin{array}{l}\frac{d(C_0+pC_1+p^2C_2+\dots)}{dt}+[(C_0+pC_1+p^2C_2+\dots)(-a.(R_0+pR_1+p^2R_2+\dots)\\ +\frac{a}{K}.(C_0+pC_1+p^2C_2+\dots).(R_0+pR_1+p^2R_2+\dots)+m])\end{array}\right]=0$$

(3.10) $$(1-p)\left[\frac{d(R_0+pR_1+p^2R_2+\dots)}{dt}+(R_0+pR_1+p^2R_2+\dots)(r.R-r+a.C-\frac{a}{K}C^2)(t=0)\right]$$
$$+p\left[\begin{array}{l}\frac{d(R_0+pR_1+p^2R_2+\dots)}{dt}+[(R_0+pR_1+p^2R_2+\dots)(r.(R_0+pR_1+p^2R_2+\dots)-r\\ +a.(C_0+pC_1+p^2C_2+\dots)-\frac{a}{K}(C_0+pC_1+p^2C_2+\dots)^2)]\end{array}\right]=0$$

(3.11) $$(1-p)\left[\frac{d(B_0+pB_1+p^2B_2+\ldots)}{dt}+(B_0+pB_1+p^2B_2+\ldots)(-b+\frac{bB_{\{i,j\}}}{K_B}+d)(t=0)-\mu_{\{i,j\}}(t)(t=0)\right]$$

$$+p\left[\frac{d(B_0+pB_1+p^2B_2+\ldots)}{dt}+(B_0+pB_1+p^2B_2+\ldots)(-b+\frac{b(B_0+pB_1+p^2B_2+\ldots)}{K_B}+d)-\mu_{\{i,j\}}(t)\right]=0$$

(3.12) $$(1-p)\left[\frac{d(H_0+pH_1+p^2H_2+\ldots)}{dt}+(H_0+pH_1+p^2H_2+\ldots)(-h+\frac{hH_{\{i,j\}}}{K_H}+\gamma B_{\{i,j\}}+\beta)(t=0)-\alpha I_{\{i,j\}}\right]$$

$$+p\left[\begin{array}{l}\frac{d(H_0+pH_1+p^2H_2+\ldots)}{dt}+(H_0+pH_1+p^2H_2+\ldots)(-h+\frac{h(H_0+pH_1+p^2H_2+\ldots)}{K_H}\\ +\gamma(B_0+pB_1+p^2B_2+\ldots)+\beta)-\alpha I_{\{i,j\}}\end{array}\right]=0$$

Comparing the coefficients of p^0 of equations (3.9), (3.10), (3.11) and (3.12) we get

(3.13) $$p^0 : \frac{dC_0}{dt}+C_0(-a.R_0+\frac{a}{K}.C_0.R_0+m)=0$$

(3.14) $$p^0 : \frac{dR_0}{dt}+R_0(r.R_0-r+a.C_0-\frac{a}{K}C_0^2)=0$$

(3.15) $$p^0 : \frac{dB_0}{dt}+B_0(-b+\frac{bB_0}{K_B}+d)-\mu_{\{i,j\}}(t)=0$$

(3.16) $$p^0 : \frac{dH_0}{dt}+H_0(-h+\frac{hH_0}{K_H}+\gamma B_0+\beta)-\alpha I_{\{i,j\}}=0$$

Using the initial condition, the solution of the equations (2.1), (2.2), (2.3) and (2.4) is given as follows:

(3.17) $$C=C_0\,e^{-(aR_0+\frac{a}{K}C_0R_0+m)t}$$

(3.18) $$R=R_0\,e^{-(rR_0-r+aC_0-\frac{a}{K}C_0^2)t}$$

(3.19) $$B_{\{i,j\}}=\frac{\mu_{\{i,j\}}(t)}{-b+\frac{bB_0}{K_B}+d}+\left(B_0-\frac{\mu_{\{i,j\}}(t)}{-b+\frac{bB_0}{K_B}+d}\right)e^{-(-b+\frac{B_0}{K_B}+d)t}$$

(3.20) $$H_{\{i,j\}}=\frac{\alpha I_{\{i,j\}}}{-h+\frac{hH_0}{K_H}+\gamma B_0+\beta}+\left(H_0-\frac{\alpha I_{\{i,j\}}}{-h+\frac{hH_0}{K_H}+\gamma B_0+\beta}\right)e^{-(-h+\frac{hH_0}{K_H}+\gamma B_0+\beta)t}$$

3.2 SOLUTION USING HOMOTOPY PERTURBATION METHOD

Homotopy is an important part of differential topology [14]. Homotopy Perturbation Method is applied to find the travelling wave solutions of non-linear wave equations. This method was first proposed by J.Huan He in 1998. Ganji et al. have applied this perturbation technique successfully in different linear, non-linear, homogeneous, non-homogeneous, coupled, de-coupled and different problems related to physics [15].

