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Perceptualization Of Parametric Study Onimprecise Queuing In A Two-Stage Model With No Waiting Line Between Channels

N. Rajeswariand W. Ritha

Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli– 620 002. TamilNadu, India

ABSTRACT:

Thisarticleproposesageneralproceduretoconstructthemembershipfunctions of the performance measures in queuing systems when the inter arrival rateandservice timeare fuzzynumbers.We explore a proposedfuzzyqueuingmodelwith two consecutive stations (stages), each with a single server, where there is noqueue at station 2 and no limitations at station 1. A FCFS service discipline exists inwhich theinput streamis poisson having fuzzyarrival rate□. Anycustomer'sservice timeatserver(i=1,2)isexponentialwithfuzzyparameter □$_i$. Asaresult,the ~ membershipfunctionsofthesteadystateperformancemeasuresinatwo–stagemodel fuzzy queuing system is obtained from distinct values of □, using a parametricprogrammingapproach.Theefficacyoftheproposedapproachisvalidatedwithnumericalillustration.

Keywords: A two–stagemodelfuzzyqueuingsystem,pentagonalfuzzynumbers,

□-cutapproach,membershipfunctions,parametricprogramming.

1. Introduction and Literature review:

In queuing theory, the study of waiting lines is crucial. To analyse sojourntime, we utilise a variety of models and approaches. Mathematical study of queuespaves the path for shorter wait times and shorter lines. Transportation engineering,service industry, production, healthcare, and information processing systems are just afew of the domains where queuing models are used. Recent advancements in domainslike as communication and computer systems prompted the creation of new models ofqueuing theory. A tandem queuing system is one of these types that are required. Thishasbeen the subject of several major research.

The mean waiting time and mean client number for two tandem channels(servers) which are poisson arrival and exponential service time were provided in [1].The mean customer number, waiting time distribution and probabilities of differentnumbers of the tandem queuing system at each level of the poisson arrival and theexponential service time of the tandem queuing system were determined in [2]. It wasdemonstratedin[3]thatifthesystem'sarrivalsareapoissonprocesswithafuzzy parameter,thenthesystem'soutputisalsoapoissonprocesswithafuzzyparameter$\tilde{\lambda}$.

A more sophisticated case including network analysis was studied in [4]. Theassumption in queuing theory is that service channels are homogeneous. However, ithas been observed that in real-world queuing systems, service is provided. There areinstances when channels are heterogeneous. understanding such system is critical forprovidingsolutions to both theoretical andtechnical problems.

Homogeneoussystemhavebeencomparedtoheterogeneoussystemsforperformance measures in [5-9], under the assumption that the total of service rates isfixed. In [10], the efficacy measures for tandem queues with blocking were computedusing an approximation approach and then simulated. The tandem queues with oneserver in the first queue, and n $\geq$ 1 servers in the second queue, where the arrivals tothesystemwithapoissonprocesswithparameterλandthereisnowaitingroom betweenthetwostages,werestudiedandcertainprobabilityforthenumberof customerswereobtainedin[11].

Blockedtandemqueuingsystemhavebeenexploredintheliteratureforseveral similar models to ours, and probability of number of customers have beenderived.Thefindingsofperformancemeasureshavebeenderivedusingapproximationsands imulations.Therearenorestrictionsforthefirststage,nowaiting space between the two stages, and no blocking in our suggested approach. (i-ecustomers leave the system after having service in the first stage if the second stage isbusy). By evaluating our suggested technique, we may theoretically get probabilitiesof being-nonbeing of consumers in the first and second stages, performance measuresandthe optimal values ofthesemeasures.

We also compared some of the outcomes by using simulation results. In reallife,duetoresponsibilities,urgency,andunavailability,thereiswaitingcaseinthe servicesystems,thelossofdesiredcharacteristicsmayoccuratthestartoftheprocess. As in our suggested paradigm, there isa second stage. Consequently, wehavedecidedtoconstructsuch amodel and analyzeit.

