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Exploration Of Ranking Based On Statistical Beta Distribution For A Heterogeneous Server Imprecise Queue With A Slow Server Threshold

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Abstract: This research article intends to explore a markovian queue with two heterogeneous servers. In this case, the second server has a service threshold. The steady-state findings for this model have been acquired. In the fuzzy numbers for defuzzification, ranking algorithms play a significant role. The core concept is to use ranking based on themean value of statisticalbeta distribution to convert the fuzzy arrival rate and service rate into crisp numbers. It analyzes the crisp ranking system on R and provides an algorithm for ranking fuzzy numbers. Some specific models, performance measures and mathematical solutions are computed for different number of servers.

Keywords: Triangular fuzzy number, fuzzy ranking algorithm, statistical Beta distribution, Heterogeneous server fuzzy queue, slow server, threshold, steady state, performance measures, defuzzification.

I. Introduction and review of the Literature

The majority of work on multi-server fuzzy queuing systems in the literature presume that the servers are homogeneous.This is only valid if the service process is controlled by either manually or mechanically.

The stationary distribution has a non-constructiveexistence theorem. It was provided in terms of general input and general service time and it was presented in Kiefer and Wolfowitz (1955).Kendall (1958) provided an equilibrium analysis for the generic input with exponential service time and servers. The busy period distribution for the M/M/S queue was obtained by Karlin and MC. Gregor (1958). Krishnamoorthi (1963) explores a

Poisson queue with two heterogeneous servers that violates the First-in-First-out (FIFO) principle.

Heffer (1969) has analyzed the distribution of $M/E_k/S$ queue waiting times. Singh (1970) discussed a Markovian queuing system with balking and two heterogeneous servers was investigated. The author of this research determines the slower server's capacity and obtains the optimal service rates. Singh (1973) described a Markovian queue in which the number of servers is determined by the length of the queue.

In 1981, Neuts and Takahashi pointed out analytical findings are intractable for a queuing system with two heterogeneous servers. Despite this, some academics have concentrated their study on queues with two heterogeneous servers. Desmit (1983 a,b) proposed a method for determining the distribution of $GI/H_2/S$ queue lengths and waiting times. He simplified the problem to the solution of Wiener – Hopf-type equations, then solved the system using a factorization method.

Lin and Kumar (1984) focused at the optimal control of a queuing system with two heterogeneous servers. Rubinovitch (1985 a,b) investigated a heterogeneous two-channel queuing system.In his initial article, he presented three simple models and stated when the slower server should be discarded based on the expectednumber of customer in the system. He looked at a queuing model with a stalling approach in his second study. In 1999, Abou-El-Ata and Shawky proposed a simplified method for determining when to discard the slower server in a two-channel heterogeneous queue.

Barcelo (2003) calculated an approximation for the average waiting time of $M/H_{2b}/S$ queue. An approximation analysis for M/G/C queues has obtained by Shin and Moon (2009).Zhernovyi (2011) investigated a queue with service mode switching and input flow threshold blocking. Markovian queue with switching of service mode was examined by Kopytko and Zhernovyi(2011). Arkat and Farahani (2014) has used a partial fraction decomposition approach to the $M/H_2/2$ queue.

Kalyanaraman and Senthilkumar (2018) analyzed a Markovian queue with two heterogeneous servers and switching service models. The authors discussed heterogeneous server Markovian queue with limited admissibility and reneging and also analyzed a two heterogeneous server queue with restricted admissibility. In the actual world, models of queuing systems with varying levels of service intensity are used to analyse telecommunication process.

This paper aims to use statistical Beta distribution for defuzzifying and ranking fuzzy numbers, since it is the only distribution function that is bounded to (0,1) and is zero outside this interval. In addition, parameters of this distribution could be set such that the resulting left spread, right spread and mode fully match their corresponding values in the fuzzy number transformed to the interval (0,1). We initiate the defuzzification process of a fuzzy number by obtaining the mean value of its corresponding Beta distribution. We then use simple arithmetic operations to determine the crispnumber corresponding to that fuzzy number. We also provide a very simple algorithm for ordering fuzzy numbers based on their corresponding crisp real values and domains.