To find the solution of equation (2.1), (2.2), (2.3), (2.4) we construct the homotopy as follows:

(3.21) $$(1-p)\left[\frac{dC}{dt}+mC\right]+p\left[\frac{dC}{dt}+C(-a.R+\frac{a}{K}.C.R+m)\right]=0$$

(3.22) $$(1-p)\left[\frac{dR}{dt}-rR\right]+p\left[\frac{dR}{dt}+R(r.R-r+a.C-\frac{a}{K}C^2)\right]=0$$

(3.23) $$(1-p)\left[\frac{dB_{\{i,j\}}}{dt}-bB_{\{i,j\}}+d\,B_{\{i,j\}})\right]+p\left[\frac{dB_{\{i,j\}}}{dt}+B_{\{i,j\}}(-b+\frac{bB_{\{i,j\}}}{K_B}+d)-\mu_{\{i,j\}}(t)\right]=0$$

(3.24) $$(1-p)\left[\frac{dH_{\{i,j\}}}{dt}-hH_{\{i,j\}}+\beta H_{\{i,j\}}\right]+p\left[\frac{dH_{\{i,j\}}}{dt}+H_{\{i,j\}}(-h+\frac{hH_{\{i,j\}}}{K_H}+\gamma B_{\{i,j\}}+\beta)-\alpha I_{\{i,j\}}\right]=0$$

The solution of equations (2.1), (2.2), (2.3) and (2.4) are written as a power series as follows:

(3.25) $$C=C_0+pC_1+p^2C_2+\ldots$$

(3.26) $$R=R_0+pR_1+p^2R_2+\ldots$$

(3.27) $$B_{\{i,j\}}=B_0+pB_1+p^2B_2+\ldots$$

(3.28) $$H_{\{i,j\}}=H_0+pH_1+p^2H_2+\ldots$$

Substituting the equations (3.25), (3.26), (3.27), (3.28) in (3.21), (3.22), (3.23) and (3.24) respectively

(3.29) $$(1-p)\left[\frac{d(C_0+pC_1+p^2C_2+\ldots)}{dt}+m(C_0+pC_1+p^2C_2+\ldots)\right]$$

$$+p\left[\begin{array}{l}\frac{d(C_0+pC_1+p^2C_2+\ldots)}{dt}+[(C_0+pC_1+p^2C_2+\ldots)(-a.(R_0+pR_1+p^2R_2+\ldots)\\ +\frac{a}{K}.(C_0+pC_1+p^2C_2+\ldots).(R_0+pR_1+p^2R_2+\ldots)+m])\end{array}\right]=0$$

(3.30) $$(1-p)\left[\frac{d(R_0+pR_1+p^2R_2+\ldots)}{dt}-r(R_0+pR_1+p^2R_2+\ldots)\right]$$

$$+p\left[\begin{array}{l}\frac{d(R_0+pR_1+p^2R_2+\ldots)}{dt}+[(R_0+pR_1+p^2R_2+\ldots)(r.\left(R_0+pR_1+p^2R_2+\ldots\right)-r\\ +a.(C_0+pC_1+p^2C_2+\ldots)-\frac{a}{K}(C_0+pC_1+p^2C_2+\ldots)^2])\end{array}\right]=0$$

(3.31) $$(1-p)\left[\frac{d(B_0+pB_1+p^2B_2+\ldots)}{dt}+(B_0+pB_1+p^2B_2+\ldots)(-b+d)\right]$$

$$+p\left[\frac{d(B_0+pB_1+p^2B_2+\ldots)}{dt}+(B_0+pB_1+p^2B_2+\ldots)(-b+\frac{b(B_0+pB_1+p^2B_2+\ldots)}{K_B}+d)-\mu_{\{i,j\}}(t)\right]=0$$

(3.32)
$$(1-p)\left[\frac{d(H_0+pH_1+p^2H_2+\ldots)}{dt}+(H_0+pH_1+p^2H_2+\ldots)(-h+\beta)\right]$$
$$+p\left[\begin{array}{l}\frac{d(H_0+pH_1+p^2H_2+\ldots)}{dt}+(H_0+pH_1+p^2H_2+\ldots)(-h+\frac{h(H_0+pH_1+p^2H_2+\ldots)}{K_H}\\+\gamma(B_0+pB_1+p^2B_2+\ldots)+\beta)-\alpha I_{\{i,j\}}\end{array}\right]=0$$

Comparing the coefficients of p^0, p^1 of equations (3.29), (3.30), (3.31) and (3.32) we get

