Common Fixed Point Theorem Sin $\mathfrak{R}$-Partial B-Metricspaces

Dr. A. Leema Maria Prakasam , Dr. A. Marypriya Dharsini And Dr. A. Jennie Sebasty Pritha

PG and Research Department of Mathematics, Holy Cross College (Autonomous), Trichy-620 002 E-mail:leemamaria15@gmail.com

Abstract: This paper deals with the mappings having the unique common fixed pointtheorem in$\mathfrak{R}$-partialb-metricspaces withthe property of $\mathfrak{R}$-preservingand$\mathfrak{R}$-propertymap.

Keywords: Partial metric spaces, b metric spaces,$\mathfrak{R}$-partialb-metricspaces.

Mathematics Subject Classfication: Primary 47H10; Secondary 54H25

1 INTRODUTION AND PRELIMINARIES

Due to the fact that Fixed Point theorem performs a vital role for dissimilarmathematicalmodelstoacquireitssolutions.Ithasabroadrangeofapplicationsinvariousmathematicalfields.

In1989,Backhtininitiatedtheideaofb-metricspace.Theconceptof"Common Fixed point theorems in b-metric space" was further extended byS.Czerwik[1] in the year 1993.Czerwik discovered to generalize the view ofcontraction mapping principle. In 1994, the idea of Partial metric space wasdiscovered by Matthews[2].In 2014, The concept of "Partial metric spaces" was further extended to "Partial b-metric spaces" by Shukla[3] and obtained some results in fixed point theorems. Newly, Gordgiet al. [4] discovered the concept of orthogonal sets and some results inFixed Point for contraction mappings. Presently, in 2020 Muhammad UsmanAli, YajingGuo, FahimUddin, HassenAydi, Khalil JavedandZhenhua Ma[8]steppedfurtherinthefieldofcommonfixedpointtheoremsforproposedcontractionin$\mathfrak{R}$–partialb-metricspace.

A diverse number of researchers took intense attention in the generalizedversion ofmetric spacesas well asfor common fixedpointtheorems.

Definition 1.1[1]

LetSbea nonemptysetand g≥1.Suppose a mappingd :S×S→ $\mathbb{R}^+$

satisfiesthefollowingconditionsforalls,t,w∈S:

(A1) d(s, t) = 0 if and only if s = t ;(A2)

d(s,t)= d(t,s);

(A3) d(s, t)≤ g[d(s,w)+d(w,t)].

Thendisknownasb-metriconS,and(S,d)iscalledab-metricspacewith coefficient g.

Definition1.2[2]

LetSbeanonemptyset.Letδ:S×S→$\mathbb{R}^+$satisfythefollowingconditionsforall s,t,w∈S:

(B1) s =t ifandonlyifδ(s,s) =δ(s,t)=δ(t,t);

(B2) δ(s, s)≤δ(s, t);

(B3) δ(s, t)=δ(t,s);

(B4) δ(s, t) ≤ δ(s, w) + δ(w, t) – δ(w, w)Then(S,δ) iscalleda partial metricspace.

Definition1.3[3]

Apartialb-metricS≠∅isafunctionσ:S×S⟶$\mathbb{R}^+$suchthatforall s,t,w∈S,andforsomeg≥1,wehave

(C1) s = t if and only if σ(s, s) = σ(s, t) = (t, t) ;

(C2) σ(s,s)≤σ(s,t)

(C3) σ(s,t)= (t,s);

(C4) σ(s,t) ≤g[σ(s,w)+σ(w,t)]– σ(w,w).

A partial metric space is denoted with (S, σ, g). The number t iscalled thecoefficientof(S,σ,g).

Remark1.4[3]

It is clear that each and every partial metric space is a partial b -metricspace having coefficient g =1 and each b -metric space is a partial b -metricspace with the identical constant and also having zero self-distance. However,theconverseofthisnotionnolongerholds.

