

# Multi Objective Multi Item Fuzzy Trans Shipment Problem With Congestion Charge

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## ABSTRACT

In current scenario supply, demand and transportation cost per unit of the commodity in multi objective transshipment problem for multi items may be rarely specified precisely due to uncertain conditions. Vehicular emission generates harmful air pollutants in urban areas which leads to severe health hazardous. In this paper, a multi objective profit transshipment problem for multi items has been framed and solved in fuzzy environment, it emphasizes on fuzzy methodology to solve transshipment problem to minimize the travel time and maximize the profit by reducing emission charge during traffic congestion. Unit transportation cost, environmental protection cost, transportation time and congestion charge during peak time and non peak time, loading and unloading time, total supply and demand, selling and purchasing prices are all considered as triangular type-2 fuzzy numbers. Then the multi objective profit transshipment problem has been transformed into single objective transshipment problem by applying Fuzzy Goal programming Technique (FGPT) and Weighted Fuzzy Goal Programming Technique (WFGPT) and the corresponding model is solved using Generalized Reduced Gradient (GRG) method LINGO-(18.0). Numerical illustration has been given to show the efficiency of the proposed model.

**Keywords:** Type-2 triangular fuzzy variable, CV reduction methods, Congestion charge, Profit transshipment problem, Fuzzy Goal Programming Technique, Weighted Fuzzy Goal Programming Technique

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## 1. Introduction

In present scenario, several establishments in societies have a compulsion to find a superior way to satisfy the needs of the customers in cost effective manner. The classical transportation problem is a particular case of linear programming problem which deals with the dispersion of the commodities from source to destination. The Transportation problem was originally developed by Hitchcock [11] in 1941 and solution to this problem is derived using simplex method. The transportation problem can be modeled as a standard linear programming problem which can then be solved by the simplex

method. The aim of this problem is to minimize the transportation cost and time required for transportation activities. As an alternative to the direct shipment, the commodity frequently moves from one source to another source or one destination to another destination before reaching its designated destination. This type of transportation problem is known as Transshipment problem. In transshipment problem, all the sources and destinations can travel through another sources in any direction. Suppose that the company wishes to judge the transportation plan in advance for the next month, they need to ask for some experts knowledge or to consider the statistical analysis of previous transportation activities. Owing to the current complex environmental conditions during transportation activities some parameters are treated as uncertain variables to meet the current situation. Saad and Abbas discussed the solutions algorithm for solving transportation problem in fuzzy environment [18]. Chanas et al proposed a fuzzy linear programming model for solving transportation problem with crisp cost coefficient and fuzzy supply and demand values [4]. The tool to manage this imprecise conditions fuzzy methodology considering fuzzy demand and fuzzy supply constraints involving fuzzy triangular variable is used. Chanas and Kuchta [6] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers and developed an algorithm for obtaining the optimal solution. Zimmermann [23] in 1978 applied fuzzy set theory concept with some membership function to solve multi-objective transportation problem. In 1975 the concept of fuzzy set called as type-2 fuzzy set was introduced by Zadeh [22] and the extension of ordinary fuzzy set called as type-2 fuzzy set was introduced by Zadeh[20] in 1975. Few Reduction method is used to convert type-2 fuzzy variable to type-1 fuzzy variable and then defuzzification is carried out using the concept of centroid method or by using geometric defuzzification method. Karnik and Mendel [13] introduced a method for type reduction via the concept of a centroid of a type-2 fuzzy set. In 2011 Qin et al [17] introduced three kinds of critical value reduction methods called as optimistic critical value, pessimistic critical value and critical value of type-2 fuzzy variables and the expected value for type-2 triangular fuzzy variable was established. Bellmann and Zadeh suggested fuzzy programming model to make decision in fuzzy environment [2]. Fuzzy transportation problem is the problem of minimizing fuzzy valued objective function with supply and demand. The Goal programming technique and weighted goal programming technique was first formulated by Charnes and Cooper in 1990 [3]. In order to measure a fuzzy event, Zadeh [20] defined a concept of possibility measure as a counterpart of probability measure in 1978. Then the possibility measure was studied by Klir [12], Dubois and Prade [8]. Liu Liu [14] proposed credibility measure. Oheigeartaigh [16] proposed an algorithm for solving

transportation problems in which the capacities and requirements are fuzzy sets with triangular membership functions. In 2017, Dutta and Jana [7] formulated the expectations of the reductions of type-2 trapezoidal fuzzy variables and applied it to a multi objective solid transportation problem via goal programming technique. In 2017, D.K Jana et al [7] proposed a comparative study on credibility measures of type-1 and type-2 fuzzy variables and applied it to multi objective profit transportation problem via goal programming technique. In 2017, Nikky and Ravinder [15] proposed a max-min method for solving transshipment problem with mixed constraints.

Traffic congestion is the vital problem during transportation activity. It indirectly contributes to the environmental pollution because of the emission of GHG gases from the vehicles which in turn cause harmful damages to the ozone layer. It also contributes to many other negative effects such as more fuel expenditure, delay in delivery of goods and also causes severe health hazardous to the travelers. In order to reduce these congestion related problem during transportation activity, the congestion cost is charged on a particular route in which the transportation process is carried out. The congestion charge varies during peak time (i.e., after 8 a.m. and before 8 p.m.) will have a charge doubled than the normal charge and non-peak time will have normal charge (i.e., before 8 a.m. and after 8 p.m.). This type of toll charging is already in practice in many countries [18]. If this strategy is implemented in India, the negative impact of traffic congestion gets much reduced. Therefore encourages the timely shipment of commodities from source to destination and drastically reduces the emission of chemical pollutants from vehicles which in turn increases the profit and the time is minimized during transportation process. In section-2, the preliminaries of Type-2 Fuzzy set have been discussed. In section-3 the Fuzzy Goal Programming Technique (FGPT) and Weighted Goal Programming Technique (WFGPT) have been explained. In section-4, the mathematical notations and assumptions used in this paper has been given. In section-5 the formulation of Type-2 Fuzzy Multi Objective Multi Items Profit Transshipment Problem (T2FMOMIPTSP) with congestion charge. Section-6 detailed the solution procedure for T2FMOMIPTSP. Section-7 discusses the numerical example for the proposed T2FMOMIPTSP model. Section-8 gives the conclusion.

## 2. Preliminaries

In this section, some basic definition on T2 fuzzy sets have been discussed.