We investigate a queue with heterogeneous servers in this article. Additionally, on the slow servers, there is also a threshold policy. The queuing system in steady state was reviewed and examined in section 2. In section 3, we derive various performance measures relevant to the model discussed in section 2. By using some specific models in section 4, section 5 describes the proposed method of defuzzifying trapezoidal fuzzy numbers using the mean value of Beta distribution. The numerical findings for the model discussed in this article are obtained in section 6.

II. Mathematical model and analysis

We consider an M/M/2 queuingsystem with a single, infinitely long waiting line. Customers arrive at the system according to a poisson process with fuzzy arrival rate $\tilde{\lambda}$. The service time of customers follows two different exponential distributinos with fuzzy service rate $\tilde{\mu}_1, \tilde{\mu}_2$ respectively corresponding to the two servers. Also $\tilde{\mu}_1 > \tilde{\mu}_2$.

When there are less than k customers in the system, the first server operates normally, while the second server remains in an optimum state. Once the system size reaches k, the second server also starts work.

Each customer is served only one server at a time and the queue discipline is first come first served. For the $P_n(t)$ be the probability that there are n customers in the system at time t. $n \geq 0, t \geq 0.$ In steady state, $\lim_{t\to\infty} P_n(t) = P_n$, Using general birth death arguments the following difference equation has been derived.

$$\tilde{\lambda} P_0 = \tilde{\mu}_1 P_1 \qquad \text{... (1)}$$

$$\left(\tilde{\lambda}+\tilde{\mu}_1\right)P_n=\tilde{\lambda}P_{n-1}+\tilde{\mu}_1P_{n+1};n=1,2,...k-1 \quad ...(2)$$

$$\left(\tilde{\lambda}+\tilde{\mu}_1\right)P_k=\tilde{\lambda}P_{k-1}+\tilde{\mu}P_{k+1}; \quad ...(3)$$

$$\left(\tilde{\lambda}+\tilde{\mu}\right)P_n=\tilde{\lambda}P_{n-1}+\tilde{\mu}P_{n+1};n=k+1.k+2,... \quad ...(4)$$

For the analysis, the following probability generating function has been defined.

$$P(z)=\sum_{n=0}^{\infty}P_nz^n;|z|\le 1 \quad ...(5)$$

Multiplying equations (1) – (4), by suitable power of z an summing over n = 0 to ∞, and then using equation (5), we get p(z)

$$\therefore P(z)=\frac{(\tilde{\mu}_1-\tilde{\mu})\sum_{n=1}^{k}p_nz^n-\tilde{\mu}p_0}{\tilde{\lambda}z-\tilde{\mu}} \quad ...(6)$$

Equation (6), can be written as

$$P(z)=\frac{(\tilde{\mu}_1-\tilde{\mu})}{\tilde{\mu}}\sum_{n=1}^{k}P_nz^n-\left(1-\frac{\tilde{\lambda}z}{\tilde{\mu}}\right)^{-1}+P_0\left(1-\frac{\tilde{\lambda}z}{\tilde{\mu}}\right)^{-1} \quad ...(7)$$

Equating the co-efficient of z^n on both sides on equation (7), we get

$$P_n=\left(\frac{\tilde{\lambda}}{\tilde{\mu}_1}\right)^{-1}P_0,n=1,2,...,k$$

$$=\left(\frac{\tilde{\lambda}}{\tilde{\mu}_1}\right)^{k}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^{n-k}P_0,n=k+1,k+2,... \quad ...(8)$$

Using the normalization condition $\sum_{n=0}^{\infty}P_n=1,$ we get,

$$P_0=\frac{\tilde{\mu}_1\left(\tilde{\mu}-\tilde{\lambda}\right)(\tilde{\mu}_1-\lambda)}{\tilde{\mu}_1^{k+1}\left(\tilde{\mu}-\tilde{\lambda}\right)-\tilde{\lambda}^{k+1}(\tilde{\mu}-\tilde{\mu}_1)} \quad ...(9)$$

The probability distribution in steady state of the model described in this article is represented by equations (8) and (9) combined, and it is clear that the stability condition is $\frac{\tilde{\lambda}}{\tilde{\mu}}<1.$