Definition1.5[5]

LetSbeanonemptyset.Asubset$\mathfrak{R}$ofI^2iscalledabinaryrelationonS.Then,foranys,t∈S,wesaythat“sis$\mathfrak{R}$-relatedtot”,thatis,s$\mathfrak{R}$t,or
“srelatestotunder$\mathfrak{R}$”ifandonlyif(s,t)∈$\mathfrak{R}$.(s,t)∉$\mathfrak{R}$meansthat“sisnot$\mathfrak{R}$-relatedtot”or“sisnotrelatedtotunder$\mathfrak{R}$”.

Definition1.6[5]

Abinaryrelation$\mathfrak{R}$definedonanonemptysetSiscalled

(a) reflexiveif(s,s)∈$\mathfrak{R}$As∈S;

(b) irreflexiveif(s,s)∉$\mathfrak{R}$forsomes∈S;

(c) symmetricif(s,t)∈$\mathfrak{R}$implies(t,s)∈$\mathfrak{R}$∀s,t∈S;

(d) antisymmetricif(s, t)∈$\mathfrak{R}$and(t,s)∈$\mathfrak{R}$implys=t∀ s,t∈S;

(e) transitiveif(s, t)∈$\mathfrak{R}$and(t,w)∈$\mathfrak{R}$imply(i,w)∈$\mathfrak{R}$∀s,t,w∈S;

(f) preorderif$\mathfrak{R}$isbothreflexiveandtransitive;

(g) partialorderif$\mathfrak{R}$isreflexive,antisymmetric,andtransitive.

Definition1.7[6]

Let Sbeanonemptysetand let $\mathfrak{R}$beabinaryrelationonS.

(a) Asequence$\{s_m\}$is$\mathfrak{R}$-sequenceif∀ m∈ℕ,$s_m \mathfrak{R} s_{m+1}$

(b) AmapP:S⟶Siscalledan$\mathfrak{R}$-preservingif∀s,t∈S,s$\mathfrak{R}$timpliesPs$\mathfrak{R}$Pt

Definition1.8[6]

Let (S, d) be a metric space and $\mathfrak{R}$be a binary relation on S.

Then (S,d,$\mathfrak{R}$)iscalledan$\mathfrak{R}$-metricspace.

Definition1.9[7]

AmappingP:S⟶Sis$\mathfrak{R}$-continuousats_0∈Sifforevery$\mathfrak{R}$–sequence$\{s_m\}_m$ ∈ℕinSwith$s_m \longrightarrow s_0$,wegetP(s_m)⟶P(s_0).Thus,Pis $\mathfrak{R}$-continuousonSif Pis$\mathfrak{R}$-continuousateverys_0∈S.

Definition1.10[8]

LetS ≠∅andbe a reflexive binary relationonS,which is denotedas(S,$\mathfrak{R}$).Amap$\sigma_{\mathfrak{R}}$:S×S⟶$\mathbb{R}^+$iscalledan$\mathfrak{R}$-partialb-metricthesetS,ifthe followingconditions are satisfied foralls, t, w∈Switheither (s $\mathfrak{R}$t or t $\mathfrak{R}$s),either(s$\mathfrak{R}$worw$\mathfrak{R}$s)andeither(w$\mathfrak{R}$tort$\mathfrak{R}$w)

(D1) s=tifandonlyif$\sigma_{\mathfrak{R}}(s,s)=\sigma_{\mathfrak{R}}(s,t)=\sigma_{\mathfrak{R}}\mathfrak{R}(t,t)$;

(D2) $\sigma_{\mathfrak{R}}(s,s)\leq\sigma_{\mathfrak{R}}(s,t)$;

(D3) $\sigma_{\mathfrak{R}}(s,t)=\sigma_{\mathfrak{R}}(t,s)$;

(D4) $\sigma_{\mathfrak{R}}(s,t)\leq g[\sigma_{\mathfrak{R}}(s,w)+\sigma_{\mathfrak{R}}(w,t)]-\sigma_{\mathfrak{R}}(w,w)$.whereg≥1.