**Definition 1.** Type 2 triangular fuzzy variable [7]

A type-2 triangular fuzzy variable  $\tilde{\xi}$  is denoted by  $\tilde{\xi} = (r_1, r_2, r_3, \theta_r, \theta_l)$  where  $r_1, r_2, r_3$  are real values and  $\theta_r, \theta_l \in [0, 1]$  are two parameters characterizing the degree of uncertainty that  $\xi$  takes a value in  $\mathbb{R}$ . For  $x \in [r_1, r_2]$  the secondary possibility function  $\mu_{\tilde{\xi}}(x)$  of  $\tilde{\xi}$  is defined in the form

$$\mu_{\tilde{\xi}}(x) = \left( \frac{x-r_1}{r_2-r_1} + \theta_l \min \left\{ \frac{x-r_1}{r_2-r_1} - \frac{r_2-x}{r_2-r_1} \right\}, \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} + \theta_r \min \left\{ \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} \right\} \right) \text{ For } x$$

$\in [r_1, r_2]$ , the secondary possibility distribution function  $\mu_{\tilde{\xi}}^*$ ,

$$\mu_{\tilde{\xi}}^* = \left( \frac{r_3-x}{r_3-r_2} + \theta_l \min \left\{ \frac{r_3-x}{r_3-r_2} - \frac{x-r_2}{r_3-r_2} \right\}, \frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2} + \theta_r \min \left\{ \frac{r_3-x}{r_3-r_2}, \frac{x-r_2}{r_3-r_2} \right\} \right)$$

**Definition 2.** Type-2 Fuzzy set ([10])

Let  $X$  be a Universal set. Then a type-2 fuzzy set denoted by  $\tilde{A}$  is characterized by a type-2 membership function of the form  $\tilde{\mu}_{\tilde{A}}(x, u)$ , where  $x \in X$  and  $J_x$  denoting the primary membership of  $x$  such that  $u \in J_x \subseteq [0, 1]$ ,

$$\tilde{A} = \{((x, u), \tilde{\mu}_{\tilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}, \text{ in which } 0 \leq \tilde{\mu}_{\tilde{A}}(x, u) \leq 1. \tilde{A} \text{ can be}$$

expressed as  $\tilde{A} = \int_{x \in X} \int_{u \in J_x} \tilde{\mu}_{\tilde{A}}(x, u) / (x, u), J_x \in [0, 1]$ , where  $\int$  denotes the union of  $x$  and  $u$ .

### 3. Fuzzy Methodology

**Method 1.** Fuzzy Goal Programming Technique [7]

The fuzzy Goal Programming Technique (FGPT) is introduced to solve linear and non-linear multi objective programming problems (MOPPs) by converting them into single objective optimization problems.

The MOPPs can be considered with  $m$ -objective functions  $f_i, i = 1, 2, \dots, m$  and may be written as  $\{\min[f_1(x), f_2(x), \dots, f_m(x)]$

Let us consider that decision makers have fixed the membership function  $\mu_k(f_k(x))$  and given the goal membership function value ( $k = 1, 2, \dots$ ).

Let us consider the following programming problem as

$$\min \sum_{i=1}^m d_i^-$$

subject to the constraints  $\mu_k(f_k(x)) + d_i^+ - d_i^- = \bar{\mu}_k$

$$d_i^+ \cdot d_i^- = 0, d_i^+ \cdot d_i^- \geq 0, k = 0, 1, 2, \dots, m$$

$d_i^+ \geq 0, d_i^- \geq 0$ , where  $d_i^+, d_i^-$  denotes the positive and negative deviations respectively.

## Method 2. Weighted Fuzzy Goal Programming Technique [7]

The Weighted Goal Programming Technique (WFGPT) is introduced to solve linear and non-linear multi objective programming problems (MOPPS) by converting them into single objective optimization problems.

The MOPPs can be considered with m-objective functions  $f_i, i = 1, 2, \dots, m$  and may be written as  $\{\min[f_1(x), f_2(x), \dots, f_m(x)]$

Let us consider that decision makers have fixed the membership function  $\mu_k(f_k(x))$  and given the weighted goal membership function value ( $k = 1, 2, \dots, m$ ).

Let us consider the following programming problem as

$$\min \sum_{i=1}^m w(d_i^- + d_i^+)$$

subject to the constraints  $w_k(f_k(x)) + w_i^+ d_i^+ - w_i^- d_i^- = \bar{w}k$

$$d_i^+ \cdot d_i^- = 0, d_i^+ \cdot d_i^- \geq 0, k = 0, 1, 2, \dots, m$$

$d_i^+ \geq 0, d_i^- \geq 0$ , where  $d_i^+, d_i^-$  denotes the positive and negative deviations respectively.

## 4. Notations and Assumptions

This section defines the mathematical notations and assumptions used in this paper. The following notations are used in this paper.

W+F-number of sources (indexed  $i = 1, 2, \dots, W+F$ ), W+F-number of destinations (indexed  $j = 1, 2, \dots, W+F$ ), I-number of items (indexed  $k = 1, 2, \dots, I$ )  $e_{ij}$ -total environmental protection cost  
 $a_{ik}$ -supply (in litres)  $b_{jk}$ -demand (in litres)  $c_{ijk}$ -unit transportation cost from  $i$ th source to  $j$ th destination (in paise/litre),  $d_{ijk}$ -loading and unloading time from  $i$ th source to  $j$ th destination,  $x_{ijk}$ -the amount (in litres) to be transported for  $i$ th source to  $j$ th destination,  $s_{jk}$ -the selling price of the product at  $j$ th destination (in paise/litre),  $p_{ik}$ -the purchasing price of the product at  $i$ th source (in paise/litres). (a)  $(PT)_{ijk}$ -congestion charge during peak time, (b)  $(NT)_{ijk}$ -congestion charge during non-peak time,  $f_1$ -total profit in the problem (in paise),  $f_2$ -total transportation time (in minutes)

#### 4.1 Assumptions

In the T2FPMOMITSP the following assumption is made.

If a commodity of the company is to be transported to different destination then the company has to pay the environmental protection cost. Hence the following binary indicator is introduced as

$$y_{ijk} = \begin{cases} 1, & \text{if } x_{ijk} \neq 0 \\ 0, & \text{if } x_{ijk} = 0 \end{cases} \quad (1)$$

#### 5. Formation of T2FPMOMITSP

In this proposed model, the following conditions are optimized separately during peak time and non-speak time.