(3.33) $$p^0 : \frac{dC_0}{dt}+mC_0=0$$ (3.34)

$$p^0 : \frac{dR_0}{dt}-rR_0=0$$

(3.35) $$p^0 : \frac{dB_0}{dt}+B_0(-b+d)=0$$

(3.36) $$p^0 : \frac{dH_0}{dt}+H_0(-h+\beta)=0$$

(3.37) $$p^1 : \frac{dC_1}{dt}+mC_1=C_ie^{-mt}R_ie^{rt}(a-\frac{a}{K}C_ie^{-mt})$$

(3.38) $$p^1 : \frac{dR_1}{dt}-rR_1=-r(R_ie^{rt})^2-\frac{a}{K}(C_ie^{-mt})^2(R_ie^{rt})$$

(3.39) $$p^1 : \frac{dB_1}{dt}+B_1(-b+d)=\mu_{\{i,j\}}(t)-\frac{b}{K_B}(B_ie^{(b-d)})^2$$

(3.40) $$p^1 : \frac{dH_1}{dt}+H_1(-h+\beta)=\alpha I_{\{i,j\}}-\frac{h}{K_H}H_i{}^2e^{2(h-\beta)t}-\gamma B_ie^{(b-d)t}H_ie^{(h-\beta)t}$$

The solution of the equations (2.1), (2.2), (2.3) and (2.4) is given as follows:

(3.41) $$C=C_ie^{-mt}+aC_iR_i\left[\frac{e^{r-mt}}{r}+\frac{C_i\,e^{(r-2m)t}}{(r-m)}\right]+\left[-aC_iR_i\left(\frac{1}{r}+\frac{C_i}{k(r-m)}\right)\right]e^{-mt}$$

(3.42) $$R=R_ie^{rt}+\left[-R_i{}^2e^{2rt}+\frac{a}{2mk}C_i{}^2R_ie^{r-2mt}\right]+\left[R_i{}^2-\frac{a}{2mk}C_i{}^2R_i\right]e^{rt}$$

(3.43) $$B_{\{i,j\}}=B_ie^{(b-d)t}+\left[\frac{\mu_{\{i,j\}}(t)}{(d-b)}-\frac{bB_i{}^2e^{2(b-d)t}}{K_B(b-d)}\right]+\left[\frac{bB_i{}^2}{K_B(b-d)}-\frac{\mu_{\{i,j\}}(t)}{(d-b)}\right]e^{-(d-b)t}$$

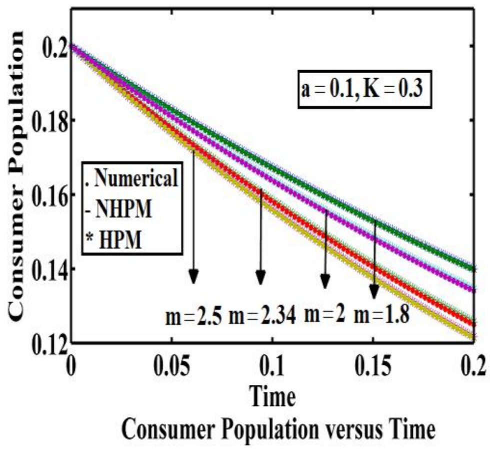
(3.44) $$H_{\{i,j\}}=H_ie^{(h-\beta)t}+\left[\frac{\alpha I_{\{i,j\}}}{\beta-h}-\frac{hH_i{}^2e^{2(h-\beta)t}}{K_H(h-\beta)}-\frac{\gamma B_iH_i}{(b-d)}e^{(b-d+h-\beta)t}\right]+\left[\frac{\gamma B_iH_i}{(b-d)}+\frac{hH_i{}^2}{K_H(h-\beta)}-\frac{\alpha I_{\{i,j\}}}{\beta-h}\right]e^{-(\beta-h)t}$$

4.RESULTS AND DISCUSSIONS

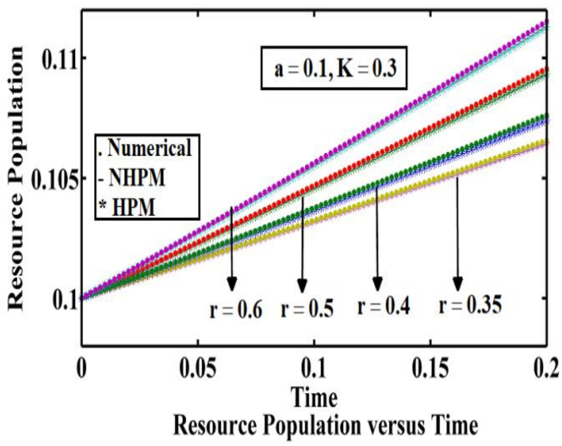
The analytical solution of the consumer population (2.1), resource population (2.2), the size of population of Bacillus thuringiensis (2.3) and the size of population of coffee rust spores Hemileia vastatrix (2.4) are computed using New Homotopy Perturbation Method and

Homotopy Perturbation Method which are given in equations (3.17) – (3.20) & (3.41) – (3.44) and is compared with its numerical simulation.

