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Optimization Of Fuzzy Inventory Model Using Lagragian Method In Decagonal Fuzzy Number

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Abstract

In this paper, a Inventory model for an item with different objective and constraints under uncertain environment. All the cost parameters involved in the study are represented by fuzzy numbers. As a result the total cost function is ultimately obtained as fuzzy. Later on this cost function is defuzzified the decagonal fuzzy number to obtain a crisp cost function with allowed variations. In this study fuzzy inventory problems are solved by using Lagrangian method with beta distribution . To determine the optimum value which is the minimize total cost. Finally, numerical examples are discussed with the proposed model.

Keywords : Decagonal fuzzy number, Beta distribution , Lagrangian method

INTRODUCTION

Operation research use logical analysis and analytical techniques to study the behavior of a system. It is a scientific approach to problem solving for executive decision making which requires the formulation of mathematical, economic and statistical models for decision and control problems to deal with situations arising out of risk and uncertainty.

Inventory was first introduced by Harris 1915. It is very important in the current scenario. For future production and sales it constitutes goods in more stock. Components of inventory are currency, the ready packed goods for sales.

A fuzzy set can be mathematically described by assigning numbers. The membership grades are present in real values ranging from o to 1 in a closed interval. A fuzzy set is a collection of distinct elements with varying degrees of relevance, as described here.

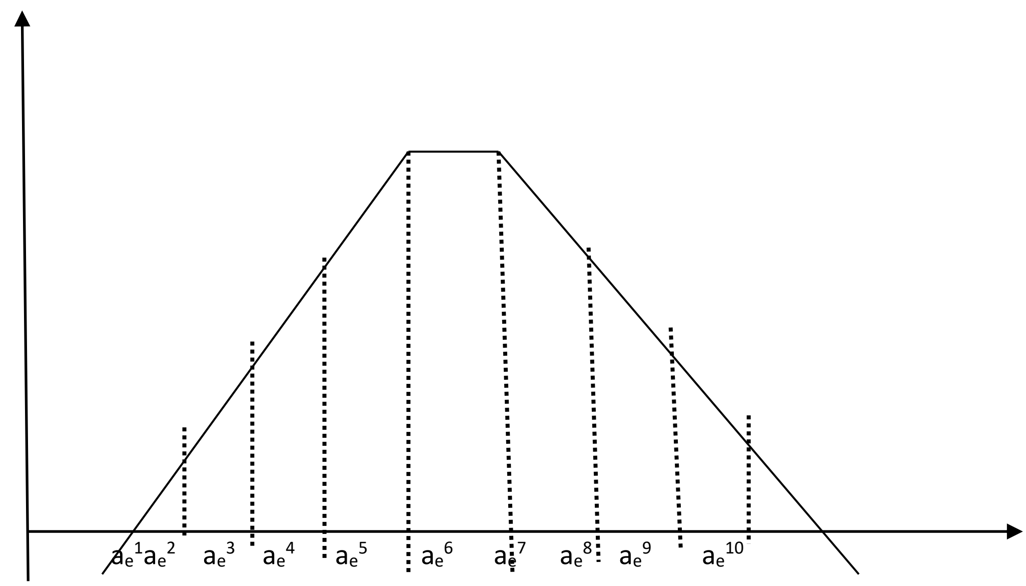
Hence, discussed about optimization of fuzzy inventory model using lagragian method in decagonal fuzzy number with beta distribution. The parameters holding cost, ordering cost, cycle plan and demand are decagonal fuzzy number. . To determine the optimum value which is the minimize total cost. The proposed method illustrated with numerical examples.

DECAGONAL FUZZY NUMBER

A Decagonal fuzzy number $\widetilde{D}$ can be defined as$\left(\alpha_{eb}^{1}, \alpha_{eb}^{2}, \alpha_{eb}^{3}, \alpha_{eb}^{4}, \alpha_{eb}^{5}, \alpha_{eb}^{6}, \alpha_{eb}^{7}, \alpha_{eb}^{8}, \alpha_{eb}^{9}, \alpha_{eb}^{10}\right)$