ResearcherssuchasLiandLee[16],Buckley[17],NegiandLee[18],Kaufmann [20], Chen [21 & 22] have used zadeh's extension principle to investigatefuzzy queues. Kao et al [23] developed a parametric linear programming technique togeneratethemembershipfunctionsofthesysteminfuzzyqueues.Thequeuingsystem may be studied using recent advancements in fuzzy numbers using randomvariables.Theconceptoffuzzyprobabilities,forexample,wasintroducedbyZadeh., L.A [24]. Here we use a non linear programming approach to analyse a two stagemodelfuzzy queuing systemwith no waiting linebetweenchannels.

2. Preliminaries:

1) FuzzySet: Afuzzyset$\tilde{A}$isdefinedby $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A; \mu_{\tilde{A}}(x) \in [0,1]\}$

where $\mu_{\tilde{A}}(x)$iscalledgradeofmembershipofxin$\tilde{A}$.

2) α-Cut:

Theα-cutforafuzzyset$\tilde{A}$areshownby$A_\alpha$for$\alpha \in [0,1]$aredefinedto be :

$A_\alpha = \{x / \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$, whereXistheuniversalset.Upperandlowerboundsof

anyα-cut$A_\alpha$areshownbysup$A_\alpha$andinf$A_\alpha$respectively.Sup$A_\alpha$isdenotedbyA_α^U

andinf $A_\alpha$is denotedbyA_α^L.

3) Fuzzy Number: Afuzzynumber$\tilde{A}$isasubsetofreallineR,withthe membership function $\mu_{\tilde{A}}$ satisfying the following properties.

(i) $\mu_{\tilde{A}}(x)$ispiecewisecontinuousinitsdomain

(ii) $\tilde{A}$isnormal(i.e)thereisa $x_0 \in \tilde{A}$such that $\mu_{\tilde{A}}(x_0) = 1$.

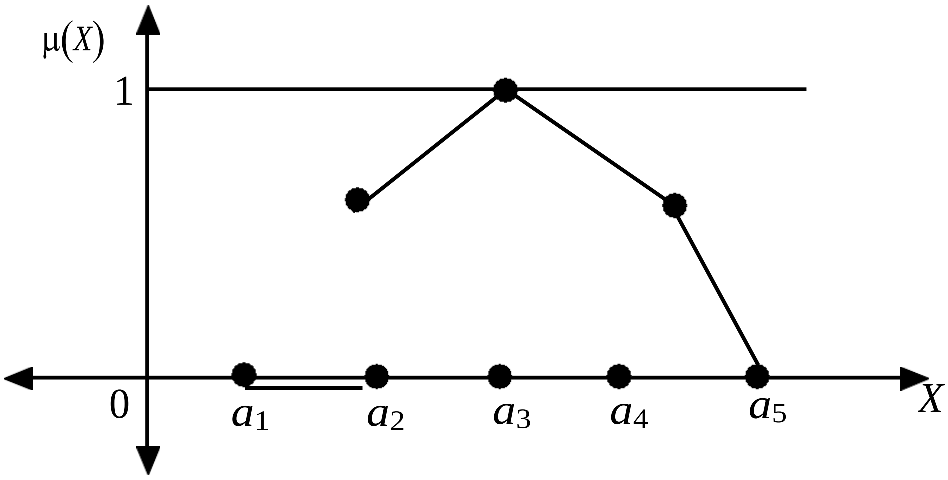
(iii) $\tilde{A}$isconvex(i.e) $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, $\forall x_1, x_2$ in X.

4) Pentagonal fuzzy number:

A pentagon fuzzy number is a 5-tuple subset of a real number having fiveparameters (a_1, a_2, a_3, a_4, a_5) where a_3 is the middle point and (a_1, a_2) and (a_4, a_5) arethe left and right sidepoints of a_3.