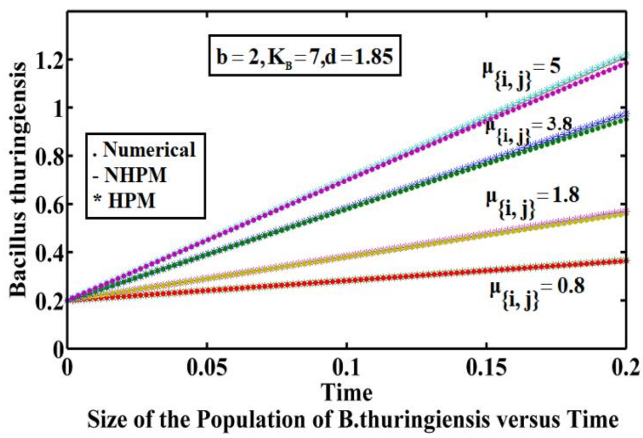
From Figure (1a) notice that as raise the value of the parameter m (i.e., mortality rate), the consumer population (i.e., bacteria population) tends down for the fixed values of a and K. From Figure (1b) observe that when boosting up the value of growth rust of coffee rust r little slower, infer that the rate of change of resource population (i.e., coffee rust population) extends up with respect to time for other values of the parameter (effective rate a and carrying capacity K). From Figure (1c) expanding the values of the parameter rate of irrigation of bacteria into the plantation $\mu_{\{i,j\}}$, the size of the population of the bacteria Bacillus thuringiensis B157 grows rapidly for different values of b, K_B and d.



(a)



(b)



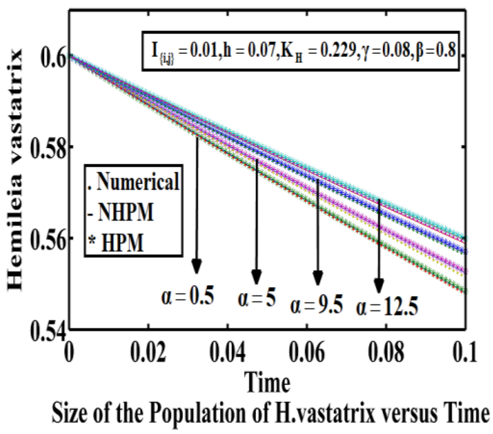
(c)

FIGURE 1: Graphs representing the comparison of the Consumer population C, the Resource population R and the size of the population of bacteria Bacillus thuringiensis $B_{\{i, j\}}$ of the equations (2.1,3.17,3.41),(2.2,3.18,3.42) and (2.3,3.19,3.43) for the various values of (a) Mortality rate m, (b)

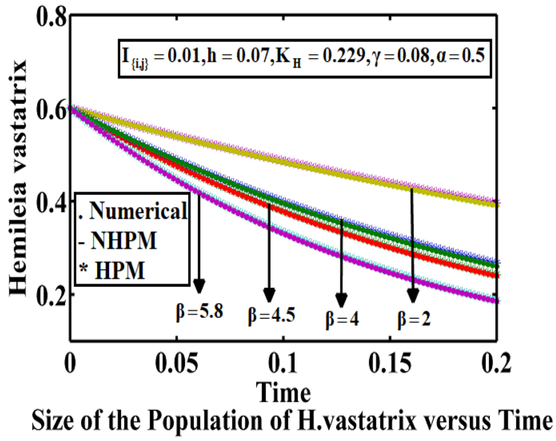
Growth rate r and (c) Rate of irrigation of bacteria into the plantation $\mu_{\{i,j\}}$.The solid line (---) and the star (***) represents the analytical solution using New Homotopy Perturbation and Homotopy Perturbation Method. The dotted line (….) represents the numerical simulation.

