

An Approach For Solving Fuzzy Transportation Problem Using Allocation Table Method

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Abstract

In this paper ,we use allocation table method for solving fuzzy transportation problem using Robust Ranking method. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal or any LR fuzzy number. Thus some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, supply and demands, in to crisp quantities by using method and then by using the classical algorithms we solve and obtain the solution of the problem. This method has systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function. At the end, this method is illustrated with a numerical example.

Keywords: Fuzzy Numbers, Fuzzy Transportation problem, ranking of fuzzy numbers

1. Introduction

In 1941, **Hitchcock** was the one who is American mathematician and physicist developed the transportation problem. In 1951, **Dantzing** gave effective methods for determining a solution. The theory of fuzzy set introduced by Zadeh in 1965 has achieved successful applications in various fields. In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a “**Robust Ranking Technique**”. method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, in to crisp quantities by using “**Robust Ranking Technique**” method and then by using algorithms we solve and obtain the solution of the problem. A numerical example is also included.

2. Preliminaries:

Definition 2.1

Let S be a universal set. A **Fuzzy set** which is defined on S is denoted by \tilde{B} shall be written as a collection of ordered pairs,

$\tilde{B} = \{(x, \tilde{B}(x)) : x \in S\}$ where $\tilde{B}(x)$ is a membership function.

DEFINITION 2.2

For a fuzzy number \tilde{B} , the **triangular fuzzy number** can be represented by $\tilde{B}(d_1, d_2, d_3; 1)$ with a membership function $\tilde{B}(x)$ is given by,

$$\tilde{B}(x) = \begin{cases} \frac{x-d_1}{d_2-d_1}, & d_1 \leq x \leq d_2 \\ 1, & x = d_2 \\ \frac{d_3-x}{d_3-d_2}, & d_2 \leq x \leq d_3 \\ 0, & \text{otherwise} \end{cases}$$

DEFINITION 2.3

For a fuzzy number \tilde{B} , the **trapezoidal fuzzy number** can be represented by $\tilde{B}(d_1, d_2, d_3, d_4; 1)$ with a membership function $\tilde{B}(x)$ is given by,

$$\tilde{B}(x) = \begin{cases} \frac{x-d_1}{d_2-d_1}, & d_1 \leq x \leq d_2 \\ 1, & d_2 \leq x \leq d_3 \\ \frac{d_4-x}{d_4-d_3}, & d_3 \leq x \leq d_4 \\ 0, & \text{otherwise} \end{cases}$$

DEFINITION 2.4

Two **triangular fuzzy numbers** satisfies the properties of **addition and subtraction** which can be given as,

$$(d_1, d_2, d_3) + (f_1, f_2, f_3) = (d_1 + f_1, d_2 + f_2, d_3 + f_3)$$

$$(d_1, d_2, d_3) - (f_1, f_2, f_3) = (d_1 - f_3, d_2 - f_2, d_3 - f_1)$$

DEFINITION 2.5

Two **trapezoidal fuzzy numbers** satisfies the properties of **addition and subtraction** which can be given as,

$$(d_1, d_2, d_3, d_4) + (f_1, f_2, f_3, f_4) = (d_1 + f_1, d_2 + f_2, d_3 + f_3, d_4 + f_4)$$

$$(d_1, d_2, d_3, d_4) - (f_1, f_2, f_3, f_4) = (d_1 - f_4, d_2 - f_3, d_3 - f_2, d_4 - f_1)$$

ROBUST RANKING TECHNIQUE:

The fuzzy numbers can be transformed into crisp values practicing the method “**Robust Ranking Technique**”. This Robust Ranking Technique suffices additive and linearity properties also affords the results which is dependable with man’s instinct. The Robust Ranking index of a convex fuzzy \tilde{d} is determined by, $R(\tilde{d}) = \int_0^1 (0.5)(d_\alpha^L, d_\alpha^U) d\alpha$

where $(d_\alpha^L, d_\alpha^U) = \{[(d_2 - d_1)\alpha + d_1] - [d_3 - (d_3 - d_2)\alpha]\}$

2.6 FUZZY TRANSPORTATION PROBLEM REPRESENTATION:

The fuzzy transportation problem can be symbolized by,

	1	...	N	SUPPLY
1	\tilde{G}_{11}	...	\tilde{G}_{1n}	\tilde{C}_1
.
M	\tilde{G}_{m1}	...	\tilde{G}_{mn}	\tilde{G}_m
DEMAND	\tilde{d}_1	...	\tilde{d}_n	

2.7 FUZZY ALLOCATION TABLE METHOD ALGORITHM:

STEP-1: From the specific fuzzy transportation problem, frame a fuzzy transportation problem. Convert the triangular fuzzy number into a crisp value by using the “**RobustRanking Method**”,

STEP-2: Make assured that the fuzzy transportation problem is balanced or not. In case, the fuzzy transportation problem is not balanced, construct it as a balanced one by introducing a dummy row or dummy column.

STEP-3: In the fuzzy transportation problem, out of every cost cells choose the smallest odd cost. Multiply all the cost cells by $\frac{1}{2}$. If at all, the fuzzy transportation

problem has no odd cost cell and pursue it on multiplying by $\frac{1}{2}$ on each cost cell until it attain a smallest odd cost in the cost cells.

STEP-4: Next frame a recent table, where this table is called as **Fuzzy Allocation Table**. Subtract the chosen smallest odd cost only from the odd cost cells of the fuzzy transportation table and hold the selected smallest odd cost as it was without subtracting and the following cell values are to be known as **Fuzzy Allocation Cell Value** in the fuzzy allocation table.