$$
\mu_{\tilde{A}_h}(x) = \begin{cases}
\frac{1}{4}\left(\frac{x - a_{eb}^{1}}{a_{eb}^{2} - a_{eb}^{1}}\right), & \text{if} a_{eb}^{1} \leq x \leq a_{eb}^{2} \\
\frac{1}{4} + \frac{1}{4}\left(\frac{x - a_{eb}^{2}}{a_{eb}^{3} - a_{eb}^{2}}\right), & \text{if} a_{eb}^{2} \leq x \leq a_{eb}^{3} \\
\frac{1}{2} + \frac{1}{4}\left(\frac{x - a_{eb}^{3}}{a_{eb}^{4} - a_{eb}^{3}}\right), & \text{if} a_{eb}^{3} \leq x \leq a_{eb}^{4} \\
\frac{3}{4} + \frac{1}{4}\left(\frac{x - a_{eb}^{4}}{a_{eb}^{5} - a_{eb}^{4}}\right), & \text{if} a_{eb}^{4} \leq x \leq a_{eb}^{5} \\
1, & \text{if} a_{eb}^{5} \leq x \leq a_{eb}^{6} \\
1 - \frac{1}{4}\left(\frac{x - a_{eb}^{6}}{a_{eb}^{7} - a_{eb}^{6}}\right), & \text{if} a_{eb}^{6} \leq x \leq a_{eb}^{7} \\
\frac{3}{4} - \frac{1}{4}\left(\frac{x - a_{eb}^{7}}{a_{eb}^{8} - a_{eb}^{7}}\right), & \text{if} a_{eb}^{7} \leq x \leq a_{eb}^{8} \\
\frac{1}{2} + \frac{1}{4}\left(\frac{x - a_{eb}^{8}}{a_{eb}^{9} - a_{eb}^{8}}\right), & \text{if} a_{eb}^{8} \leq x \leq a_{eb}^{9} \\
\frac{1}{4}\left(\frac{a_{eb}^{10} - x}{a_{eb}^{10} - a_{eb}^{9}}\right), & \text{if} a_{eb}^{9} \leq x \leq a_{eb}^{10} \\
0, & \text{otherwise}
\end{cases}
$$

GRAPHICAL REPRESENTATION IN DECAGONAL NUMBER



LAGRAGIAN METHOD

The solution of a constrained optimization problem can often be found by using the so-called Lagragian method. We define the Lagragian as

$$L(x, \lambda) = f(x) + \lambda(b - g(x)).$$

DEFUZZIFICATION OF DECAGONAL FUZZY NUMBER

$$\widetilde{B} = \frac{-5S_1 + 6S_2 + 6S_3 + 6S_4 - S_5 - S_6 + 6S_7 + 6S_8 - 7S_9 + 8S_{10}}{24}$$

Arithmetic operations on Decagonal fuzzy number

Let (α_e, α_b),and $\left(\gamma_g, \gamma_j\right)$ are the two terms $\left(\alpha_{eb}^1, \alpha_{eb}^2, \alpha_{eb}^3, \alpha_{eb}^4, \alpha_{eb}^5, \alpha_{eb}^6, \alpha_{eb}^7, \alpha_{eb}^8, \alpha_{eb}^9, \alpha_{eb}^{10}, \right) \left(\gamma_{gj}^1, \gamma_{gj}^2, \gamma_{gj}^3, \gamma_{gj}^4, \gamma_{gj}^5, \gamma_{gj}^6, \gamma_{gj}^7, \gamma_{gj}^8, \gamma_{gj}^8, \gamma_{gj}^{10}\right)$

Addition

$$(\alpha_e, \alpha_b) + \left(\gamma_g, \gamma_j\right) =$$

$$\left(\alpha_{eb}^1, \alpha_{eb}^2, \alpha_{eb}^3, \alpha_{eb}^4, \alpha_{eb}^5, \alpha_{eb}^6, \alpha_{eb}^7, \alpha_{eb}^8, \alpha_{eb}^9, \alpha_{eb}^{10}, \right) + \left(\gamma_{gj}^1, \gamma_{gj}^2, \gamma_{gj}^3, \gamma_{gj}^4, \gamma_{gj}^5, \gamma_{gj}^6, \gamma_{gj}^7, \gamma_{gj}^8, \gamma_{gj}^8, \gamma_{gj}^{10}\right)$$