From Figure (2d) note that as the immigration rate of coffee rust α swells up, the size of the population of H.vastatrix reduces with an increase in time for the

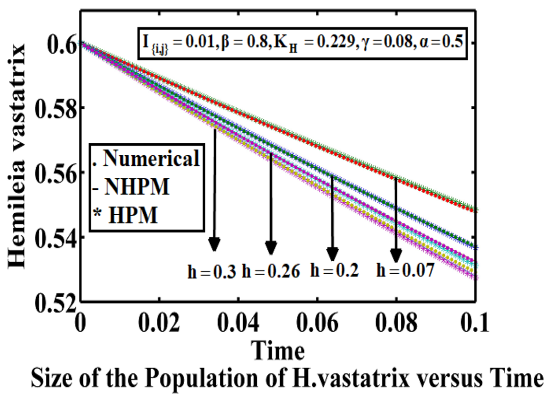
values of parameters $I_{\{i,j\}}, \beta, \gamma, K_H$ and h. From Figure (2e) for greater values of emigration rate of coffee rust β, the size of the population of H.vastatrix declines down rapidly leaving other parameters $I_{\{i,j\}}, \alpha, \gamma, K_H$ and h. From figure (2f) notice that when the natural growth rate of rust h gradually rises, the size of H.vastatrix decreases slowly for various values of $I_{\{i,j\}}, \alpha, \beta, \gamma$ and K_H. From Figure (2g) states that increasing the value of the parameter $I_{\{i,j\}}$ number of spores in coffee leaf, the rate of change in the size of population of Hemileia vastatrix multiplies up tremendously with respect to time and for other values of the parameters $\alpha, \beta, \gamma, K_H$ and h. From Figure (2h) as the carrying capacity of coffee rust K_H mounts up, the population of Hemileia vastatrix decreases slowly for different values of $I_{\{i,j\},}, \alpha, \beta, \gamma$ and h.



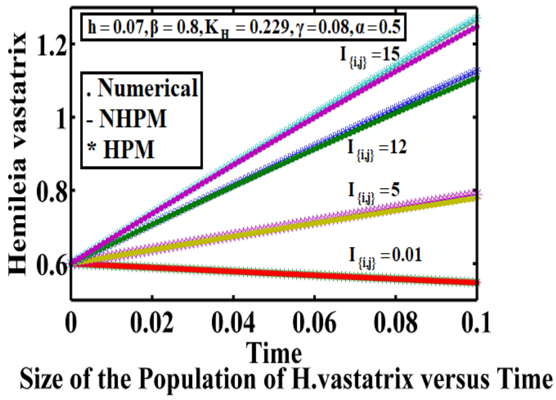
(d)



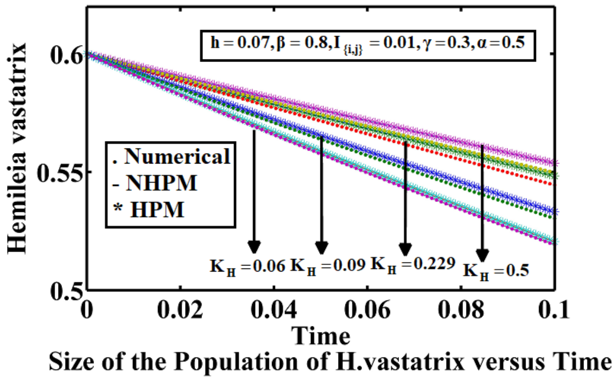
(e)



(f)



(g)



(h)

FIGURE 2: Graphs representing the comparison of the size of the population of coffee rust spores Hemileia vastatrix $H_{\{i,j\}}$of the equations (2.4,3.20,3.44) for the various values of (d) Immigration rate of coffee rust α ,

(e) Emigration rate of coffee rust β, (f)Natural growth rate of rust h ,(g) Number of spores $I_{\{i,j\}}$ and

(h) Carrying capacity of coffee rust K_H .The solid line (---) and the star (***) represents the analytical solution using New Homotopy Perturbation and Homotopy Perturbation Method. The dotted line (….) represents the numerical simulation.

The tables (2) - (9) represents the comparison between the analytic solution of New Homotopy Perturbation Method and Homotopy Perturbation Method with numerical simulation. From all these tables we observe that New Homotopy Perturbation Method converges to its solution faster than the Homotopy Perturbation Method and the maximum average error is 1.18% for all the parameters.

TABLE 2: COMPARISON OF CONSUMER POPULATION C WITH NUMERICAL RESULT FOR VARIOUS VALUES OF m AND FIXED VALUES OF $a=0.1$ AND $K=0.3$