STEP-5: Initially, commence the allocation on the chosen smallest odd cell in the fuzzy allocation table on step-4. Eliminate the column if the demand is fulfilled and eliminate the row if the supply is fulfilled.

STEP-6: At once diagnose the lowest fuzzy allocation cell value and at the position of the selected fuzzy allocation cell values in the fuzzy allocation table, allocate the minimum supply/demand. In case, if there is a tie on choosing the fuzzy allocation cell values, select the allocation cell value where lowest fuzzy allocation can be made. Besides in the case of same allocation in fuzzy allocation cell, the cost cell of the fuzzy transportation table formed in step-1 is corresponded by preferring the minimum cost cell. In addition, if the cost cell and allocation are identical, in such case, select the closer cell to the minimum of supply/demand which is to be allocated. Besides, if the demand is fulfilled eliminate the column and if the supply is fulfilled eliminate the row.

STEP-7: Till the supply and demand gets drained, continually proceed step-6.

STEP-8: Next change this fuzzy allocation to the initial fuzzy transportation table.

STEP-9: From the fuzzy transportation table, at last estimate the total fuzzy transportation cost.

	STORES			FUZZY QUANTITY

NUMERICAL EXAMPLES

EXAMPLE-1:

produces four vehicles and it own three branches B_1, B_2, B_3 whose weekly yield quantities are (3,4,5), (2,3,4) and (7,8,9) pieces of vehicles respectively. The crew provide the vehicles to its three stores placed at D_1, D_2 and D_3 whose demands are (1,2,3), (8,9,10) and (3,4,5). For a single piece of vehicles, the fuzzy transportation cost of the fuzzy transportation problem is given as,

BRANCHES	D ₁	D ₂	D ₃	
B ₁	(6,7,8)	(7,8,9)	(5,6,7)	(3,4,5)
B ₂	(8,9,10)	(1,2,3)	(3,4,5)	(2,3,4)
B ₃	(4,5,6)	(5,6,7)	(2,3,4)	(7,8,9)

Find out the initial basic solution on shifting the vehicles from the branches to stores.

SOLUTION:

The fuzzy transportation problem is given by,

The fuzzy transportation problem can be written as the mathematical form given below,

$$\text{Min } Z = R(6,7,8)y_{11} + R(7,8,9)y_{12} + R(5,6,7)y_{13} +$$

$$R(8,9,10)y_{21} + R(1,2,3)y_{22} + R(3,4,5)y_{23} +$$

$$R(4,5,6)y_{31} + R(5,6,7)y_{32} + R(2,3,4)y_{33}$$

Let us convert this triangular fuzzy number into a crisp number, by using **“Robust Ranking Method”**.

The Robust ranking method formula is given as,

$$R(\tilde{d}) = \int_0^1 (0.5)(d_{\alpha}^L d_{\alpha}^U) d\alpha$$

$$\text{where } (d_{\alpha}^L d_{\alpha}^U) = \{[(d_2 - d_1)\alpha + d_1] - [d_3 - (d_3 - d_2)\alpha]\}$$

BRANCHES	STORES			FUZZY QUANTITY
	D ₁	D ₂	D ₃	
B ₁	(6,7,8)	(7,8,9)	(5,6,7)	(3,4,5)

B_2	(8,9,10)	(1,2,3)	(3,4,5)	(2,3,4)
B_3	(4,5,6)	(5,6,7)	(2,3,4)	(7,8,9)

Rank of every supply:

$$R(3,4,5) = 4, \quad R(2,3,4) = 3, \quad R(7,8,9) = 8$$

Rank of every demand:

$$R(1,2,3) = 2, \quad R(8,9,10) = 9, \quad R(3,4,5) = 4$$

By substituting these values, we attain crisp transportation problem in the fuzzy transportation problem in the below table,

BRANCHES	STORES			FUZZY SUPPLY
	D_1	D_2	D_3	
B_1	7	8	6	4
B_2	9	2	4	3
B_3	5	6	3	8
FUZZY DEMAND	2	9	4	

Total supply = Total demand = 15

∴ The given fuzzy transportation problem is a balanced fuzzy transportation problem.

BRANCHES	STORES			FUZZY SUPPLY
	D_1	D_2	D_3	
B_1	4	8	6	4

B₂	6	2	4	3
B₃	2	6	3	8
FUZZY DEMAND	2	9	4	

(The smallest odd cost had been chosen and all the odd cost had subtracted by smallest odd cost)

BRANCHES	STORES				FUZZY SUPPLY
	D ₁	D ₂		D ₃	
B₁	4		4	6	4
B₂	6		3	4	3
B₃	2		2	4	8
FUZZY DEMAND	2	9		4	

(In the fuzzy allocation table, the fuzzy allocation had formed for several cells)

BRANCHES	STORES				FUZZY SUPPLY
	D ₁	D ₂		D ₃	
B₁	7	4	8	6	4
B₂	9	3	2	4	3
B₃	2 5	2 6	4 3		8
FUZZY DEMAND	2	9		4	

(In accordance with fuzzy allocation table method, the initial basic feasible solution had formed)

Total fuzzy transportation cost

$$= (4 \times 8) + (2 \times 5) + (3 \times 2) + (2 \times 6) + (4 \times 3)$$

$$= 72$$

Conclusion:

In this paper, the fuzzy allocation table method is used to determine the initial basic feasible solution. The triangular fuzzy numbers are transformed into crisp values by the method of Robust Ranking technique. Finally, we found that the solution of fuzzy transportation was the reliable tool with high accuracy. Numerical examples are demonstrated by the fuzzy allocation table method. Recently, ATM is one of the useful techniques as preliminary solving tools for transportation problems.

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