Subtraction

$$(\alpha_e, \alpha_b) - \left(\gamma_g, \gamma_j\right)$$

$$= \left(\alpha_{eb}^1, \alpha_{eb}^2, \alpha_{eb}^3, \alpha_{eb}^4, \alpha_{eb}^5, \alpha_{eb}^6, \alpha_{eb}^7, \alpha_{eb}^8, \alpha_{eb}^9, \alpha_{eb}^{10}, \right) - \left(\gamma_{gj}^1, \gamma_{gj}^2, \gamma_{gj}^3, \gamma_{gj}^4, \gamma_{gj}^5, \gamma_{gj}^6, \gamma_{gj}^7, \gamma_{gj}^8, \gamma_{gj}^8, \gamma_{gj}^{10}\right)$$

Multiplication $(\alpha_e, \alpha_b) \times \left(\gamma_g, \gamma_j\right) = \left(\alpha_{eb}^1, \alpha_{eb}^2, \alpha_{eb}^3, \alpha_{eb}^4, \alpha_{eb}^5, \alpha_{eb}^6, \alpha_{eb}^7, \alpha_{eb}^8, \alpha_{eb}^9, \alpha_{eb}^{10}, \right) \times \left(\gamma_{gj}^1, \gamma_{gj}^2, \gamma_{gj}^3, \gamma_{gj}^4, \gamma_{gj}^5, \gamma_{gj}^6, \gamma_{gj}^7, \gamma_{gj}^8, \gamma_{gj}^8, \gamma_{gj}^{10}\right)$

Division $(\alpha_e, \alpha_b) / \left(\gamma_g, \gamma_j\right) = \left(\alpha_{eb}^1, \alpha_{eb}^2, \alpha_{eb}^3, \alpha_{eb}^4, \alpha_{eb}^5, \alpha_{eb}^6, \alpha_{eb}^7, \alpha_{eb}^8, \alpha_{eb}^9, \alpha_{eb}^{10}, \right) / \left(\gamma_{gj}^1, \gamma_{gj}^2, \gamma_{gj}^3, \gamma_{gj}^4, \gamma_{gj}^5, \gamma_{gj}^6, \gamma_{gj}^7, \gamma_{gj}^8, \gamma_{gj}^8, \gamma_{gj}^{10}\right)$

NOTATIONS

V = Order quantity for per length of a cycle

W = Inventory item in a total demand

Z = Ordering cost for one month

Y = Cycle plan for per month

U = Holding cost for per month

Q_s = Total inventory item of the minimum product

$\widetilde{W}$ = Fuzzy inventory item in annual demand

$\widetilde{Y}$ = Cycle plan in fuzzy product

$\tilde{Z}$ = Fuzzy ordering cost in per unit period

$\widetilde{U}$ = Fuzzy holding cost per month

$\widetilde{V}$ = Fuzzy order quantity per month

$\widetilde{Q_S}$ = Fuzzy total inventory product

Assumption

Assumption are considered to be as:

- Demand is considered as a nature in fuzzy inventory
- Time is always to be plan as a constant value
- Holding cost is considered as a nature in fuzzy inventory
- Ordering cost are also nature

Mathematical Model Formulation

$T_I = \frac{JO}{X} + \frac{XPE}{2}$ ----------------------(1)

We have to find the optimum solution ,differentiate we get$\frac{\partial(\tilde{T}_I)}{\partial X}$

$X^* = \sqrt{\frac{2JO}{PE}}$ -----------------------(2)

Now we using the formula of decagonal number of defuzzification

$$(\widetilde{D}) = \frac{-5S_1 + 6S_2 + 6S_3 + 6S_4 - S_5 - S_6 + 6S_7 + 6S_8 - 7S_9 + 8S_{10}}{24}$$

$$X^* = \sqrt{\frac{2\begin{bmatrix}-5J_1O_1+6J_2O_2+6J_3O_3+6J_4O_4-J_5O_5-\\J_6O_6+6J_7O_7+6J_8O_8-7J_9O_9+8J_{10}O_{10}\end{bmatrix}}{\begin{matrix}-5P_1E_1+6P_2E_2+6P_3E_3+6P_4E_4-P_5E_5-\\P_6E_6+6P_7E_7+6P_8E-7PE_9+8P_{10}E_{10}\end{matrix}}} \text{------------}(3)$$