t	$m=1.8$					$m=2$					$m=2.34$					$m=2.5$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00
0.04	0.1860	0.1861	0.05	0.1862	0.11	0.1845	0.1846	0.05	0.1848	0.16	0.1820	0.1822	0.11	0.1823	0.16	0.1808	0.1810	0.11	0.1812	0.22
0.08	0.1730	0.1732	0.12	0.1734	0.23	0.1702	0.1705	0.18	0.1707	0.29	0.1656	0.1659	0.18	0.1662	0.36	0.1635	0.1638	0.18	0.1641	0.37
0.12	0.1614	0.1612	0.12	0.1615	0.06	0.1570	0.1574	0.25	0.1578	0.51	0.1507	0.1511	0.27	0.1515	0.53	0.1478	0.1482	0.27	0.1487	0.61
0.16	0.1501	0.1500	0.06	0.1505	0.27	0.1454	0.1453	0.07	0.1458	0.28	0.1377	0.1376	0.07	0.1382	0.36	0.1343	0.1341	0.15	0.1347	0.30
0.2	0.1397	0.1396	0.07	0.1401	0.29	0.1340	0.1342	0.15	0.1347	0.52	0.1253	0.1253	0.00	0.1260	0.56	0.1217	0.1214	0.25	0.1221	0.33
	Average Error		0.07		0.16	Average Error		0.12		0.29	Average Error		0.11		0.33	Average Error		0.16		0.31

TABLE 3: COMPARISON OF RESOURCE POPULATION R WITH NUMERICAL RESULT FOR VARIOUS VALUES OF r AND FIXED VALUES OF $a=0.1$ AND $k=0.3$

t	$r=0.35$					$r=0.4$					$r=0.5$					$r=0.6$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.1000	0.1000	0.00	0.1000	0.00	0.1000	0.1000	0.00	0.1000	0.00	0.1000	0.1000	0.00	0.1000	0.00	0.1000	0.1000	0.00	0.1000	0.00
0.04	0.1013	0.1013	0.00	0.1013	0.00	0.1015	0.1014	0.10	0.1014	0.10	0.1019	0.1018	0.10	0.1018	0.10	0.1022	0.1022	0.00	0.1022	0.00
0.08	0.1025	0.1025	0.00	0.1025	0.00	0.1029	0.1029	0.00	0.1029	0.00	0.1037	0.1036	0.10	0.1036	0.10	0.1044	0.1044	0.00	0.1044	0.00
0.12	0.1038	0.1038	0.00	0.1038	0.00	0.1044	0.1044	0.00	0.1044	0.00	0.1056	0.1055	0.10	0.1055	0.10	0.1067	0.1066	0.09	0.1067	0.00
0.16	0.1052	0.1051	0.10	0.1051	0.10	0.1060	0.1059	0.09	0.1059	0.09	0.1075	0.1074	0.09	0.1074	0.09	0.1092	0.1090	0.18	0.1090	0.18
0.2	0.1066	0.1064	0.19	0.1065	0.09	0.1076	0.1074	0.19	0.1074	0.19	0.1096	0.1093	0.27	0.1093	0.27	0.1115	0.1113	0.18	0.1113	0.18

	Average Error		0.05		0.03	Average Error		0.06		0.06	Average Error		0.11		0.11	Average Error		0.08		0.06

TABLE 4: COMPARISON OF THE SIZE OF THE POPULATION OF BACILLUS THURINGIENSIS $B_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF $\mu_{\{i,j\}}$

AND FIXED VALUES OF $b=2, K_B=7$ AND $d=1.85$

t	$\mu_{\{i,j\}}=0.8$					$\mu_{\{i,j\}}=1.8$					$\mu_{\{i,j\}}=3.8$					$\mu_{\{i,j\}}=5$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00	0.2000	0.2000	0.00	0.2000	0.00
0.04	0.2332	0.2321	0.47	0.2328	0.17	0.2737	0.2721	0.58	0.2730	0.26	0.3548	0.3523	0.70	0.3532	0.45	0.4020	0.4004	0.40	0.4013	0.17
0.08	0.2649	0.2642	0.26	0.2659	0.38	0.3455	0.3445	0.29	0.3464	0.26	0.5068	0.5051	0.34	0.5073	0.10	0.6034	0.6015	0.31	0.6039	0.08
0.12	0.2975	0.2965	0.34	0.2991	0.53	0.4194	0.4172	0.52	0.4202	0.19	0.6591	0.6586	0.08	0.6624	0.50	0.8031	0.8034	0.04	0.8077	0.57
0.16	0.3303	0.3290	0.39	0.3325	0.67	0.4904	0.4902	0.04	0.4944	0.82	0.8100	0.8125	0.31	0.8183	1.02	1.0000	1.0060	0.60	1.0126	1.26
0.2	0.3641	0.3615	0.71	0.3661	0.55	0.5588	0.5634	0.82	0.5692	1.86	0.9516	0.9671	1.63	0.9752	2.48	1.1850	1.2093	2.05	1.2189	2.86
	Average Error		0.36		0.38	Average Error		0.38		0.57	Average Error		0.51		0.76	Average Error		0.57		0.82