Then,(S,$\mathfrak{R}$,$\sigma_{\mathfrak{R}}$,g)iscalled$\mathfrak{R}$-partialb-metricspacewiththecoefficientg≥1.

Definition1.11[8]

Let$\{s_m\}$bea$\mathfrak{R}$-sequencein(S,$\mathfrak{R}$,$\sigma_{\mathfrak{R}}$,g)thatis,$s_m \mathfrak{R} s_{m+1}$ or $s_{m+1} \mathfrak{R} s_m$ forevery m∈ℕ.Then,

(a) $\{s_m\}$isaconvergentsequencetosome s∈Siflim$_{m\to\infty}\sigma_{\mathfrak{R}}(s_m,s)=\sigma_{\mathfrak{R}}(s,s)ands_m \mathfrak{R}$sforeachm≥k.

(b) $\{s_m\}$isCauchyiflim$_{m,n\to\infty}\sigma_{\mathfrak{R}}(s_m,s_n)$existsandisfinite.

Definition1.12[8]

Let$(S,\mathfrak{R},\sigma_{\mathfrak{R}},g)$issaidtobe$\mathfrak{R}$–completeifforeveryCauchy$\mathfrak{R}$–sequenceinS,thereiss∈Swith$\lim_{m,n\to\infty}\sigma_{\mathfrak{R}}(s_m, s_n)=\lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, s)=\sigma_{\mathfrak{R}}(s,s)ands_n\ \mathfrak{R}s$foreachm ≥k.

Definition1.13[8]

We say that P:S⟶SisanR-propertymap,ifforanyiterative$\mathfrak{R}$–sequence

$\{s_m : s_m = P^m s, s\in S\}in(S,\mathfrak{R},\sigma_{\mathfrak{R}},g)$with$\lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, s)=\sigma_{\mathfrak{R}}(s,s)$,$s_m\ \mathfrak{R}s$for

Somem≥kand$\lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, Ps)\le\sigma_{\mathfrak{R}}(s,s)$,wehavethat$s\mathfrak{R}PtorPs\mathfrak{R}s$

2 MAINRESULT

Theorem2.1:

Let $(S,\mathfrak{R},\sigma_{\mathfrak{R}},g)$bean$\mathfrak{R}$-complete$\mathfrak{R}$-partialb-metricspacewith the coefficient g≥1andpermit$s_0\in S$besuchthat$s_0\ \mathfrak{R}t$foreveryt∈S. Let P:S⟶Sbe an$\mathfrak{R}$-preservingand$\mathfrak{R}$-propertymapsatisfyingthefollowing:

$$\sigma_{\mathfrak{R}}(Ps, Pt)\le k[\sigma_{\mathfrak{R}}(s,Pt)+\sigma_{\mathfrak{R}}(t,Ps)]\ \text{foralls,t}\in S\ \text{withs}\ \mathfrak{R}t, \tag{1}$$

wherek $\in[0,1/g)$.Then,Shaveafixedpoint$s^*\in S$and$\sigma_{\mathfrak{R}}(s^*,s^*)=0$.

Proof:

As$s_0\in S$issuchthat$s_0\ \mathfrak{R}t$foreveryt∈S,then by using the -preservingnature of P, weconstructan$\mathfrak{R}$-sequence$\{s_m\}$suchthat$s_m = s_{m-1} = p^m s_0$ and$s_{m-1}\ \mathfrak{R}s_m$ foreverym$\in\mathbb{N}$.

Weconsider$s_m\ne s_{m+1}$ foreverym$\in\mathbb{N}\cup\{0\}$.