$$(T_I) = \frac{1}{24}\begin{bmatrix}-5\left(\frac{J_1O_1}{X_{10}}+\frac{XP_1E_1}{2}\right)+6\left(\frac{J_2O_2}{X_9}+\frac{XP_2E_2}{2}\right)+6\left(\frac{J_3O_3}{X_8}+\frac{XP_3E_3}{2}\right)\\+6\left(\frac{J_4O_4}{X_7}+\frac{XP_4E_4}{2}\right)-\left(\frac{J_5O_5}{V_6}+\frac{XP_5E_5}{2}\right)-\left(\frac{J_6O_6}{X_5}+\frac{XP_6E_6}{2}\right)\\+6\left(\frac{J_7O_7}{X_4}+\frac{XP_7E_7}{2}\right)+6\left(\frac{J_8O_8}{X_3}+\frac{XP_8E_8}{2}\right)-7\left(\frac{J_9O_9}{X_2}+\frac{XP_9E_9}{2}\right)\\+8\left(\frac{J_{10}O_{10}}{X_1}+\frac{XP_{10}E_{10}}{2}\right)\end{bmatrix}$$

STEP 1 $\quad X_1 = \sqrt{\frac{-16J_{10}O_{10}}{5P_1E_1}}$

STEP 2

$$X_1 = X_2 = \sqrt{\frac{2(8J_{10}O_{10}-7J_9O_9)}{-5P_1E_1+6P_2E_2}}$$

Continuously the step is going on as by one to one finally the last step we get as,

STEP 10

$$X_1 = X_2 = X_3 = X_4 = X_5 = X_6 = X_7 = X_8 = X_9 = X_{10} =$$

$$\sqrt{\frac{2[8J_{10}O_{10}-7J_9O_9+6J_8O_8+6J_7O_7-J_6O_6-J_5O_5+6J_4O_4+6J_3O_3+6J_2O_2-5J_1O_1]}{-5P_1E_1+6P_2E_2+6P_3E_3+6P_4E_4-P_5E_5-P_6E_6+6P_7E_7+6P_8E_8-7P_9E_9+8P_{10}E_{10}}}$$

FUZZY INVENTORY MODEL USING LAGRAGIAN METHOD IN DECAGONAL FUZZY NUMBER

The total cost is given by

$$Q_s = \frac{WU}{V} + \frac{VZY}{2}$$

Differentiate partially with respect to V

$$V^* = \sqrt{\frac{2WU}{ZY}}\,\tilde{Q}_S == \frac{\tilde{W}\tilde{U}}{V} + \frac{V\tilde{Z}\tilde{Y}}{2}$$

Let $\widetilde{W},\widetilde{U},\tilde{Z},\widetilde{Y}$ are non negative decagonal fuzzy number

$$\widetilde{W} = (W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10},)$$

$$\widetilde{U} = (U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, U_9, U_{10})$$

$$\tilde{Z} = \left(Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_{9,} Z_{10}\right)$$

$$\widetilde{Y} = (Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10})$$

$$\widetilde{Q}_S = \begin{bmatrix} \frac{W_1U_1}{V} + \frac{VZ_1Y_1}{2}, \frac{W_2U_2}{V} + \frac{Vz_2y_2}{2}, \frac{W_3U_3}{V} + \frac{VZ_3Y_3}{2}, \frac{W_4U_4}{V} + \frac{VZ_4Y_4}{2}, \frac{W_5U_5}{V} + \\ \frac{VZ_5Y_5}{2}, \frac{W_6U_6}{V} + \frac{VZ_6Y_6}{2}, \frac{W_7U_7}{V} + \frac{VZ_7Y_7}{2}, \frac{W_8U_8}{V} + \frac{VZ_8Y_8}{2} + \frac{W_9U_9}{V}, \frac{VZ_9Y_9}{2} \\ \frac{W_{10}U}{V}, \frac{VZ_{10}Y_{10}}{2} \end{bmatrix}$$