4. Some performance Measures

In this section, some performance measures such as the probability that both the servers are ideal. Probability of both servers being busy, expected number of customers in the system, second moment of the number of customers in the system and expected waiting time of acustomers in the system have been obtained, Straight forward computations yielded these results;

(i) Probability that both the servers are idle (P_0)

$$P_0 = \frac{\tilde{\mu}_1^k\left(\tilde{\mu}-\tilde{\lambda}\right)\left(\tilde{\mu}_1-\tilde{\lambda}\right)}{\tilde{\mu}_1^{k+1}\left(\tilde{\mu}-\tilde{\lambda}\right)-\tilde{\lambda}^{k+1}\left(\tilde{\mu}-\tilde{\mu}_1\right)}$$

(ii) Probability that both the servers are busy

$$P_B = 1 - P_0\left(\frac{\tilde{\mu}_1-\tilde{\lambda}}{\tilde{\mu}_1}\right)$$

(iii) Expected number of customers in the system

$$L = P^1(1) = \frac{\tilde{\lambda}P_0}{\left(\tilde{\mu}_1\right)^k}\left[\frac{\tilde{\mu}_1\left(\tilde{\mu}_1^k-\tilde{\lambda}^k\right)-K\lambda^k\left(\tilde{\mu}_1-\tilde{\lambda}\right)}{\left(\tilde{\mu}_1-\tilde{\lambda}\right)^2}\right]$$

The following probabilities have been defined within the mathematical framework of the above mentioned model.

(iv) Second moment of number of customers in the system.

$$L_1 = \frac{\tilde{\lambda}^2P_0}{\left(\tilde{\mu}_1-\tilde{\lambda}\right)^3}\left[\frac{\tilde{\lambda}\left(\tilde{\mu}^k-\tilde{\lambda}^k\right)+2\tilde{\mu}\left(\tilde{\mu}^k-(k+1)\tilde{\lambda}^k\right)+k(k+1)\tilde{\lambda}(k-1)\left(\tilde{\mu}-\tilde{\lambda}\right)^2}{\tilde{\mu}^k\left(\tilde{\lambda}-\tilde{\mu}\right)}\right]-\frac{2\tilde{\lambda}L}{\left(\tilde{\lambda}-\tilde{\mu}\right)}$$

(v) Expected waiting time of customers in the system (using Little's Law)

$$W = \frac{L}{\tilde{\lambda}}$$

4.1 Preliminaries

Definitions.1

A fuzzy number is a fuzzy set in the form of $\tilde{a}: R \to [0,1]$ that satisfies the following conditions :

(1) $\tilde{a}$ is upper semi continuous

(2) $\tilde{a}(x)$ is zero outside the interval [l, u],

(3) there exist real numbers m_1, m_2 such that $l \le m_1 \le m_2 \le u$ and

(3.1) $\tilde{a}(x)$ is increasing on [l, m_1],

(3.2) $\tilde{a}(x)$ is decreasing on [m_2, u],

(3.3) $\tilde{a}(x)$ = 1, $m_1 \le x \le m_2$.

If $m_1 = m_2 = m,$ then the fuzzy number $\tilde{a} = (l, m, u)$ is called the triangular fuzzy number and is defined as follows.

$$\tilde{a}(x) = \begin{cases} \dfrac{x-1}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m}, & m \le x \le u \\ 0 & , otherwise \end{cases}$$

4.2 Defuzzifying and ranking fuzzy numbers using Beta distribution

The proposed distribution is the only reason for the equality of left spread, right spread and mode of beta distribution with their corresponding values in fuzzy numbers within (0,1) interval. Consider a fuzzy number $\tilde{a},$ the mean value of its corresponding Beta distribution in its domain is considered as the crisp real number corresponding to $\tilde{a},$ which based on crisp ranking system on R.