Then,by(1),weobtain

$\sigma_{\Re}(s_m, s_{m+1}) = \sigma_{\Re}(Ps_{m-1}, s_m) \le k[\sigma_{\Re}(s_{m-1}, Ps_m) + \sigma_{\Re}(s_m, Ps_{m-1})]$ for all $m \in \mathbb{N}$ (2)

Thisinequalitygives,

$\sigma_{\Re}(s_m, s_{m+1}) \le k^m[\sigma_{\Re}(s_0, Ps_1) + \sigma_{\Re}(s_1, Ps_0)]$ for all $m \in \mathbb{N}$ (3)

Wewillconsider anrandomintegerm≥1,n ≥1withn >mandalsousing(D4)on(3), weget

$$\sigma_{\Re}(s_m, s_n) \le g[\sigma_{\Re}(s_m, s_{m+1}) + \sigma_{\Re}(s_{m+1}, s_n)] - \sigma_{\Re}(s_{m+1}, s_{m+1})$$

$$\le g\sigma_{\Re}(s_m, s_{m+1}) + g^2[\sigma_{\Re}(s_{m+1}, s_{m+2}) + \sigma_{\Re}(s_{m+2}, s_n)] - \sigma_{\Re}(s_{m+2}, s_{m+2})$$

$$\le g\sigma_{\Re}(s_m, s_{m+1}) + g^2\sigma_{\Re}(s_{m+1}, s_{m+2}) + g^3[\sigma_{\Re}(s_{m+2}, s_{m+2}) + \ldots + g^{n-m}\sigma_{\Re}(s_{n-1}, s_n)$$

$$\le g\, k^m[\sigma_{\Re}(s_0, Ps_1) + \sigma_{\Re}(s_1, Ps_0)] + g^2k^{m+1}[\sigma_{\Re} s_0, s_1) + \sigma_{\Re}(s_1, Ps_0)] + g^3k^{m+2}$$

$$[\sigma_{\Re}(s_0, Ps_1) + \sigma_{\Re}(s_1, Ps_0)] + \ldots + g^{n-m}k^{n-1}[\sigma_{\Re}(s_0, Ps_1) + \sigma_{\Re}(s_1, Ps_0)]$$

$$\le gk^m[1 + gk + (gk)^2 + \ldots][\sigma_{\Re}(s_0, Ps_1) + \sigma_{\Re}(s_1, Ps_0)]$$

$$= \frac{gk}{1-gk}[\sigma_{\Re}(s_0, Ps_1)) + \sigma_{\Re}(s_1, Ps_0)]$$

Whilek∈[0, 1/g)andg≥1,itcomesfromtheaboveinequitythat

$$\lim_{m\to\infty}\sigma_{\Re}(s_m, s_n) = 0$$

Hence,$\{s_m\}$is aCauchy$\mathfrak{R}$-sequence.SinceSis$\mathfrak{R}$-complete, there exists $s^* \in S$suchthat,$\lim_{m,n\to\infty}\sigma_{\Re}(s_m, s_n) = \lim_{m\to\infty}\sigma_{\Re}(s_m, s^*) = \sigma_{\Re}(s^*, s^*)ands_m\, \mathfrak{R}\, s^*$forevery

m≥k (forsome valueofk).Thusfromthe above,we get

$0 = \lim_{m,n\to\infty}\sigma_{\Re}(s_m, s_n) = \lim_{m\to\infty}\sigma_{\Re}(s_m, s^*) = \sigma_{\Re}(s^*, s^*)ands_m\, \mathfrak{R}\, s^*$forevery

m≥k.While$s_m\, \mathfrak{R}\, s^*$foreverym≥k,from(1),weobtain

$$\sigma_{\Re}(Ps_m, Ps^*) \le k[\sigma_{\Re}(s_m, Ps^*) + \sigma_{\Re}(s^*, Ps_m)] \quad (5)$$

