Nat. Volatiles & Essent. Oils, 2021; 8(4): 9965-9978

Cost Analysis Of Fuzzy Tandem Queues Usinga Parametric Relation Pair

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Abstract:

In connection with vague data, fuzziness gravels a perplexing fragment to create uncertainty. This paper compares two different tandem models of queuing systems with cost estimates and employs fuzzy ranking based on preference relations. The characteristic measures of two distinct fuzzy tandem queues are being estimated by α -cut approach and interval arithmetic. Numerical examples are used to validate the effectiveness of the models.

Keywords: Queuing network, Tandem queue, Trapezoidal fuzzy number, α -cut, Interval- arithmetic, Characteristic measures, Total cost, Preference, Ranking fuzzy numbers, Fuzzy relations, Parametric relation pair.

1. Introduction:

The spectacular extension of queues to the fuzzy world has real-life implications for decision analysis, operations research, computer technology and abstract theory. Fuzziness is not a quxillary but one of the enhancements.Tandem queue is the simplest non-trivial network of queues. Tandem queue and tandem queuing network both are the specialized case of open queuing network. There are various applications of tandem queues to real-world cases including communication networks, production models, banking sectors, tollbooths and service stations, etc.

Nowadays, Tandem queuing model has attracted a lot of attention as a result of its practical applications in real life. Burke's proposed the theorem in 1956 to get the product form solution from the network. Baumann and Sandman analyzed the multi-server tandem queue using Markov chains. A system with two stations and general arrivals was studied by Tsiolras. Chand and Chen have developed a no-wait two-stage tandem queuing system as an admission control measure. The optimal cost of queuing systems, their fine structure, and their techniques have been explored by many researchers in several Markovian lines.

Fuzzy queuing model have been discussed by the authors likeR.J. Li and E.S. Lee, J.J. Buckley, R.S. Negi and E.S. Lee.The two types of fuzzy ranking methods are defuzzification methods and preference relation for comparing two fuzzy numbers.There is a diversity of methods in the literature of recent years. Wang presented a method for ranking triangular and trapezoidal fuzzy numbers based on a relative preference relation.Fuzzy number ranking has played a key role in fuzzy decision-making, and it has been the focus of a lot of research. There are many approaches that could be used to address the problem of fuzzy numbers. A parametric pair of regressions led to a ranking algorithm for trapezoidal fuzzy numbers presented by J. Dombi and T. Jonas.

This paper is organized as follows: The second section provides the basic definitions. The third section describes a model which figures the queue length and waiting time for two distinct tandem queues as well as its cost function. The fourth segment discusses methods for ranking trapezoidal fuzzy numbers according to preference relations. The fifth segment illustrates a numerical example. And the sixth section is the conclusion.

2. Preliminaries:

Definition: 2.1

The α - cut of a fuzzy number $\tilde{C}$ is defined as $\tilde{C}=\{ x:\phi_{\tilde{C}}(x)\geq\alpha,\ \alpha\in[0,1]\}$

Definition: 2.2

A fuzzy number $\tilde{C}$ is said to be trapezoidal fuzzy number if and only if there exists

real numbers, $c_1\leq c_2\leq c_3\leq c_4$, such that:

$$\phi_{\tilde{c}}(x)=\begin{cases}\dfrac{x-c_1}{c_2-c_1}, & c_1\leq x\leq c_2\\ 1, & c_2\leq x\leq c_3\\ \dfrac{x-c_4}{c_3-c_4}, & c_3\leq x\leq c_4\\ 0, & otherwise\end{cases}$$

Denoted by $\tilde{c}=(c_1,c_2,c_3,c_4)$ or $\tilde{c}=(c_1/c_2/c_3/c_4)$.

2.3. Interval Analysis Arithmetic:

Let I_1 and I_2 be two interval numbers defined by ordered pair of real numberswith lower and upper bounds. Consider $I_1 = [p,q],\ p < q$ and $I_1 = [r,s],\ r < s$

$I_1 * I_2 = [p,\ q]*[r,\ s]$, where $* = [+,\ -,\ \times,\ \div]$ symbolically.

(i) $I_1 + I_2 = [p+r,\ q+s]$

(ii) $I_1 - I_2 = [p-s,\ q-r]$

(iii) $I_1 \bullet I_2 = [\min(pr, ps, qr, qs), \max(pr, ps, qr, qs)]$

(iv) $I_1 \div I_2 = [p,q] \bullet \left[\frac{1}{s}, \frac{1}{r}\right]$, provided $s, r \neq 0$

(v) $\alpha[p,q] = \begin{cases} [\alpha p, \alpha q], & \alpha > 0 \\ [\alpha q, \alpha p], & \alpha < 0 \end{cases}$

3. Two Distinct Tandem queuing models

3.1. Three stage fuzzy tandem queuing Model:

A tandem line with three service stations are considered wherein with a fuzzy arrival rate $\tilde{\lambda}$ the passengers from outside entrance to go the station S_1,afterthat the passengers should go to the station S_2, from the station S_2 for getting administration, before the station S_3 they join the queue. The passengers leave the lining framework solely after the culmination of administration at station S_3. All the states follow service time as exponential distribution with fuzzy service rate and they are denoted by $\tilde{\mu}_1$, $\tilde{\mu}_2$ and $\tilde{\mu}_3$ respectively. The limit of the line space is attempted to be infinite and at stations S_1, S_2 and S_3. Administration time follows outstanding conveyances with boundaries respectively.It is likewise to be noticed that at a time one customer can benefit from the assistance from each station.