- (i) Maximize the total profit (ii) Minimize the total transportation time The T2FPMOMITSP is formulated as

$$\max f_1 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left\{ (s_{jk} - p_{ik} - c_{ijk} - c_{ijk}) x_{ijk} - e_{ijk} \times y_{ijk} \right\} \quad (2)$$

$$\min f_2 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left\{ t_{ijk} \cdot y_{ijk} + d_{ijk} \cdot x_{ijk} \right\} \quad (3)$$

subject to the constraints:

$$\sum_{j=1}^{W+F} \sum_{k=1}^I x_{ijk} = a_{ik} + L, \forall i = 1, 2, \dots, W, \quad (4)$$

$$\sum_{i=1}^{W+F} \sum_{k=1}^I x_{ijk} \leq b_{jk} + L, \forall j = W + 1, W + 2, \dots, W + F \quad (5)$$

$$x_{ijk} \geq 0 \text{ where } L = \sum a_{ik} \quad (6)$$

Similarly, the T2FPTSP can also be formulated for non-peak time.

Suppose the commodity in transshipment problem is to be transported to a destination and no expert knowledge about the transportation cost, supply and demand. This uncertainty is examined by considering type-2 triangular fuzzy cost. The objective functions  $f_i (i = 1, 2, 3, 4)$  consists of selling price  $(s)_{jk} = (s_{jk}^1, s_{jk}^2, s_{jk}^3, s_{jk}^{\theta_l}, s_{jk}^{\theta_r})$ , purchasing price  $(p)_{ik} = (p_{ik}^1, p_{ik}^2, p_{ik}^3, p_{ik}^{\theta_l}, p_{ik}^{\theta_r})$ , transportation cost  $(c)_{ijk} = (c_{ijk}^1, c_{ijk}^2, c_{ijk}^3, c_{ijk}^{\theta_l}, c_{ijk}^{\theta_r})$ , total environment protection cost  $\tilde{e}_{ijk} = (e_{ijk}^1, e_{ijk}^2, e_{ijk}^3, e_{ijk}^{\theta_l}, e_{ijk}^{\theta_r})$ , loading time and unloading time  $(d)_{ijk} = (d_{ijk}^1, d_{ijk}^2, d_{ijk}^3, d_{ijk}^{\theta_l}, d_{ijk}^{\theta_r})$ , transportation time  $(t)_{ijk} = (t_{ijk}^1, t_{ijk}^2, t_{ijk}^3, t_{ijk}^{\theta_l}, t_{ijk}^{\theta_r})$ , congestion charge during peak time  $(PT)_{ijk} = ((PT)_{ijk}^1, (PT)_{ijk}^2, (PT)_{ijk}^3, (PT)_{ijk}^{\theta_l}, (PT)_{ijk}^{\theta_r})$ , congestion charge during non-peak time  $(NT)_{ijk} = ((NT)_{ijk}^1, (NT)_{ijk}^2, (NT)_{ijk}^3, (NT)_{ijk}^{\theta_l}, (NT)_{ijk}^{\theta_r})$ , supply  $a_{ik} = (a_{ik}^1, a_{ik}^2, a_{ik}^3, a_{ik}^{\theta_l}, a_{ik}^{\theta_r})$ , demand  $b_{jk} = (b_{jk}^1, b_{jk}^2, b_{jk}^3, b_{jk}^{\theta_l}, b_{jk}^{\theta_r})$ .

## 6. Solution procedure for T2FPMOMITSP

The Type-2 fuzzy variable can be reduced to non fuzzy variable using critical value reduction methods (CV).

Special cases of CV methods are discussed as follows:

## 6.1 Optimistic expected value

Using the generalized expectation theory [5], the objective functions and the constructions for T2FPMOMITSP can be written as:

$$\max \tilde{f}_1 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ \frac{s_{jk}^1 + s_{jk}^3}{2} - \frac{(s_{jk}^1 - 2s_{jk}^2 + s_{jk}^3) \ln(1 + \frac{s_{jk}^{\theta_r}}{2})}{2s_{jk}^{\theta_r}} \right. \right. \\ \left. \left. - \frac{p_{jk}^1 + p_{jk}^3}{2} - \frac{(p_{ik}^1 - 2p_{ik}^2 + p_{ik}^3) \ln(1 + \frac{p_{ik}^{\theta_r}}{2})}{2p_{ik}^{\theta_r}} \right. \right. \\ \left. \left. - \frac{c_{ijk}^1 + c_{ijk}^3}{2} - \frac{(c_{ijk}^1 - 2c_{ijk}^2 + c_{ijk}^3) \ln(1 + \frac{c_{ijk}^{\theta_r}}{2})}{2c_{ijk}^{\theta_r}} \right\} \times x_{jik} \right. \\ \left. - \frac{e_{ijk}^1 + e_{ijk}^3}{2} - \frac{(e_{ijk}^1 - 2e_{ijk}^2 + e_{ijk}^3) \ln(1 + \frac{e_{ijk}^{\theta_r}}{2})}{2e_{ijk}^{\theta_r}} \times y_{ijk} \right] \quad (7)$$

$$\min \tilde{f}_2 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ \frac{(t)_{ijk}^1 + (t)_{ijk}^3}{2} - \frac{((t)_{ijk}^1 - 2(t)_{ijk}^2 + (t)_{ijk}^3) \ln(1 + \frac{(t)_{ijk}^{\theta_r}}{2})}{2(pt)_{ijk}^{\theta_r}} \right\} y_{ijk} \right. \\ \left. + \left\{ \frac{d_{ijk}^1 + d_{ijk}^3}{2} - \frac{(d_{ijk}^1 - 2d_{ijk}^2 + d_{ijk}^3) \ln(1 + \frac{d_{ijk}^{\theta_r}}{2})}{2d_{ijk}^{\theta_r}} \right\} x_{ijk} \right] \quad (8)$$

Subject to the constraints

$$\sum_{j=1}^{W+F} \sum_{k=1}^I x_{ijk} = \frac{a_{ik}^1 + a_{ik}^3}{2} - \frac{(a_{ik}^1 - 2a_{ik}^2 + a_{ik}^3) \ln(1 + \frac{a_{ik}^{\theta_r}}{2})}{2a_{ik}^{\theta_r}} + L, \forall j \quad (9)$$

$$\sum_{i=1}^W \sum_{k=1}^I x_{ijk} = \frac{b_{jk}^1 + b_{jk}^3}{2} - \frac{(b_{jk}^1 - 2b_{jk}^2 + b_{jk}^3) \ln(1 + \frac{b_{jk}^{\theta_r}}{2})}{2b_{jk}^{\theta_r}} + L, \forall i \quad (10)$$

$$x_{ijk} \geq 0 \text{ where, } L = \sum a_{ik}$$

Similarly the objective function  $f_3$  and  $f_4$ , their corresponding constraints for the proposed T2FPMOMITSP during non-peak time can also be formulated.