For Triangular fuzzy number

Let $\tilde{a} = (l, m, u)$, the ranking function and corresponding crisp real number $\mu_{\tilde{a}}$ is defined by $R(\tilde{a}) = \mu_{\tilde{a}} = \dfrac{l+m+u}{3}$

Numerical study

In this section, we present some numerical illustrations corresponding to the model discussed in this paper. For the numerical illustration, let us take the value of $\tilde{\lambda}$ from 0.1 to 1, k = 5;

Let us contemplate the service rates are triangular fuzzy numbers $\tilde{\mu}_1 = (3, 3.9, 4.8)$ and $\tilde{\mu}_2 = (3, 3.5, 4)$, (Table 1, Graph 1)

$$\therefore R(\tilde{\mu}_1) = \frac{3+3.9+4.8}{3} = \frac{11.7}{3} = 3.9$$
$$\& R(\tilde{\mu}_2) = \frac{3+3.5+4}{3} = \frac{10.5}{3} = 3.5$$

arrival rate ($\tilde{\lambda}$)	Probability of down time (P_0)	Probability of all the server are busy (P_B)	Second moment of number of customers in the system (L_1)
0.1	0.974359	0.050625	0.001163
0.2	0.948718	0.099934	0.004804
0.3	0.923077	0.147929	0.011172
0.4	0.897436	0.194609	0.020548
0.5	0.871794	0.239974	0.033249
0.6	0.846152	0.284025	0.049629
0.7	0.820509	0.326762	0.070077
0.8	0.794863	0.368186	0.095027
0.9	0.769213	0.408298	0.124961
1.0	0.743556	0.447099	0.160432

(Table 1): k = 5, $\tilde{\mu}_1$ = 3.9, $\tilde{\mu}_2$ = 3.5 ($\tilde{\lambda}$ various from 0.1 to 1)

Similarly for Table 2, Graph 2; Let us contemplate the service rates are triangular fuzzy numbers.

$\tilde{\mu}_1 = (3, 3.7, 4.4), \tilde{\mu}_2 = (3, 3.3, 3.6)$

$$\therefore R(\tilde{\mu}_1) = \frac{3+3.7+4.4}{3} = \frac{11.1}{3} = 3.7$$

$$\&R(\tilde{\mu}_2) = \frac{3+3.3+3.6}{3} = \frac{9.9}{3} = 3.3$$

$\tilde{\lambda}$	P_0	P_B	L_1
0.1	0.972973	0.053324	0.001279
0.2	0.945946	0.105186	0.005270
0.3	0.918919	0.155588	0.012226
0.4	0.891892	0.204529	0.022438
0.5	0.864864	0.252009	0.036230
0.6	0.837836	0.298030	0.053961
0.7	0.810805	0.342590	0.072972
0.8	0.783771	0.385693	0.102870
0.9	0.756731	0.427339	0.134972
1.0	0.729680	0.467531	0.172911

(Table : 2) $k = 5, \tilde{\mu}_1 = 3.7, \tilde{\mu}_2 = 3.3$, ($\tilde{\lambda}$ various from 0.1 to 1)

For Table 3, Graph 3; Let us take $\tilde{\mu}_1 = (3, 3.5, 4)$, & $\tilde{\mu}_2 = (3, 3.1, 3.2)$ are triangular fuzzy numbers.

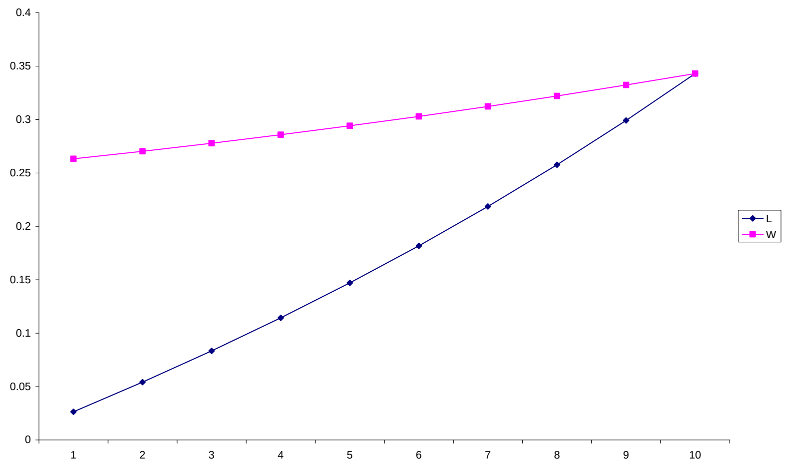
$$\therefore R(\tilde{\mu}_1) = \frac{3+3.5+4}{3} = \frac{10.5}{3} = 3.5$$