Defuzzification for Beta distribution of decagonal fuzzy number is

$$\beta(\widetilde{B}) = \frac{-5S_1 + 6S_2 + 6S_3 + 6S_4 - S_5 - S_6 + 6S_7 + 6S_8 - 7S_9 + 8S_{10}}{24}$$

$$\beta(\widetilde{Q}_S) = \frac{1}{24v}\begin{bmatrix} -5W_1U_1 + 6W_2U_2 + 6W_3U_3 + 6W_4U_4 - W_5U_5 - \\ W_6U_6 + 6W_7U_7 + 6W_8U_8 - 7W_9U_9 + 8W_{10}U_{10} \end{bmatrix} + \frac{1}{48}\begin{bmatrix} -5Z_1Y_1 + 6Z_2Y_2 + 6Z_3Y_3 + 6Z_4Y_4 - Z_5Y_5 - \\ Z_6Y_6 + 6Z_7Y_7 + 6Z_8Y_8 - 7Z_9Y_9 + 8Z_{10}Y_{10} \end{bmatrix}$$

$$V^* = \sqrt{\frac{2\begin{bmatrix} -5W_1U_1 + 6W_2U_2 + 6W_3U_3 + 6W_4U_4 - W_5U_5 - \\ W_6U_6 + 6W_7U_7 + 6W_8U_8 - 7W_9U_9 + 8W_{10}U_{10} \end{bmatrix}}{\begin{matrix} -5Z_1Y_1 + 6Z_2Y_2 + 6Z_3Y_3 + 6Z_4Y_4 - Z_5Y_5 - \\ Z_6Y_6 + 6Z_7Y_7 + 6Z_8Y_8 - 7Z_9Y_9 + 8Z_{10}Y_{10} \end{matrix}}}$$

Now applying a lagragian method,

$\widetilde{Q}_S = \frac{\widetilde{W}\widetilde{U}}{V} + \frac{V\tilde{Z}\widetilde{Y}}{2}$ With $V = V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}$

$0 < V_1 < V_2 < V_3 < V_4 < V_5 < V_6 < V_7 < V_8 < V_9 < V_{10}$

$$\widetilde{Q}_S = \begin{bmatrix} \frac{W_1U_1}{V} + \frac{VZ_1Y_1}{2}, \frac{W_2U_2}{V} + \frac{Vz_2y_2}{2}, \frac{W_3U_3}{V} + \frac{VZ_3Y_3}{2}, \frac{W_4U_4}{V} + \frac{VZ_4Y_4}{2}, \frac{W_5U_5}{V} + \\ \frac{VZ_5Y_5}{2}, \frac{W_6U_6}{V} + \frac{VZ_6Y_6}{2}, \frac{W_7U_7}{V} + \frac{VZ_7Y_7}{2}, \frac{W_8U_8}{V} + \frac{VZ_8Y_8}{2} + \frac{W_9U_9}{V}, \frac{VZ_9Y_9}{2} \\ \frac{W_{10}U}{V}, \frac{VZ_{10}Y_{10}}{2} \end{bmatrix}$$

Now applying beta distribution method,

$$\beta(\widetilde{Q}_S) = \frac{1}{24}\left[\begin{array}{c} -5\left(\frac{W_1U_1}{V_{10}} + \frac{VZ_1Y_1}{2}\right) + 6\left(\frac{W_2U_2}{V_9} + \frac{VZ_2Y_2}{2}\right) + 6\left(\frac{W_3U_3}{V_8} + \frac{VZ_3Y_3}{2}\right) \\ +6\left(\frac{W_4U_4}{V_7} + \frac{VZ_4Y_4}{2}\right) - \left(\frac{W_5U_5}{V_6} + \frac{VZ_5Y_5}{2}\right) - \left(\frac{W_6U_6}{V_5} + \frac{VZ_6Y_6}{2}\right) \\ +6\left(\frac{W_7U_7}{V_4} + \frac{VZ_7Y_7}{2}\right) + 6\left(\frac{W_8U_8}{V_3} + \frac{VZ_8Y_8}{2}\right) - 7\left(\frac{W_9U_9}{V_2} + \frac{VZ_9Y_9}{2}\right) \\ +8\left(\frac{W_{10}Y_{10}}{V_1} + \frac{VZ_{10}Y_{10}}{2}\right) \end{array}\right]$$

Numerical Examples

Fuzzy model :

Let us consider the following data:

Y = 20 month, W = 300 year , Z = Rs.50/- per month, U = 100/- per month.