Afuzzynumbers $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ issaidto bepentagon fuzzy number ifits membership function is:

$$
\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ L_1(x) = \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ L_2(x) = \dfrac{x - a_2}{a_2 - a_1}, & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ R_1(x) = \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ R_2(x) = \dfrac{a_5 - x}{a_5 - a_4}, & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}
$$


μ(X)
1
0
a1
a2
a3
a4
a5
X

3. Model description:

The measures of performance and optimization of performance measures :The meansojourn time: LetTbea randomvariablethatdescribesthesojourntimeofcustomersinthesystem.Usingthelawoftotal expectation,wecanwriteitisasfollows.

$$E(T) = E(T/A)P(A) + E(T/\bar{A})P(\bar{A}) \quad \text{... (1)}$$

whereP(A)isthe probabilityofthelossofacustomer.Nowit isclearthat

$$E(T/A) = \frac{1}{\tilde{\mu}_1-\tilde{\lambda}},\; E(T/\bar{A}) = \frac{1}{\tilde{\mu}_1-\tilde{\lambda}} + \frac{1}{\tilde{\mu}_2} \quad \text{... (2)}$$

Thus, $$E(T) = \frac{\tilde{\mu}_1+\tilde{\mu}_2}{(\tilde{\mu}_1-\tilde{\lambda})(\tilde{\mu}_2+\tilde{\lambda})} \quad \text{... (3)}$$

Ourmainresultsabouttheproblemofminimizingthemeansojourntimecanbeexplainedby the following theorem.

Theorem :

Ifsumof two service rates $\tilde{\mu}_1+\tilde{\mu}_2=\tilde{\mu}$ isfixed,thenthemeansojourntimeof thistandemsystemattainsitsminimumvaluefor $\tilde{\mu}_1 = \frac{\tilde{\mu}}{2}$ and $\tilde{\mu}_2 = \frac{\tilde{\mu}}{2}$

Proof:

Wewill provethetheorem by usingthefollowing inequality.

$$\left(\prod_{i=1}^{m} a_i\right)^{\frac{1}{m}} \le \frac{1}{m}\sum_{i=1}^{m} a_i,\; a_i \ge 0,\; m \in Z^{+}. \quad \text{... (4)}$$

Frominequality(4),we have

$$\text{... (5)} \quad \frac{1}{(\tilde{\mu}_1-\lambda)(\tilde{\mu}_2+\lambda)} \ge \frac{4}{\tilde{\mu}^2}$$

Ifwereplacetheexpressions $(\tilde{\mu}_1-\tilde{\lambda})(\tilde{\mu}_2+\tilde{\lambda})$ and $\frac{4}{\tilde{\mu}^2}$ inequality(3),weobtainthe

1 2

minimumvalueof E(T)as follows:

minE□T□□ $\frac{4}{\tilde{\mu}}$, ... (6)

wheretheequality(6)isprovidedwith □□ □□and□ □ □□

1 2 2

2

The mean number of customers:

LetNbetherandomvariablethatdescribesthenumberofcustomersinthesystem.

� 1

E�N������n₁�n₂��Pn₁,n₂

n1�0n

$$2 \text{ÿ} 0 \quad \frac{\tilde{\lambda}(\tilde{\mu}_1+\tilde{\mu}_2)}{(\tilde{\mu}_1-\tilde{\lambda})(\tilde{\mu}_2+\tilde{\lambda})} \qquad \text{...(7a)}$$

E�N��

(or)

E�N���E�T�

...(7b)

Themeannumberofcustomersinthesystemisoptimizedfromtheorem1andtheequality(7b)as below:

$$\min E(N) = \min \tilde{E}(T) = \frac{4}{\tilde{\mu}} \qquad \text{... (8)}$$

4. Problemformulation:

4.1. Atwo–stagemodelfuzzyqueuingsystemwithnowaitinglinebetweenchannels :

Letthearrivalrateandserviceratesareapproximatelyknownandcanbe representedbythefuzzynumbers,�,�₁,�₂ respectively.Let���x�,���y₁�,���y₂� denotethemembershipfunctionof�,�₁,�₂ respectively.Wethenhavethefollowing fuzzysets,

���x,���x��|x�X�,

�₁��y1,��1�y1��|y1�Y1�,

�₂��y2,��2�y2��|y2�Y2�

whereX,Y1,Y2arethecrispuniversalsetsofthearrivalrateandservicerateswith two consecutive stations (stages), andcorrespondingmembership functions. ���x �,���

y_1□

and

□ □y_2□

are

Letf(x,y_1,y_2)denotethesystemcharacteristicofinterest.Since□,$□_1$,$□_2$ ˜ ~ ~

fuzzynumbers.f(x,y_1,y_2)isalsoafuzzynumber.Byzadeh'sextensionprinciple,themembershipfunction ofthesystem characteristicf(x,y_1, y_2) is definedas.