## 6.2 Pessimistic expected value

Using the generalized expectation theory [5], the objective functions and the constraints for T2FPMOMITSP can be written as:

$$\max \tilde{f}_1 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ s_{jk}^2 - \frac{(s_{jk}^1 - 2s_{jk}^2 + s_{jk}^3) \ln(1 + \frac{s_{jk}^{\theta_r}}{2})}{2s_{jk}^{\theta_r}} \right. \right. \\ \left. \left. - p_{jk}^2 - \frac{(p_{ik}^1 - 2p_{ik}^2 + p_{ik}^3) \ln(1 + \frac{p_{ik}^{\theta_r}}{2})}{2p_{ik}^{\theta_r}} - c_{ijk}^2 - \frac{(c_{ijk}^1 - 2c_{ijk}^2 + c_{ijk}^3) \ln(1 + \frac{c_{ijk}^{\theta_r}}{2})}{2c_{ijk}^{\theta_r}} \right\} x_{ijk} \right. \\ \left. - e_{ijk}^2 - \frac{(e_{ijk}^1 - 2e_{ijk}^2 + e_{ijk}^3) \ln(1 + \frac{e_{ijk}^{\theta_r}}{2})}{2e_{ijk}^{\theta_r}} \times y_{ijk} \right] \quad (11)$$

$$\min \tilde{f}_2 = \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ (t)_{ijk}^2 - \frac{((t)_{ijk}^1 - 2(t)_{ijk}^2 + (t)_{ijk}^3) \ln(1 + \frac{(t)_{ijk}^{\theta_r}}{2})}{2(t)_{ijk}^{\theta_r}} \right\} y_{ijk} \right. \\ \left. + \left\{ d_{ijk}^2 - \frac{(d_{ijk}^1 - 2d_{ijk}^2 + d_{ijk}^3) \ln(1 + \frac{d_{ijk}^{\theta_r}}{2})}{2d_{ijk}^{\theta_r}} \right\} x_{ijk} \right] \quad (12)$$

and with subject to the constraints (9) and (10)

Similarly the objective function  $f_3$  and  $f_4$ , their corresponding constraints for the proposed T2FPMOMITSP during non-peak time can also be formulated.

### 6.3 Expected critical value

Using the generalized expectation theory [5], the objective functions and the constraints for T2FPMOMITSP can be written as:

$$\begin{aligned} \max \tilde{f}_1 = & \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ \frac{s_{jk}^1 + 2s_{jk}^2 + s_{jk}^3}{4} + \frac{s_{jk}^1 + 2s_{jk}^2 + s_{jk}^3}{8} \right. \right. \\ & \left. \left\{ \frac{1}{s_{jk}^{\theta_r}} - \frac{1}{s_{jk}^{\theta_l}} - \frac{(1+s_{jk}^{\theta_r}) \ln(1+s_{jk}^{\theta_r})}{s_{jk}^{\theta_r^2}} + \frac{(1+s_{jk}^{\theta_l}) \ln(1+s_{jk}^{\theta_l})}{s_{jk}^{\theta_l^2}} \right\} \right. \\ & \left. - \frac{p_{jk}^1 + 2p_{jk}^2 + p_{jk}^3}{4} + \frac{p_{jk}^1 + 2p_{jk}^2 + p_{jk}^3}{8} \right. \\ & \left. \left\{ \frac{1}{p_{jk}^{\theta_r}} - \frac{1}{p_{jk}^{\theta_l}} - \frac{(1+p_{jk}^{\theta_r}) \ln(1+p_{jk}^{\theta_r})}{p_{jk}^{\theta_r^2}} + \frac{(1+p_{jk}^{\theta_l}) \ln(1+p_{jk}^{\theta_l})}{p_{jk}^{\theta_l^2}} \right\} \right. \\ & \left. - \frac{c_{ijk}^1 + 2c_{ijk}^2 + c_{ijk}^3}{4} + \frac{c_{ijk}^1 + 2c_{ijk}^2 + c_{ijk}^3}{8} \right. \\ & \left. \left\{ \frac{1}{c_{ijk}^{\theta_r}} - \frac{1}{c_{ijk}^{\theta_l}} - \frac{(1+c_{ijk}^{\theta_r}) \ln(1+c_{ijk}^{\theta_r})}{c_{ijk}^{\theta_r^2}} + \frac{(1+c_{ijk}^{\theta_l}) \ln(1+c_{ijk}^{\theta_l})}{c_{ijk}^{\theta_l^2}} \right\} \times x_{ijk} \right. \\ & \left. - \frac{e_{ijk}^1 + 2e_{ijk}^2 + e_{ijk}^3}{4} + \frac{e_{ijk}^1 + 2e_{ijk}^2 + e_{ijk}^3}{8} \right. \\ & \left. \left\{ \frac{1}{e_{ijk}^{\theta_r}} - \frac{1}{e_{ijk}^{\theta_l}} - \frac{(1+e_{ijk}^{\theta_r}) \ln(1+e_{ijk}^{\theta_r})}{e_{ijk}^{\theta_r^2}} + \frac{(1+e_{ijk}^{\theta_l}) \ln(1+e_{ijk}^{\theta_l})}{e_{ijk}^{\theta_l^2}} \right\} \times y_{ijk} \right] \quad (13) \\ \min \tilde{f}_2 = & \sum_{i=1}^{W+F} \sum_{j=1}^{W+F} \sum_{k=1}^I \left[ \left\{ \frac{(t)_{ijk}^1 + 2(t)_{ijk}^2 + (t)_{ijk}^3}{4} + \frac{(t)_{ijk}^1 + 2(t)_{ijk}^2 + (t)_{ijk}^3}{8} \right. \right. \end{aligned}$$

$$\left\{ \frac{1}{(t)_{ijk}^{\theta_r}} - \frac{1}{(t)_{ijk}^{\theta_l}} - \frac{(1 + (t)_{ijk}^{\theta_r}) \ln(1 + (t)_{ijk}^{\theta_r})}{(t)_{ijk}^{\theta_r^2}} + \frac{(1 + (t)_{ijk}^{\theta_l}) \ln(1 + (t)_{ijk}^{\theta_l})}{(t)_{ijk}^{\theta_l^2}} \right\} \times y_{ijk} \\ + \left\{ \frac{d_{ijk}^1 + 2d_{ijk}^2 + d_{ijk}^3}{4} + \frac{d_{ijk}^1 + 2d_{ijk}^2 + d_{ijk}^3}{8} \right. \\ \left. \left\{ \frac{1}{d_{ijk}^{\theta_r}} - \frac{1}{d_{ijk}^{\theta_l}} - \frac{(1 + d_{ijk}^{\theta_r}) \ln(1 + d_{ijk}^{\theta_r})}{d_{ijk}^{\theta_r^2}} + \frac{(1 + d_{ijk}^{\theta_l}) \ln(1 + d_{ijk}^{\theta_l})}{d_{ijk}^{\theta_l^2}} \right\} \right\} x_{ijk} \quad (14)$$

and with subject to the constraints (9) & (10). Similarly the objective function  $f_3$  and  $f_4$  with their corresponding constraints for the proposed T2FPMOMITSP during non-peak time can also be formulated.