$V^* = \sqrt{\frac{2WU}{ZY}} = \sqrt{60} = 7.7 \cong 8$

$Q_S = \frac{WU}{V} + \frac{VZY}{2} = 7750$

Crisp model :

Let us consider the following data:

U = (95.1, 96.1, 97.1, 98.1, 99.1, 101.1, 102.1, 103.1, 104.1, 105.1)

Z = (43, 44, 45, 46, 47, 48, 49, 51, 52, 53)

Y = (13, 14, 15, 16, 17, 18, 19, 21, 22, 23)

W = (0, 50, 100, 150, 200, 250, 350, 400, 450, 500)

$$V_1 = \sqrt{\frac{2[8W_{10}U_{10} - 7W_9U_9 + 6W_8U_8 + 6W_7U_7 - W_6U_6 - W_5U_5 + 6W_4U_4 + 6W_3U_3 + 6W_2U_2 - 5W_1U_1]}{-5Z_1Y_1 + 6Z_2Y_2 + 6Z_3Y_3 + 6Z_4Y_4 - Z_5Y_5 - Z_6Y_6 + 6Z_7Y_7 + 6Z_8Y_8 - 7Z_9Y_9 + 8Z_{10}Y_{10}}}$$

$V_1 = \sqrt{\frac{1531220}{21460}} = 8.4 \cong 8$

$$\beta(\widetilde{Q}_S) = \frac{1}{24v}\left[\begin{array}{c} -5W_1U_1 + 6W_2U_2 + 6W_3U_3 + 6W_4U_4 - W_5U_5 - \\ W_6U_6 + 6W_7U_7 + 6W_8U_8 - 7W_9U_9 + 8W_{10}U_{10} \end{array}\right] +$$

$$\frac{1}{48}\begin{bmatrix}-5Z_1Y_1 + 6Z_2Y_2 + 6Z_3Y_3 + 6Z_4Y_4 - Z_5Y_5 - \\ Z_6Y_6 + 6Z_7Y_7 + 6Z_8Y_8 - 7Z_9Y_9 + 8Z_{10}Y_{10}\end{bmatrix}$$

= 4173 + 3577

$$\beta(\tilde{Q}_S) = 7750$$

CONCLUSION

In many real life cases, the decision data of human judgments with preferences are often vague so that the traditional ways of using crisp values are inadequate also using fuzzy numbers such as triangular, trapezoidal are not suitable in few case where the uncertainties arises in ten different points in such cases Decagonal Fuzzy Number can be used to solve the problems.

The optimization of fuzzy inventory model using lagragian method in decagonal fuzzy number with beta distribution. The parameters holding cost, ordering cost, cycle plan and demand are decagonal fuzzy number. The optimum results of fuzzy model were defuzzifiedand it will increase the total profit and also the fuzzy model permits flexibility in the system inputs. The notations are taken as decagonal fuzzy numbers, Lagrangian method has been applied with beta distribution.

The arithmetic operations of decagonal fuzzy numbers are also proposed to get the expected result.This fuzzy model assists in determining the optimal order quantity amidst the existing fluctuations to finding theminimum total cost for the inventory model derived both crisp and fuzzy sense with beta distribution

REFERENCES

1. J. Jon Arockiaraj and N. Sivasankari, A decagonal fuzzy number and its vertex method, International Journal of Mathematics and its Applications, Vol. 4, 283-292.
2. P. Kirtiwant, Ghadle and A. Priyanka, Pathade, Solving transportation problem with generalized hexagonal and generalized octagonal fuzzy numbers by ranking method, Global Journal of Pure and Applied Mathematics, Vol. 13, 6367-6376, 2017.
3. A.VictorDevadoss and A. Felix. (2015), A New Decagonal Fuzzy Number under Uncertain Linguistic Environment, International Journal of Mathematics And its Applications Volume 3, Issue 1 (2015), 89–97.
4. P. Rajarajeshwari, A.S. Sudha and R. Karthika, A new Operation on Hexagonal Fuzzy Number, International Journal of Fuzzy Logic Systems, 3(3)(2013), 15-26.

5. A.SahayaSudha and M. Revathy, A new ranking on hexagonal fuzzy numbers, International Journal of Fuzzy Logic Systems (IJFLS), Vol. 6 (2016), 1-8.
6. H.J Zimmermann, Fuzzy Set Theory and its Application, Fourth Edition, Springer, (2011).