This inequality and therefore above findings imply

$$\lim_{m\to\infty}\sigma_{\Re}(s_{m+1}, Ps^*) \le k[\sigma_{\Re}(s^*, s^*) + \sigma_{\Re}(s^*, s_{m+1})]$$

$$\le k[\sigma_{\Re}(s^*, s^*) + \sigma_{\Re}(s^*, s^*)]$$

=0

While S is a $\mathfrak{R}$-property map, hence we obtain $s^*\mathfrak{R}Ps^*$ or $Ps^*\mathfrak{R}s^*$. Without loss of generality, we are going to take $s^*\mathfrak{R}Ps^*$. Then, by using $(D4)$ with (1), we obtain the subsequent for every $m\geq k$.

$$\sigma_{\mathfrak{R}}(s^*,Ps^*)\leq g\sigma_{\mathfrak{R}}(s^*,s_{m+1})+g\sigma_{\mathfrak{R}}(s_{m+1},Ps^*)-\sigma_{\mathfrak{R}}(s_{m+1},s_{m+1})$$

$$\leq g\sigma_{\mathfrak{R}}(s^*,s_{m+1})+g\sigma_{\mathfrak{R}}(Ps_m,Ps^*)$$

$$\leq g\sigma_{\mathfrak{R}}(s^*,s_{m+1})+g\,k[\sigma_{\mathfrak{R}}(s_m,Ps^*)+\sigma_{\mathfrak{R}}(s^*,Ps_m)]$$

When m tends to infinity, the above inequality gives $\sigma_{\mathfrak{R}}(s^*,Ps^*)=0$.

Therefore we obtain $\sigma_{\mathfrak{R}}(s^*,Ps^*)=0$, $\sigma_{\mathfrak{R}}(s^*,s^*)=0$ and $\sigma_{\mathfrak{R}}(Ps^*,Ps^*)=0$.

Hence, $s^*=Ps^*$, that is, s^* is a fixed point

Theorem2:

Let $(S,\mathfrak{R},\sigma_{\mathfrak{R}},g)$ be a $\mathfrak{R}$-complete $\mathfrak{R}$-partial b-metric space having constant $g\geq 1$ and permit $s_0\in S$ be such that $s_0\,\mathfrak{R}\,t$ for every $t\in S$. Permit $P:S\longrightarrow S$ be $\mathfrak{R}$-preserving and an $\mathfrak{R}$-property map satisfying the subsequent

$$\sigma_{\mathfrak{R}}(Ps,Pt)\leq a\sigma_{\mathfrak{R}}(s,Ps)+b\sigma_{\mathfrak{R}}(s,Pt)+c\sigma_{\mathfrak{R}}(s,t) \text{ for all } s,t\in S \text{ with } i\mathfrak{R}j, \qquad (1)$$

where $a,b,c\in[0,1/g)$. Then, S have a fixed point $s^*\in S$ and $\sigma_{\mathfrak{R}}(s^*,s^*)=0$.

Proof:

As $s_0\in S$ is such that $s_0\,\mathfrak{R}\,t$ for every $t\in S$, then by using the $\mathfrak{R}$-preserving

nature of P, we construct an $\mathfrak{R}$-sequence $\{s_m\}$ such that $s_m=Ps_{m-1}=p^m s_0$

and $s_{m-1}\,\mathfrak{R}\,s_m$ for every $m\in\mathbb{N}$. We inspect $s_m\neq s_{m+1}$ for every $m\in\mathbb{N}\cup\{0\}$.