$$\&R(\tilde{\mu}_2) = \frac{3+3.1+3.2}{3} = \frac{9.3}{3} = 3.1$$

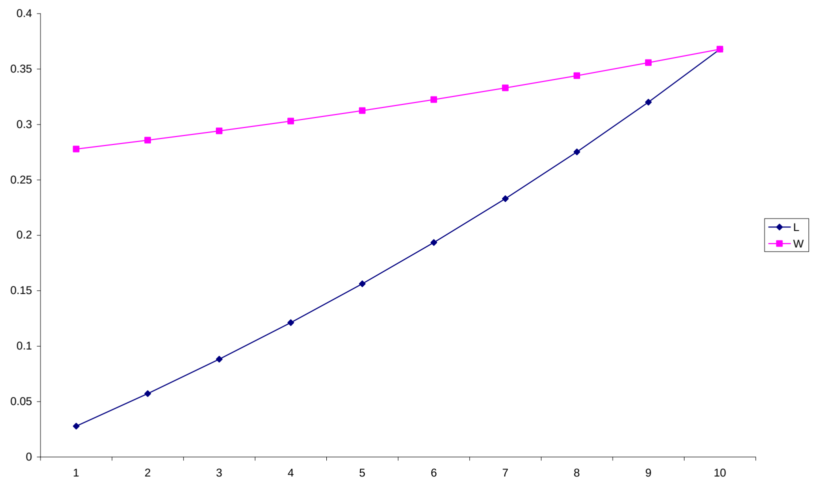
$\tilde{\lambda}$	P_0	P_B	L_1
0.1	0.971429	0.056327	0.001411
0.2	0.942857	0.111020	0.005955
0.3	0.914286	0.164082	0.013944
0.4	0.885714	0.215510	0.025803

0.5	0.857142	0.265307	0.038039
0.6	0.828568	0.313472	0.058309
0.7	0.799991	0.360007	0.081802
0.8	0.771409	0.404913	0.110188
0.9	0.742818	0.448192	0.143914
1.0	0.714212	0.489849	0.183549

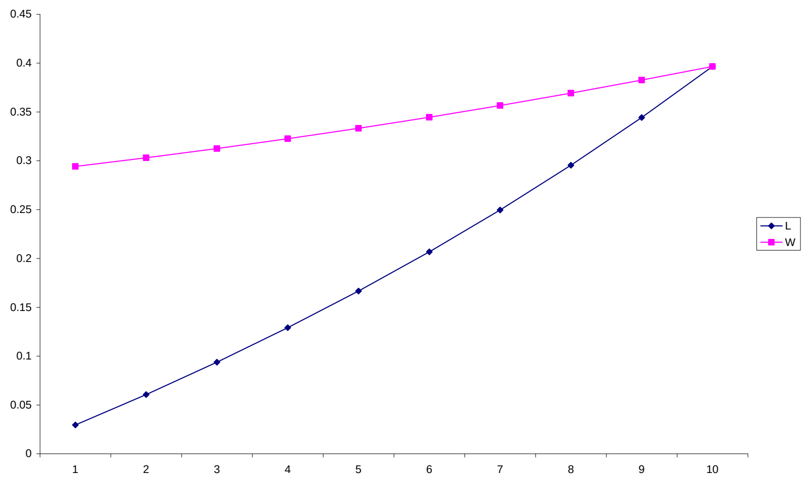
(Table : 3)k = 5, $\tilde{\mu}_1$ = 3.5, $\tilde{\mu}_2$ = 3.1 ($\tilde{\lambda}$ various from 0.1 to 1). The values of P_0, P_B and L_1 are presented in the tables and the values of L and W are presented in the graphs.



Graph - 1



Graph – 2



Graph – 3

Conclusion

Thus this article highlights the performance measures of heterogeneous server with a slow server threshold fuzzy queuing model and ranking triangular fuzzy numbers on the basis of statistical beta distribution. The projected ranking methods can rank fuzzy numbers in practical reality. We used numerical examples to demonstrate the performance of our method and this method not only produces crisp results, but it also does so with more precision than most other methods.

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