## 7. Case Study

### 7.1 Input data

In order to examine the validity of the proposed model a case study is done in milk centres functioning in trichy and thanjavur districts based on the secondary data. Considering seven sources ( $W = 7$ ) and seven receiving milk centres from these sources as destinations ( $F = 7$ ), the two milk varieties such as Standardized milk and Double toned milk is transported for daily sales. Owing to the real life uncertain situations, the transportation problem cannot be analyzed using the crisp value so that the parameters used in the transportation problem are considered as type-2 fuzzy variable. The type-2 fuzzy transportation cost, total environment protection cost, congestion charge during peak-time and non-peak time and transportation time for a unit quantity of the commodity from the  $i^{\text{th}}$  source to the  $j^{\text{th}}$  destination and the selling prices and purchasing prices, the total supply in each source and total demand in each destination, for multi items ( $l = 2$ ) have been assumed and the values are given in the following table.

Table: 1.1-Input data for T2FPMOMITSP

| Item-1                                    | Item-2                                    |
|-------------------------------------------|-------------------------------------------|
| $\widetilde{s}_{11} = (40,41,42,0.5,0.5)$ | $\widetilde{s}_{12} = (44,45,46,0.5,0.5)$ |
| $\widetilde{s}_{21} = (40,41,42,0.5,0.5)$ | $\widetilde{s}_{22} = (44,45,46,0.5,0.5)$ |
| $\widetilde{s}_{31} = (40,41,42,0.5,0.5)$ | $\widetilde{s}_{32} = (44,45,46,0.5,0.5)$ |

|                                                 |                                              |
|-------------------------------------------------|----------------------------------------------|
| $\widetilde{s}_{41} = (40,41,42,0.5,0.5)$       | $\widetilde{s}_{42} = (44,45,46,0.5,0.5)$    |
| $\widetilde{s}_{51} = (40,41,42,0.5,0.5)$       | $\widetilde{s}_{52} = (44,45,46,0.5,0.5)$    |
| $\widetilde{s}_{61} = (40,41,42,0.5,0.5)$       | $\widetilde{s}_{62} = (44,45,46,0.5,0.5)$    |
| $\widetilde{s}_{71} = (40,41,42,0.5,0.5)$       | $\widetilde{s}_{72} = (44,45,46,0.5,0.5)$    |
| $\widetilde{p}_{11} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{12} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{21} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{22} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{31} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{32} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{41} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{42} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{51} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{52} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{61} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{62} = (27,28,29,0.5,0.5)$    |
| $\widetilde{p}_{71} = (27,28,29,0.5,0.5)$       | $\widetilde{p}_{72} = (27,28,29,0.5,0.5)$    |
| $\widetilde{a}_{11} = (1056,1057,1058,0.5,0.5)$ | $\widetilde{a}_{12} = (262,263,264,0.5,0.5)$ |
| $\widetilde{a}_{21} = (1063,1064,1065,0.5,0.5)$ | $\widetilde{a}_{22} = (264,265,266,0.5,0.5)$ |
| $\widetilde{a}_{31} = (706,707,708,0.5,0.5)$    | $\widetilde{a}_{32} = (175,176,177,0.5,0.5)$ |
| $\widetilde{a}_{41} = (706,707,708,0.5,0.5)$    | $\widetilde{a}_{42} = (175,176,177,0.5,0.5)$ |
| $\widetilde{a}_{51} = (706,707,708,0.5,0.5)$    | $\widetilde{a}_{52} = (175,176,177,0.5,0.5)$ |
| $\widetilde{a}_{61} = (706,707,708,0.5,0.5)$    | $\widetilde{a}_{62} = (175,176,177,0.5,0.5)$ |
| $\widetilde{a}_{71} = (706,707,708,0.5,0.5)$    | $\widetilde{a}_{72} = (175,176,177,0.5,0.5)$ |
| $\widetilde{b}_{11} = (706,707,708,0.5,0.5)$    | $\widetilde{b}_{12} = (175,176,177,0.5,0.5)$ |
| $\widetilde{b}_{21} = (706,707,708,0.5,0.5)$    | $\widetilde{b}_{22} = (175,176,177,0.5,0.5)$ |
| $\widetilde{b}_{31} = (880,881,882,0.5,0.5)$    | $\widetilde{b}_{32} = (218,219,220,0.5,0.5)$ |
| $\widetilde{b}_{41} = (911,912,913,0.5,0.5)$    | $\widetilde{b}_{42} = (226,227,228,0.5,0.5)$ |
| $\widetilde{b}_{51} = (792,793,794,0.5,0.5)$    | $\widetilde{b}_{52} = (196,197,198,0.5,0.5)$ |
| $\widetilde{b}_{61} = (772,773,774,0.5,0.5)$    | $\widetilde{b}_{62} = (192,193,194,0.5,0.5)$ |
| $\widetilde{b}_{71} = (882,883,884,0.5,0.5)$    | $\widetilde{b}_{72} = (119,120,121,0.5,0.5)$ |
| $\widetilde{c}_{111} = (0,0,0,0,0)$             | $\widetilde{c}_{112} = (0,0,0,0,0)$          |
| $\widetilde{c}_{121} = (40,41,42,0.5,0.5)$      | $\widetilde{c}_{122} = (40,41,42,0.5,0.5)$   |