Then,by(1),weobtain

$$\sigma_{\mathfrak{R}}(s_m, s_{m+1}) = \sigma_{\mathfrak{R}}(s_{m+1}, Ps_m) \leq a\sigma_{\mathfrak{R}}(s_{m+1}, Ps_{m-1}) + b\sigma_{\mathfrak{R}}(s_{m-1}, Ps_m) + c\sigma_{\mathfrak{R}}(s_{m-1}, s_m)] \quad (2)$$

forallm$\in \mathbb{N}$.Thisinequitygives,

$$\sigma_{\mathfrak{R}}(s_m, s_{m+1}) \leq a^m\sigma_{\mathfrak{R}}(s_0, Ps_0) + b^m\sigma_{\mathfrak{R}}(s_0, Ps_1) + c^m\sigma_{\mathfrak{R}}(s_0, s_1) \quad (3)$$

forallm$\in\mathbb{N}$.wewillinspectanrandomintegerm$\geq$1,n$\geq$1withn>manduse$(D4)$on(3),weget

$$\sigma_{\mathfrak{R}}(s_m, s_n) \leq g[\sigma_{\mathfrak{R}}(s_m, s_{m+1}) + \sigma_{\mathfrak{R}}(s_{m+1}, s_m)] - \sigma_{\mathfrak{R}}(s_{m+1}, s_{m+1})$$

$$\leq g\sigma_{\mathfrak{R}}(s_m, s_{m+1}) + g^2[\sigma_{\mathfrak{R}}(s_{m+1}, s_{m+2}) + \sigma_{\mathfrak{R}}(s_{m+2}, s_n)] - \sigma_{\mathfrak{R}}(s_{m+1}, s_{m+1})$$

$$\leq g\sigma_{\mathfrak{R}}(s_m, s_{m+1}) + g^2\sigma_{\mathfrak{R}}(s_{m+1}, s_{m+2}) + g^3[\sigma_{\mathfrak{R}}(s_{m+2}, s_{m+3})] + \ldots\ldots + g^{n-m}\sigma_{\mathfrak{R}}(s_{n-1}, s_n)$$

$$\leq g[a^m\sigma_{\mathfrak{R}}(s_0, Ps_0) + b^m\sigma_{\mathfrak{R}}(s_0, Ps_0) + c^m\sigma_{\mathfrak{R}}(s_0, Ps_0)] +$$

$$g^2[a^{m+1}\sigma_{\mathfrak{R}}(s_0, Ps_0) + b^{m+1}\sigma_{\mathfrak{R}}(s_0, Ps_0) + c^{m+1}\sigma_{\mathfrak{R}}(s_0, Ps_0)] +$$

$$g^3[a^{m+2}\sigma_{\mathfrak{R}}(s_0, Ps_0) + b^{m+2}\sigma_{\mathfrak{R}}(s_0, Ps_1) + c^{m+2}\sigma_{\mathfrak{R}}(s_0, Ps_0)] + \ldots\ldots +$$

$$g^{n-m}[a^{n-1}\sigma_{\mathfrak{R}}(s_0, Ps_0) + b^{n-1}\sigma_{\mathfrak{R}}(s_0, Ps_0) + c^{n-1}\sigma_{\mathfrak{R}}(s_0, Ps_0)]$$

$$\leq g a^m [1+ga+(ga)^2+\ldots]\sigma_{\mathfrak{R}}(s_0, s_1) + gb^m[1+gb+(gb)^2+\ldots]\sigma_{\mathfrak{R}}(s_0, s_1) + gc^m[1+gc+(gc)^2+]\sigma_{\mathfrak{R}}(s_0, s_1)$$

$$= \frac{ga^m}{1-ga}\sigma_{\mathfrak{R}}(s_0, s_1) + \frac{gb^m}{1-gb}\sigma_{\mathfrak{R}}(s_0, s_1) + \frac{gc^m}{1-gc}\sigma_{\mathfrak{R}}(s_0, s_1)$$