TABLE 5: COMPARISON OF THE SIZE OF THE POPULATION OF HEMILEIA VASTATRIX $H_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF α AND FIXED VALUES OF $I_{\{i,j\}}=0.01, h=0.07, K_H=0.229, \gamma=0.08$ AND $\beta=0.8$

t	$\alpha=0.5$					$\alpha=5$					$\alpha=9.5$					$\alpha=12.5$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00
0.02	0.5890	0.5890	0.00	0.5890	0.00	0.5898	0.5899	0.02	0.5900	0.03	0.5908	0.5908	0.00	0.5908	0.00	0.5914	0.5914	0.00	0.5914	0.00
0.04	0.5782	0.5783	0.02	0.5784	0.03	0.5798	0.5801	0.05	0.5801	0.05	0.5818	0.5818	0.00	0.5819	0.02	0.5830	0.5830	0.00	0.5831	0.02
0.06	0.5677	0.5677	0.00	0.5679	0.04	0.5700	0.5704	0.07	0.5705	0.09	0.5730	0.5730	0.00	0.5731	0.02	0.5747	0.5748	0.02	0.5749	0.03
0.08	0.5574	0.5574	0.00	0.5576	0.04	0.5604	0.5609	0.09	0.5611	0.12	0.5643	0.5643	0.00	0.5646	0.05	0.5666	0.5666	0.00	0.5669	0.05

0.1	0.5478	0.5472	0.11	0.5475	0.05	0.5515	0.5515	0.00	0.5519	0.07	0.5563	0.5558	0.09	0.5562	0.02	0.5591	0.5587	0.07	0.5591	0.00
	Average Error		0.02		0.03	Average Error		0.04		0.06	Average Error		0.02		0.02	Average Error		0.02		0.02

TABLE 6: COMPARISON OF THE SIZE OF THE POPULATION OF HEMILEIA VASTATRIX $H_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF β AND FIXED VALUES OF $I_{\{i,j\}} = 0.01, h = 0.07, K_H = 0.229, \gamma = 0.08$ AND $\alpha = 0.5$

t	$\beta = 2$					$\beta = 4$					$\beta = 4.5$					$\beta = 5.8$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00
0.04	0.5498	0.5512	0.25	0.5513	0.27	0.5071	0.5088	0.34	0.5091	0.39	0.4971	0.4988	0.34	0.4991	0.40	0.4711	0.4735	0.51	0.4739	0.59
0.08	0.5041	0.5064	0.46	0.5069	0.56	0.4287	0.4315	0.65	0.4324	0.86	0.4123	0.4146	0.56	0.4156	0.80	0.3703	0.3737	0.92	0.3748	1.22
0.12	0.4623	0.4652	0.63	0.4663	0.87	0.3623	0.3660	1.02	0.3676	1.46	0.3424	0.3447	0.67	0.3464	1.17	0.2917	0.2950	1.13	0.2968	1.75
0.16	0.4242	0.4274	0.75	0.4291	1.16	0.3061	0.3105	1.44	0.3128	2.19	0.2850	0.2866	0.56	0.2890	1.40	0.2306	0.2329	1.00	0.2352	1.99
0.2	0.3910	0.3927	0.43	0.3951	1.05	0.2607	0.2634	1.04	0.2664	2.19	0.2400	0.2384	0.67	0.2413	0.54	0.1855	0.1839	0.86	0.1866	0.59
	Average Error		0.42		0.65	Average Error		0.75		1.18	Average Error		0.47		0.72	Average Error		0.74		1.02

TABLE 7: COMPARISON OF THE SIZE OF THE POPULATION OF HEMILEIA VASTATRIX $H_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF h AND FIXED VALUES OF $I_{\{i,j\}} = 0.01, \beta = 0.8, K_H = 0.229, \gamma = 0.08$ AND $\alpha = 0.5$

t	$h = 0.07$					$h = 0.2$					$h = 0.26$					$h = 0.3$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00
0.02	0.5890	0.5890	0.00	0.5890	0.00	0.5865	0.5866	0.02	0.5866	0.02	0.5853	0.5854	0.02	0.5854	0.19	0.5846	0.5847	0.02	0.5848	0.03
0.04	0.5782	0.5783	0.02	0.5784	0.03	0.5734	0.5734	0.00	0.5735	0.02	0.5712	0.5712	0.00	0.5712	0.00	0.5697	0.5697	0.00	0.5700	0.05

0.06	0.5677	0.5677	0.00	0.5677	0.00	0.5607	0.5606	0.02	0.5607	0.00	0.5576	0.5574	0.04	0.5573	0.05	0.5555	0.5552	0.05	0.5555	0.00
0.08	0.5574	0.5574	0.00	0.5576	0.04	0.5484	0.5481	0.05	0.5482	0.04	0.5450	0.5438	0.22	0.5437	0.24	0.5417	0.5410	0.13	0.5413	0.07
0.1	0.5478	0.5473	0.09	0.5475	0.05	0.5368	0.5358	0.19	0.5359	0.17	0.5323	0.5306	0.32	0.5304	0.36	0.5292	0.5272	0.38	0.5273	0.34
	Average Error		0.02		0.02	Average Error		0.05		0.04	Average Error		0.10		0.14	Average Error		0.10		0.08