|                                            |                                            |
|--------------------------------------------|--------------------------------------------|
| $\widetilde{c}_{131} = (10,11,12,0.5,0.5)$ | $\widetilde{c}_{132} = (10,11,12,0.5,0.5)$ |
| $\widetilde{c}_{141} = (40,41,42,0.5,0.5)$ | $\widetilde{c}_{142} = (40,41,42,0.5,0.5)$ |
| $\widetilde{c}_{151} = (25,26,27,0.5,0.5)$ | $\widetilde{c}_{152} = (25,26,27,0.5,0.5)$ |
| $\widetilde{c}_{161} = (25,26,27,0.5,0.5)$ | $\widetilde{c}_{162} = (25,26,27,0.5,0.5)$ |
| $\widetilde{c}_{171} = (17,18,19,0.5,0.5)$ | $\widetilde{c}_{172} = (17,18,19,0.5,0.5)$ |
| $\widetilde{c}_{211} = (41,42,43,0.5,0.5)$ | $\widetilde{c}_{212} = (41,42,43,0.5,0.5)$ |
| $\widetilde{c}_{221} = (0,0,0,0,0)$        | $\widetilde{c}_{222} = (0,0,0,0,0)$        |
| $\widetilde{c}_{231} = (41,42,43,0.5,0.5)$ | $\widetilde{c}_{232} = (41,42,43,0.5,0.5)$ |
| $\widetilde{c}_{241} = (10,11,12,0.5,0.5)$ | $\widetilde{c}_{242} = (10,11,12,0.5,0.5)$ |
| $\widetilde{c}_{251} = (43,44,45,0.5,0.5)$ | $\widetilde{c}_{252} = (43,44,45,0.5,0.5)$ |
| $\widetilde{c}_{261} = (65,66,67,0.5,0.5)$ | $\widetilde{c}_{262} = (65,66,67,0.5,0.5)$ |
| $\widetilde{c}_{271} = (56,57,58,0.5,0.5)$ | $\widetilde{c}_{272} = (56,57,58,0.5,0.5)$ |
| $\widetilde{c}_{711} = (17,18,19,0.5,0.5)$ | $\widetilde{c}_{712} = (17,18,19,0.5,0.5)$ |
| $\widetilde{c}_{721} = (56,57,58,0.5,0.5)$ | $\widetilde{c}_{722} = (56,57,58,0.5,0.5)$ |
| $\widetilde{c}_{751} = (33,34,35,0.5,0.5)$ | $\widetilde{c}_{752} = (33,34,35,0.5,0.5)$ |
| $\widetilde{c}_{761} = (11,12,13,0.5,0.5)$ | $\widetilde{c}_{762} = (11,12,13,0.5,0.5)$ |
| $\widetilde{c}_{771} = (0,0,0,0,0)$        | $\widetilde{c}_{772} = (0,0,0,0,0)$        |
| $\widetilde{t}_{111} = (0,0,0,0,0)$        | $\widetilde{t}_{112} = (0,0,0,0,0)$        |
| $\widetilde{t}_{121} = (40,41,42,0.5,0.5)$ | $\widetilde{t}_{122} = (40,41,42,0.5,0.5)$ |
| $\widetilde{t}_{131} = (10,11,12,0.5,0.5)$ | $\widetilde{t}_{132} = (10,11,12,0.5,0.5)$ |
| $\widetilde{t}_{141} = (40,41,42,0.5,0.5)$ | $\widetilde{t}_{142} = (40,41,42,0.5,0.5)$ |
| $\widetilde{t}_{151} = (25,26,27,0.5,0.5)$ | $\widetilde{t}_{152} = (25,26,27,0.5,0.5)$ |
| $\widetilde{t}_{161} = (25,26,27,0.5,0.5)$ | $\widetilde{t}_{162} = (25,26,27,0.5,0.5)$ |
| $\widetilde{t}_{171} = (17,18,19,0.5,0.5)$ | $\widetilde{t}_{172} = (17,18,19,0.5,0.5)$ |
| $\widetilde{t}_{711} = (17,18,19,0.5,0.5)$ | $\widetilde{t}_{712} = (17,18,19,0.5,0.5)$ |
| $\widetilde{t}_{721} = (56,57,58,0.5,0.5)$ | $\widetilde{t}_{722} = (56,57,58,0.5,0.5)$ |
| $\widetilde{t}_{731} = (17,18,19,0.5,0.5)$ | $\widetilde{t}_{732} = (17,18,19,0.5,0.5)$ |

|                                               |                                               |
|-----------------------------------------------|-----------------------------------------------|
| $\widetilde{t}_{741} = (56,57,58,0.5,0.5)$    | $\widetilde{t}_{742} = (56,57,58,0.5,0.5)$    |
| $\widetilde{t}_{751} = (33,34,35,0.5,0.5)$    | $\widetilde{t}_{752} = (33,34,35,0.5,0.5)$    |
| $\widetilde{t}_{761} = (11,12,13,0.5,0.5)$    | $\widetilde{t}_{762} = (11,12,13,0.5,0.5)$    |
| $\widetilde{t}_{771} = (0,0,0,0,0)$           | $\widetilde{t}_{772} = (0,0,0,0,0)$           |
| $\widetilde{d}_{211} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{212} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{221} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{222} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{231} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{232} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{241} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{242} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{251} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{252} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{261} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{262} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{271} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{272} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{711} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{712} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{721} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{722} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{731} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{732} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{741} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{742} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{751} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{752} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{761} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{762} = (15,16,17,0.5,0.5)$    |
| $\widetilde{d}_{771} = (15,16,17,0.5,0.5)$    | $\widetilde{d}_{772} = (15,16,17,0.5,0.5)$    |
| $(\widetilde{PT})_{111} = (0,0,0,0,0)$        | $(\widetilde{PT})_{112} = (0,0,0,0,0)$        |
| $(\widetilde{PT})_{121} = (96,97,98,0.5,0.5)$ | $(\widetilde{PT})_{122} = (96,97,98,0.5,0.5)$ |
| $(\widetilde{PT})_{131} = (0,0,0,0,0)$        | $(\widetilde{PT})_{132} = (0,0,0,0,0)$        |
| $(\widetilde{PT})_{141} = (96,97,98,0.5,0.5)$ | $(\widetilde{PT})_{142} = (96,97,98,0.5,0.5)$ |
| $(\widetilde{PT})_{151} = (46,47,48,0.5,0.5)$ | $(\widetilde{PT})_{152} = (46,47,48,0.5,0.5)$ |
| $(\widetilde{PT})_{161} = (26,27,28,0.5,0.5)$ | $(\widetilde{PT})_{162} = (26,27,28,0.5,0.5)$ |
| $(\widetilde{PT})_{171} = (26,27,28,0.5,0.5)$ | $(\widetilde{PT})_{172} = (26,27,28,0.5,0.5)$ |
| $(\widetilde{PT})_{211} = (96,97,98,0.5,0.5)$ | $(\widetilde{PT})_{212} = (96,97,98,0.5,0.5)$ |