Whilea,b,c$\in$[0,1/g)andg$\geq$1,itcomesfromaboveinequitythat,

$$\lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, s_n) = 0$$

Hence,$\{s_m\}$is a Cauchy$\mathfrak{R}$-sequence.HenceSis$\mathfrak{R}$-complete,thereexistss*$\in$Ssuchthat,$\lim_{m,n\to\infty}\sigma_{\mathfrak{R}}(s_m, s_n) = \lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, s^*) = \sigma_{\mathfrak{R}}(s^*, s^*)ands_m \mathfrak{R} s^*$foreach

m$\geq$a,b,c(forsomevalue ofa,b,c).Then,fromabove,weget

$0 = \lim_{m,n\to\infty}\sigma_{\mathfrak{R}}(s_m, s_n) = \lim_{m\to\infty}\sigma_{\mathfrak{R}}(s_m, s^*) = \sigma_{\mathfrak{R}}(s^*, s^*)ands_m \mathfrak{R} s^*$forevery m≥a,b,c.While$s_m$ $\mathfrak{R}$s*foreverym≥a,b,cfrom(1),weobtain

$\sigma_{\Re}(Ps_m, Ps^*) \leq a\sigma_{\Re}(s_m, Ps_m) + b\sigma_{\Re}(s_m, Ps^*) + c\sigma_{\Re}(s_m, s^*)$ (5)

Thisinequityandthereforeabovefindingsimplicit

$\lim_{m\to\infty}\sigma_{\Re}(s_{m+1}, Ps^*) \leq a\sigma_{\Re}(s^*, s_{m+1}) + b\sigma_{\Re}(s^*, s_{m+1}) + c\sigma_{\Re}(s^*, s^*)$

$\leq a\sigma_{\Re}(s^*, s^*) + b\sigma_{\Re}(s^*, s^*) + c\sigma_{\Re}(s^*, s^*)$

$=0$

WhileSisan$\mathfrak{R}$-propertymap,soweobtain$s^*\mathfrak{R}Ps^*$(or)$Ps^*\mathfrak{R}s^*$.Withoutlossofgenerality,wewilltake$s^*\mathfrak{R}Ps^*$.Then,by utilizing(D4)with(1), weobtainthesubsequent foreverym≥a, b,c

$\sigma_{\Re}(s^*, Ps^*) \leq g\sigma_{\Re}(s^*, s_{m+1}) + g\sigma_{\Re}(s_{m+1}, Ps^*) - \sigma_{\Re}(s_{m+1}, s_{m+1})$

$\leq g\sigma_{\Re}(s^*, s_{m+1}) + g\sigma_{\Re}(Ps_m, Ps^*)$

$\leq g\sigma_{\Re}(s^*, s_{m+1}) + g[a\sigma_{\Re}(s_m, Ps_m) + b\sigma_{\Re}(s_m, Ps^*) + c\sigma_{\Re}(s_m, s^*)]$

Whenmtendstoinfinity,theaboveinequitygives$\sigma_{\Re}(s^*, Ps^*)=0$.Henceweobtain

$\sigma_{\Re}(s^*, Ps^*)=0$,$\sigma_{\Re}(s^*, s^*)= 0$ and$\sigma_{\Re}(Ps^*, Ps^*)=0$.Hence,$s^*=Ps^*$, that is ,$s^*$is a fixed pointofP.

Corollary 3.1:

Admit(S,, $\sigma_{\Re}$, g)bea$\mathfrak{R}$-complete$\mathfrak{R}$-partialb -metric space havingconstantg≥1andpermits_0 ∈Sbesuchthat$s_0\mathfrak{R}t$foreveryt∈S.Permit$P:S\longrightarrow S$be$\mathfrak{R}$-preservingandan$\mathfrak{R}$-propertymapsatisfyingthesubsequent

$\sigma_{\Re}(Ps,Pt) \leq a\sigma_{\Re}(s,Ps) + b[\sigma_{\Re}(s,Pt) + \sigma_{\Re}(s,t)] + c[\sigma_{\Re}(t, Ps) + \sigma_{\Re}(t,Pt)]$

foralls,t∈Swith$s\mathfrak{R}t$,wherea,b,c∈[0,1/g).

Then,Phaveafixedpoint $s^*\in S$and$\sigma_{\Re}(s^*, s^*)=0$.

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