□ f□x,y,y□□z□□sup min□□□x□,$□_1$□y_1□,$□_2$□y_2□/z□f□x,y_1,y_2□□,

1 2 □

wherethesupremumis takenovertheset

□□ x□X,y□Y,y□Y/0□ x □ □

□ 1 1 2 y□y 1□

2

□ 1 2 □

Letthesystemcharacteristicofinterestinthemeansojourntimeofcustomersinthesystem is

$$f□x,y_1,y_2□□ □y \frac{y_1□y_2}{□x□□y} □x□$$

1 2

Themembership functionforthe mean sojourntime of customers in the systemsis

□ □z□□supmin□ □x□,□ □y□,□ □y□/z□ $\frac{y_1□y_2}{□y□x□□y}$ □x□ □

E□T□ □□□ □1 □2

□

1 2

□ 1 2 □

Themeannumberofcustomersinthesystemis f□x,y,y□□ x□y_1□y_2□

1 2 y x y x

are the

1 2

Similarly,Themembershipfunctionforthemeannumberofcustomersinthesystemis

z supmin x, y, y /z x y1 y2

E N 1 2 y x y x

1 2

1 2

The shapes of these membership functions are difficult to envision since they are notstated in the typical ways. Here we use a mathematical programming approach toexamine at the representation problem. Based on the extension principle, parametricnonlinear programming isused to determinethe-cuts of(x,y1, y2).

Thesolutionprocedure :

Byconstructingthemembershipfunction $\tilde{E}(T)$ ofE T isonthebasisof

derivingthe -cutof E T .Denotethe -cutsof , 1, 2 ascrispintervalsas

follows.

x L,x U min x/ x , max x/ x

x X y , max y/ y

y L,y U min y/

1 1 y1 Y1 1 1 y1 Y1 1 1 1

1

1

y L,y U min y/ y , max y/ y

2 2 y2 Y2 2 2 y2 Y2 2 2 2

2

2

Theconstantarrivalratesandserviceratesaregivenasintervalswhenthemembership functions are not less than a given possibility level for . Hence theboundsofthesentervalscanbedescribedasfunctionsof andcanbeobtainedas

x L min 1 , x U max 1 , y L min 1 ,

1 1

y U max 1 , y L min 1 ,

1 1 2 2

y U max 1 .

2 2 Hence we can use the -cutsof

E(T) and E(N) tocon structi tsme mbers hip

functionusingzadeh'sextensionprinciple,toderviethemembershipfunction $\tilde{E}(T)$

weneedatleastoneofthefollowingcasestoholdsuchthat $Z = \dfrac{y_1 y_2}{y_1 x y_2 x}$

satisfies $\tilde{E}(T)$.

Case(i) : x , y1 , y2

1 2

Case(ii) : x , y1 , y2

1 2

Case(iii): x , y1 , y2

1 2

ThiscanbeaccomplishedusingparametricNLPtechniques.TheNLPtofindthe

lowerandupperboundsofthe -cutsof $\tilde{E}(T)$ forthecase(i)are:

L y y

E T 1 min 1 2

y x y x

1 2

U y y

E T 1 max 1 2

y x y x

1 2

L y y

forthecase(ii)areE T 2 min 1 2

y x y x

1 2

U y y

E T 2 max 1 2

y x y x

1 2

L y y

forthecase(iii)are $\tilde{E}(T)^3 = \min$ 1 2

y x y x

1 2

U y y

$\tilde{E}(T)^3 = \max$ 1 2

y x y x

1 2

From the definitions x, y₁ 1, y₂ 2 can be replaced by

$x = [x^L, x^U]$, $y = [y^L, y^U]$, $y = [y^L, y^U]$ respectively. Given 0 1 we

1 1 2 2 2 1

1

2

have $[x^L_{\alpha_1}, x^U_{\alpha_1}]$ $[x^L_{\alpha_2}, x^U_{\alpha_2}]$, $[y^L_{1\alpha_1}, y^U_{1\alpha_1}]$ $[y^L_{1\alpha_2}, y^U_{1\alpha_2}]$ $[y^L_{2\alpha_1}, y^U_{2\alpha_1}]$ $[y^L_{2\alpha_2}, y^U_{2\alpha_2}]$.

Hencetofindthemembershipfunction $\tilde{E}(T)$ itissufficienttofindtheleftandright

shape functions of $\eta_{E(T)}$ which is equivalent in finding the lower bound

$E(T)^L = \min$ $y_1 y_2$ y x y x suchthat $x^L x x^U, y^L_1 y_1 y^U_1, y^L_2 y_2 y^U_2$

1 2

$E(T)^U = \max$ $y_1 y_2$

 yxyx

 1 2

suchthatx^Lxx^U,y^L yy^U,

  1 1

1

$y^L \le y \le y^U$

2 2 2

Thecrispintervalobtainedaboverepresentsthe-cutsof ET. Again

applyingtheresultsofzimmermannandkaufmannandconvexitypropertiesto

ET,wehave

ET^{L}ET^U,ET^{L}ET^{U}where01.

1 1 2 2 2 1

Inboth ET^L and ET^U areinvertiblewithrespecttothenaleftshapefunction

α α

LzET$^{L\,1}$ andarightshapefunction RzET$^{U\,1}$ canbederivedfrom

whichthemembershipfunctionis constrictedas

LzET^L zET^L

 1

0

ET

RzET^U zET^U

0

1

Themembershipfunctionsofthemeannumberofcustomerscanbederivedinasimilar manner.

1.Numericalexample:

Themean sojourntimeofcustomers in the system:

Supposethearrival,serviceratesarepentagonalfuzzynumbersgivenby

□□□2,3,4,5,6□, □1□□9,10,11,12,13□, □2□□15,16,17,18,19□ perminuterespectively.

Itiseasytofindthat [□x^L,x^U[□□□2□2□,6□2□□ [□y^L,y^U[□□□9□2□,13□2□□,

□ □ 1□ 1□

[□y^L

$_{2\alpha}$ y $^U_{2\alpha}$[□□□15□2□,19□2□□

The□-cutsof $\tilde{E}$□T□

□$\tilde{E}$□T□□ ^{L} □$\frac{32□4□}{}$

□ □ 231□84□

□$\tilde{E}$□T□□ ^{U} □$\frac{24□4□}{}$

□ □ 63□84□

WiththehelpofMATLAB6.0,theinversefunctionsof $\left[□E□T□□\right]^L_\alpha$ and $\left[□\tilde{E}□T□□\right]^U_\alpha$ exist,

□ L□z□; 32 □z□ 4

yieldthemembershipfunction□ □z□□ □ 231 21

$\tilde{E}$□T □R□z□;