TABLE 8: COMPARISON OF THE SIZE OF THE POPULATION OF HEMILEIA VASTATRIX $H_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF $I_{\{i,j\}}$ AND FIXED VALUES OF $h=0.07, \beta=0.8, K_H=0.229, \gamma=0.08$ AND $\alpha=0.5$

t	$I_{\{i,j\}}=0.01$					$I_{\{i,j\}}=5$					$I_{\{i,j\}}=12$					$I_{\{i,j\}}=15$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00
0.02	0.5890	0.5892	0.03	0.5892	0.03	0.6392	0.6387	0.08	0.6388	0.06	0.7079	0.7080	0.01	0.7083	0.06	0.7378	0.7378	0.00	0.7381	0.04
0.04	0.5782	0.5787	0.09	0.5787	0.09	0.6750	0.6767	0.25	0.6771	0.31	0.8132	0.8141	0.11	0.8150	0.22	0.8722	0.8731	0.10	0.8742	0.23
0.06	0.5677	0.5683	0.11	0.5684	0.12	0.7111	0.7140	0.41	0.7149	0.53	0.9159	0.9183	0.26	0.9203	0.48	1.0030	1.0059	0.29	1.0084	0.54
0.08	0.5574	0.5581	0.13	0.5583	0.16	0.7464	0.7506	0.56	0.7522	0.78	1.0160	1.0206	0.45	1.0241	0.80	1.1300	1.1363	0.56	1.1407	0.95
0.1	0.5484	0.5481	0.05	0.5484	0.00	0.7790	0.7865	0.96	0.7890	1.28	1.1080	1.1210	1.17	1.1265	1.67	1.2480	1.2644	1.31	1.2712	1.86
	Average Error		0.07		0.07	Average Error		0.38		0.49	Average Error		0.33		0.54	Average Error		0.38		0.60

TABLE 9: COMPARISON OF THE SIZE OF THE POPULATION OF HEMILEIA VASTATRIX $H_{\{i,j\}}$ WITH NUMERICAL RESULT FOR VARIOUS VALUES OF K_H AND FIXED VALUES OF $h=0.07, \beta=0.8, I_{\{i,j\}}=0.01, \gamma=0.3$ AND $\alpha=0.5$

t	$K_H=0.06$					$K_H=0.09$					$K_H=0.229$					$K_H=0.5$				
	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPM	Error of NHPM (%)	HPM	Error of HPM (%)	Numerical	NHPMl	Error of NHPM (%)	HPM	Error of HPM (%)
0	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00	0.6000	0.6000	0.00	0.6000	0.00

0.02	0.5824	0.5832	0.14	0.5832	0.14	0.5851	0.5859	0.14	0.5859	0.14	0.5884	0.5892	0.14	0.5892	0.14	0.5896	0.5904	0.14	0.5904	0.14
0.04	0.5655	0.5668	0.23	0.5668	0.23	0.5706	0.5721	0.26	0.5722	0.28	0.5770	0.5787	0.29	0.5787	0.29	0.5793	0.5810	0.29	0.5810	0.29
0.06	0.5492	0.5509	0.31	0.5510	0.33	0.5566	0.5587	0.38	0.5588	0.40	0.5658	0.5683	0.44	0.5684	0.46	0.5691	0.5717	0.46	0.5717	0.46
0.08	0.5335	0.5355	0.37	0.5356	0.39	0.5429	0.5456	0.50	0.5458	0.53	0.5548	0.5581	0.59	0.5583	0.63	0.5591	0.5625	0.61	0.5626	0.63
0.1	0.5192	0.5205	0.25	0.5206	0.27	0.5303	0.5328	0.47	0.5331	0.53	0.5445	0.5481	0.66	0.5484	0.72	0.5498	0.5536	0.69	0.5537	0.71
	Average Error		0.22		0.23	Average Error		0.29		0.31	Average Error		0.35		0.37	Average Error		0.37		0.37

5. CONCLUSION

Thus the system of non-linear differential equations on the consumer population, resource population, the size of the population of Bacillus thuringiensis and the size of the population of coffee rust spores Hemileia vastatrix have been solved using New Homotopy Perturbation method (NHPM) & Homotopy Perturbation Method (HPM) and the precision of the approximate analytical solution has been verified by comparison with its numerical simulation. To the best of my knowledge, we conclude that New Homotopy Perturbation Method is very effective and converges to its solution quickly than Homotopy Perturbation Method.

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