|                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|
| $(\widetilde{PT})_{221} = (0,0,0,0,0)$           | $(\widetilde{PT})_{222} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{231} = (96,97,98,0.5,0.5)$    | $(\widetilde{PT})_{232} = (96,97,98,0.5,0.5)$    |
| $(\widetilde{PT})_{241} = (0,0,0,0,0)$           | $(\widetilde{PT})_{242} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{251} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{252} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{261} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{262} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{271} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{272} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{511} = (46,47,48,0.5,0.5)$    | $(\widetilde{PT})_{512} = (46,47,48,0.5,0.5)$    |
| $(\widetilde{PT})_{521} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{522} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{531} = (46,47,48,0.5,0.5)$    | $(\widetilde{PT})_{532} = (46,47,48,0.5,0.5)$    |
| $(\widetilde{PT})_{541} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{542} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{551} = (0,0,0,0,0)$           | $(\widetilde{PT})_{552} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{561} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{562} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{571} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{572} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{611} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{612} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{621} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{622} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{631} = (80,81,82,0.5,0.5)$    | $(\widetilde{PT})_{632} = (80,81,82,0.5,0.5)$    |
| $(\widetilde{PT})_{641} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{642} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{651} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{652} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{661} = (0,0,0,0,0)$           | $(\widetilde{PT})_{662} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{671} = (0,0,0,0,0)$           | $(\widetilde{PT})_{672} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{711} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{712} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{721} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{722} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{731} = (26,27,28,0.5,0.5)$    | $(\widetilde{PT})_{732} = (26,27,28,0.5,0.5)$    |
| $(\widetilde{PT})_{741} = (123,124,125,0.5,0.5)$ | $(\widetilde{PT})_{742} = (123,124,125,0.5,0.5)$ |
| $(\widetilde{PT})_{751} = (0,0,0,0,0)$           | $(\widetilde{PT})_{752} = (0,0,0,0,0)$           |
| $(\widetilde{PT})_{761} = (0,0,0,0,0)$           | $(\widetilde{PT})_{762} = (0,0,0,0,0)$           |

|                                                      |                                                      |
|------------------------------------------------------|------------------------------------------------------|
| $(\widetilde{\text{PT}})_{771} = (0,0,0,0,0)$        | $(\widetilde{\text{PT}})_{772} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{111} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{112} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{121} = (48,49,50,0.5,0.5)$ | $(\widetilde{\text{NT}})_{122} = (48,49,50,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{131} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{132} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{141} = (48,49,50,0.5,0.5)$ | $(\widetilde{\text{NT}})_{142} = (48,49,50,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{151} = (23,24,25,0.5,0.5)$ | $(\widetilde{\text{NT}})_{152} = (23,24,25,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{161} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{162} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{171} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{172} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{611} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{612} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{621} = (62,63,64,0.5,0.5)$ | $(\widetilde{\text{NT}})_{622} = (62,63,64,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{631} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{632} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{641} = (62,63,64,0.5,0.5)$ | $(\widetilde{\text{NT}})_{642} = (62,63,64,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{651} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{652} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{661} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{662} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{671} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{672} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{711} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{712} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{721} = (62,63,64,0.5,0.5)$ | $(\widetilde{\text{NT}})_{722} = (62,63,64,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{731} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{732} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{741} = (62,63,64,0.5,0.5)$ | $(\widetilde{\text{NT}})_{742} = (62,63,64,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{751} = (13,14,15,0.5,0.5)$ | $(\widetilde{\text{NT}})_{752} = (13,14,15,0.5,0.5)$ |
| $(\widetilde{\text{NT}})_{761} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{762} = (0,0,0,0,0)$        |
| $(\widetilde{\text{NT}})_{771} = (0,0,0,0,0)$        | $(\widetilde{\text{NT}})_{772} = (0,0,0,0,0)$        |
| $\widetilde{e}_{111} = (0,0,0,0,0)$                  | $\widetilde{e}_{112} = (0,0,0,0,0)$                  |
| $\widetilde{e}_{121} = (19,20,21,0.5,0.5)$           | $\widetilde{e}_{122} = (19,20,21,0.5,0.5)$           |
| $\widetilde{e}_{131} = (33,34,35,0.5,0.5)$           | $\widetilde{e}_{132} = (33,34,35,0.5,0.5)$           |
| $\widetilde{e}_{141} = (19,20,21,0.5,0.5)$           | $\widetilde{e}_{142} = (19,20,21,0.5,0.5)$           |

|                                |                                |
|--------------------------------|--------------------------------|
| $e_{151} = (10,11,12,0.5,0.5)$ | $e_{152} = (10,11,12,0.5,0.5)$ |
| $e_{161} = (12,13,14,0.5,0.5)$ | $e_{162} = (12,13,14,0.5,0.5)$ |
| $e_{171} = (11,12,13,0.5,0.5)$ | $e_{172} = (11,12,13,0.5,0.5)$ |
| $e_{211} = (19,20,21,0.5,0.5)$ | $e_{212} = (19,20,21,0.5,0.5)$ |
| $e_{221} = (0,0,0,0,0)$        | $e_{222} = (0,0,0,0,0)$        |
| $e_{231} = (19,20,21,0.5,0.5)$ | $e_{232} = (19,20,21,0.5,0.5)$ |
| $e_{241} = (40,41,42,0.5,0.5)$ | $e_{242} = (40,41,42,0.5,0.5)$ |
| $e_{251} = (20,21,22,0.5,0.5)$ | $e_{252} = (20,21,22,0.5,0.5)$ |
| $e_{261} = (30,31,32,0.5,0.5)$ | $e_{262} = (30,31,32,0.5,0.5)$ |
| $e_{271} = (27,28,29,0.5,0.5)$ | $e_{272} = (27,28,29,0.5,0.5)$ |
| $e_{611} = (12,13,14,0.5,0.5)$ | $e_{612} = (12,13,14,0.5,0.5)$ |
| $e_{621} = (30,31,32,0.5,0.5)$ | $e_{622} = (30,31,32,0.5,0.5)$ |
| $e_{631} = (12,13,14,0.5,0.5)$ | $e_{632} = (12,13,14,0.5,0.5)$ |
| $e_{641} = (30,31,32,0.5,0.5)$ | $e_{642} = (30,31,32,0.5,0.5)$ |
| $e_{651} = (21,22,23,0.5,0.5)$ | $e_{652} = (21,22,23,0.5,0.5)$ |
| $e_{661} = (0,0,0,0,0)$        | $e_{662} = (0,0,0,0,0)$        |
| $e_{671} = (15,16,17,0.5,0.5)$ | $e_{672} = (15,16,17,0.5,0.5)$ |
| $e_{711} = (11,12,13,0.5,0.5)$ | $e_{712} = (11,12,13,0.5,0.5)$ |
| $e_{721} = (27,28,29,0.5,0.5)$ | $e_{722} = (27,28,29,0.5,0.5)$ |
| $e_{731} = (11,12,13,0.5,0.5)$ | $e_{732} = (11,12,13,0.5,0.5)$ |
| $e_{741} = (27,28,29,0.5,0.5)$ | $e_{742} = (27,28,29,0.5,0.5)$ |
| $e_{751} = (17,18,19,0.5,0.5)$ | $e_{752} = (17,18,19,0.5,0.5)$ |
| $e_{761} = (11,12,13,0.5,0.5)$ | $e_{762} = (11,12,13,0.5,0.5)$ |
| $e_{771} = (0,0,0,0,0)$        | $e_{772} = (0,0,0,0,0)$        |