□ □ $\frac{4}{21}$□z □ $\frac{8}{21}$

where L□z□□ $\frac{32□231z}{4□84z}$ and

R□z□□ $\frac{24□63z}{84z□4}$

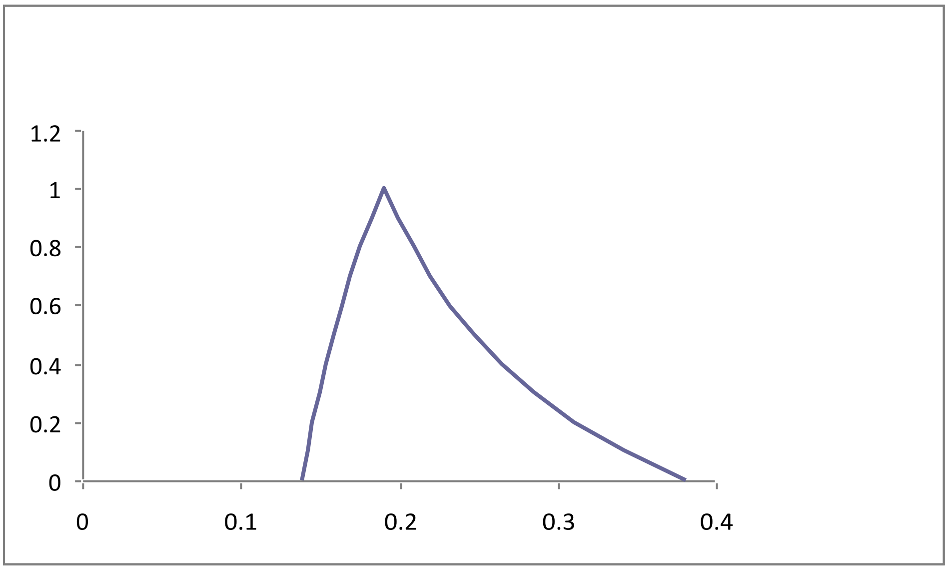
The □-cuts of arrival and service rates and fuzzy mean sojourn

ofcustomers inthe system.

α	x^L_α	x^U_α	$y^L_{1\alpha}$	$y^U_{1\alpha}$	$y^L_{2\alpha}$	$y^U_{2\alpha}$	$[E(T)]^L_\alpha$	$[E(T)]^U_\alpha$
0.0	2.0	6.0	9.0	13.0	15.0	19.0	0.13853	0.38095
0.1	2.2	5.8	9.2	12.8	15.2	18.8	0.14196	0.34174
0.2	2.4	5.6	9.4	12.6	15.4	18.6	0.14566	0.31078
0.3	2.6	5.4	9.6	12.4	15.6	18.4	0.14966	0.28571
0.4	2.8	5.2	9.8	12.2	15.8	18.2	0.15400	0.26501
0.5	3.0	5.0	10.0	12.0	16.0	18.0	0.15873	0.24762
0.6	3.2	4.8	10.2	11.8	16.2	17.8	0.16390	0.23280
0.7	3.4	4.6	10.4	11.6	16.4	17.6	0.16957	0.22003
0.8	3.6	4.4	10.6	11.4	16.6	17.4	0.17582	0.20891
0.9	3.8	4.2	10.8	11.2	16.8	17.2	0.18275	0.19913
1.0	4.0	4.0	11.0	11.0	17.0	17.0	0.19048	0.19048

E□T□at□=1is0.19048,indicatingthatthemeansojourntimeofcustomersinthe systemis0.19048. E□T□neverexceed0.38095(or)fallbelow

Moreovertherangeof0.13853



Themembership functionformeansojourntimeofcustomersin thesystem

Themean numberof customersinthe system:

Similarlythe□-cutsof $\tilde{E}$□N□are:

□8□^{2}□56□□64231□84□

$\lfloor$□$\tilde{E}$□N□□$\rfloor^L_\alpha$□

□$\tilde{E}$□N□□^{U}□8□^{2}□24□□144

□ □□ 63□84□

Theinversefunctionof $\lfloor$□E□N□□$\rfloor^L_\alpha$ and □$\lfloor$□$\tilde{E}$□N□□$\rfloor^U_\alpha$ exist,yieldthemembershipfunction

□L□Z□;64 □Z□16

{

□$\tilde{E}$□N□ □Z□□□ □R□Z□;

□

16 Z 16

231 21

21 7

where L Z 84 z 56 7056 z ^{2} 2016 z 5184 ^{1} 2

16

R□Z□□ □□84z□24□□□7056z^{2}□2016z□5184□ ^{1} 2

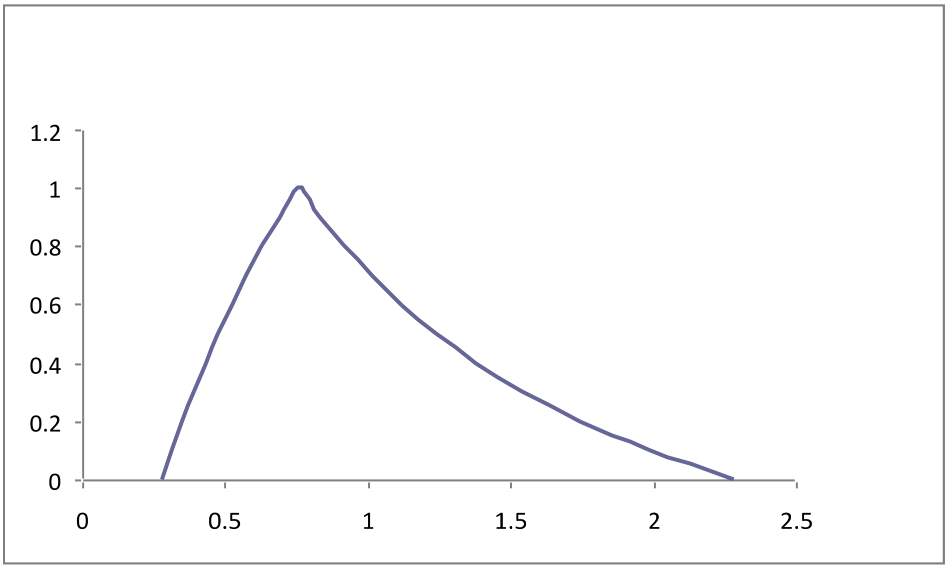
and

16

The□-cutsofarrivalandservicerates andmeantime ofcustomers inthesystem.

□	$x^{L}_{□}$	$x^{U}_{□}$	$y^{L}_{1□}$	$y^{U}_{1□}$	$y^{L}_{2□}$	$y^{U}_{2□}$	$□\tilde{E}□N□□^{L}_{□}$	$□\tilde{E}□N□□^{U}_{□}$
0.0	2.0	6.0	9.0	13.0	15.0	19.0	0.27706	2.28571
0.1	2.2	5.8	9.2	12.8	15.2	18.8	0.31231	1.98207
0.2	2.4	5.6	9.4	12.6	15.4	18.6	0.34958	1.74035
0.3	2.6	5.4	9.6	12.4	15.6	18.4	0.38912	1.54286
0.4	2.8	5.2	9.8	12.2	16.8	18.2	0.43121	1.37805
0.5	3.0	5.0	10.0	12.0	16.0	18.0	0.47619	1.23810
0.6	3.2	4.8	10.2	11.8	16.2	17.8	0.52447	1.11746
0.7	3.4	4.6	10.4	11.6	16.4	17.6	0.57654	1.01215
0.8	3.6	4.4	10.6	11.4	16.6	17.4	0.63297	0.91920
0.9	3.8	4.2	10.8	11.2	16.8	17.2	0.69447	0.83636
1.0	4.0	4.0	11.0	11.0	17.0	17.0	0.76190	0.76190

Fromthetablewefindthatthemeannumberofcustomersinthesystem □$\tilde{E}$□N□□ at □=1is0.76190,indicatingthatthemeannumberofcustomersinthesystemis 0.76190.Moreovertherangeof $\tilde{E}$□N□neverexceed2.28571orfallbelow0.27706.



Themembership functionformeannumberofcustomersin thesystem.

Conclusion:

A new queuing discipline is given for a markov model which consists of twoconsecutive channels and no waiting line between channels. In this model, steady –state conditions, the mean sojourn time, the mean number of customers are obtained.Additionally, the theorem is explained about optimization of performance measures.Here -cut and zadeh's extension principle is applied to a two-stage model imprecisequeuing system with no waiting line between channels and membership function ofmean sojourn time of customers, mean number of customers in the queuing system isconstructed using non-linear programming approach. The numerical example is alsogivento illustrate theeffectiveness of the proposedtechnique.

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