Using the input value in Table-1.1, the objective functions together with crisp constraints are solved separately using GRG technique (lingo 18.0).

The feasible expected values is obtained under Critical value reduction and the expected values  $f_i^0$  and  $f_i^1 = (i = 1, 2, 3, 4)$  are listed below:

| Peak Time                             | Non-peak Time                    |
|---------------------------------------|----------------------------------|
| $f_1^0 = 14895181$ $f_1^1 = 14527247$ | $f_3^0 = 389010$ $f_3^1 = 19966$ |
| $f_2^0 = 170136$ $f_1^2 = 165122$     | $f_4^0 = 10353$ $f_4^1 = 9996$   |

The membership functions for the objective functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  are formulated as follows

$$\mu_1(f_1(x)) = \begin{cases} 1, & \text{for } f_1(x) > 14895181 \\ \frac{f_1(x) - 14527247}{104565 - 85048}, & \text{for } 14527247 \leq f_1(x) \leq 14895181 \\ 0, & \text{for } f_1(x) < 14527247 \end{cases} \quad (15)$$

$$\mu_2(f_2(x)) = \begin{cases} 1, & \text{for } f_2(x) > 170136 \\ \frac{170136 - f_2(x)}{170136 - 165122}, & \text{for } 165122 \leq f_2(x) \leq 170136 \\ 0, & \text{for } f_2(x) < 165122 \end{cases} \quad (16)$$

Similarly, the membership functions can be formulated for the objective functions  $f_3$  and  $f_4$

The following model is formulated using fuzzy goal programming (FGPT) technique and the critical value is give as follows.

PEAK TIME:

$$\text{Maximize } [d_1^- + d_2^-]$$

subject to the constraints:

$$\begin{cases} \frac{f_1 - 14527247}{14895181 - 14527247} + d_1^+ - d_1^- = \mu_1 \\ \frac{f_2 - 170136}{170136 - 165122} + d_2^+ - d_2^- = \mu_2 \\ d_i^+ \cdot d_i^- = 0, d_i^+ \cdot d_i^- \geq 0, i = 1, 2 \\ d_i^+ \geq 0, d_i^- \geq 0, i = 1, 2 \end{cases} \quad (17)$$

Similarly the above model can be formulated for non-peak time. The above single objective function is solved using LINGO-18.0 and the optimum results are reported in the following table.

**Table: 1.2-Expected critical values via FGPT (peak time and Non-peak time)**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $f_1$    | $f_2$  | $f_3$  | $f_4$ |
|---------|---------|---------|---------|----------|--------|--------|-------|
| 0.2     | 0.1     | 0.3     | 0.4     | 14600833 | 169634 | 278297 | 10210 |
| 0.4     | 0.4     | 0.5     | 0.5     | 14637627 | 169133 | 241393 | 10175 |
| 0.6     | 0.5     | 0.7     | 0.7     | 14674420 | 168631 | 130680 | 10139 |
| 0.7     | 0.6     | 0.8     | 0.9     | 14748007 | 167629 | 93775  | 10103 |

The following model is formulated using weighted fuzzy goal programming technique (WFGPT) (for critical value)

PEAK TIME:

$$\text{Maximize } \sum_{i=1}^2 [w_i (d_i^+ + d_i^-)]$$

$$\text{subject to the constraints: } \begin{cases} \frac{f_1 - 14527247}{14895181 - 14527247} + d_1^+ - d_1^- = w_1 \\ \frac{f_2 - 170136}{170136 - 165122} + d_2^+ - d_2^- = w_2 \\ d_i^+ \cdot d_i^- = 0, d_i^+ \cdot d_i^- \geq 0, i = 1, 2 \\ d_i^+ \geq 0, d_i^- \geq 0, i = 1, 2 \end{cases} \quad (18)$$

Similarly the above model can be formulated for non-peak time. The above single objective function is solved using LINGO-18.0 and the optimum results are reported in the following table.

**Table 1.3 Expected critical values via WFGPT (peak time and non-peak time)**

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $f_1$    | $f_2$  | $f_3$  | $f_4$   |
|-------|-------|-------|-------|----------|--------|--------|---------|
| 0.1   | 0.2   | 0.3   | 0.4   | 14564040 | 169133 | 278297 | 10210.2 |

|     |     |     |     |          |        |        |         |
|-----|-----|-----|-----|----------|--------|--------|---------|
| 0.2 | 0.1 | 0.4 | 0.3 | 14600833 | 169634 | 241393 | 10245.9 |
| 0.3 | 0.4 | 0.1 | 0.2 | 14637627 | 168131 | 352106 | 10281.6 |
| 0.4 | 0.3 | 0.2 | 0.1 | 14674420 | 168432 | 315202 | 10317.3 |

From the above tables it is observed that the profit in transportation increases with respect to increase in credibility measure and the time required for transportation decreases with increase in credibility measures.

## 8. Conclusion

This paper mainly investigated multi objective profit transshipment problem with congestion charge for multi items under type-2 fuzzy environment. Using the numerical experiment for the proposed model, it is observed that the profit is increased in non-peak time than the profit obtained in peak time. Also, the transportation time is minimized in non-peak time than peak time. This paper can be extended to different types of transportation problems including price discounts, breakable items and damageable items in complex environment.

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