Characteristic Measures:

(i) Average number of passengers in the system (L_s)

$$L_s = \frac{\tilde{\lambda}}{\tilde{\mu}_1 - \tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}_2 - \tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}_3 - \tilde{\lambda}}$$

(ii) Average waiting time of passengers in the system (W_S)

$$W_s = \frac{1}{\tilde{\mu}_1 - \tilde{\lambda}} + \frac{1}{\tilde{\mu}_2 - \tilde{\lambda}} + \frac{1}{\tilde{\mu}_3 - \tilde{\lambda}}$$

(iii) Average number of passengers in the queue $\left(L_q\right)$

$$L_q = \tilde{\lambda}^2 \left[\frac{1}{\tilde{\mu}_1(\tilde{\mu}_1 - \tilde{\lambda})} + \frac{1}{\tilde{\mu}_2(\tilde{\mu}_2 - \tilde{\lambda})} + \frac{1}{\tilde{\mu}_3(\tilde{\mu}_3 - \tilde{\lambda})} \right]$$

(iv) Average waiting time of passengers in the queue $\left(W_q\right)$

$$W_q = \tilde{\lambda} \left[\frac{1}{\tilde{\mu}_1(\tilde{\mu}_1 - \tilde{\lambda})} + \frac{1}{\tilde{\mu}_2(\tilde{\mu}_2 - \tilde{\lambda})} + \frac{1}{\tilde{\mu}_3(\tilde{\mu}_3 - \tilde{\lambda})} \right]$$

The Cost function of three stage tandem queuing model is given by

$$C_T(s_v) = C_s S_v + C_w L_s\,, \quad with \ \ S_v = 3$$

$where\ C_T(s_v)$ is the expected total cost, C_s is the idle cost of each server, $C_w$is the cost related with customer's waiting time during service,$L_s$is the average number of passengers spends in the system and S_v is the total usage of servers.

3.2. Parallel four-state fuzzy tandem open queuing network:

We built a model in the form of open parallel tandem queuing network and every single queue is M/M/1 line. We have taken two tandem queues, and each queue has two states and both are placed parallel. The queue capacity is infinite. The queue has a fuzzyarrival rateas Poisson distribution denoted by $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$. The model contains four service states, and all the states follow service time as exponential distribution, and they are denoted by $\tilde{\mu}_1$, $\tilde{\mu}_2$, $\tilde{\mu}_3$ and $\tilde{\mu}_4$ respectively. The model is likewise an open queuing organization, and it implies a passenger comes from move to go into the framework and passenger leaves the framework after the assistance.

Characteristic Measures:

(i) Average number of passengers in the system $\left(L_s^{'}\right)$

$$L_s^{'} = \frac{\tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1} + \frac{\tilde{\lambda}_1}{\tilde{\mu}_2 - \tilde{\lambda}_1} + \frac{\tilde{\lambda}_2}{\tilde{\mu}_3 - \tilde{\lambda}_2} + \frac{\tilde{\lambda}_2}{\tilde{\mu}_4 - \tilde{\lambda}_2}$$

(ii) Average waiting time of the passengers in the system $\left(W_S^{'}\right)$

$$W_s^{'} = \frac{1}{\tilde{\mu}_1 - \tilde{\lambda}_1 + \tilde{\lambda}_2} + \frac{1}{\tilde{\mu}_2 - \tilde{\lambda}_1 + \tilde{\lambda}_2} + \frac{1}{\tilde{\mu}_3 - \tilde{\lambda}_2 + \tilde{\lambda}_1} + \frac{1}{\tilde{\mu}_4 - \tilde{\lambda}_2 + \tilde{\lambda}_1}$$

(iii) Average number of passengers in the queue $\left(L_q^{'}\right)$

$$L_q^{'} = \tilde{\lambda}_1^{\,2}\left[\frac{1}{\tilde{\mu}_1(\tilde{\mu}_1 - \tilde{\lambda}_1)} + \frac{1}{\tilde{\mu}_2(\tilde{\mu}_2 - \tilde{\lambda}_1)}\right] + \tilde{\lambda}_2^{\,2}\left[\frac{1}{\tilde{\mu}_3(\tilde{\mu}_3 - \tilde{\lambda}_2)} + \frac{1}{\tilde{\mu}_4(\tilde{\mu}_4 - \tilde{\lambda}_2)}\right]$$

(v) Average waiting time of passengers in the queue $\left(W_q^{'}\right)$

$$W_q^{'} = \frac{\tilde{\lambda}_1}{\tilde{\lambda}_2}\left[\frac{1}{\tilde{\mu}_1(\tilde{\mu}_1 - \tilde{\lambda}_1)} + \frac{1}{\tilde{\mu}_2(\tilde{\mu}_2 - \tilde{\lambda}_1)}\right] + \frac{\tilde{\lambda}_2}{\tilde{\lambda}_1}\left[\frac{1}{\tilde{\mu}_3(\tilde{\mu}_3 - \tilde{\lambda}_2)} + \frac{1}{\tilde{\mu}_4(\tilde{\mu}_4 - \tilde{\lambda}_2)}\right]$$

The Cost function of four state parallel tandem open queuing network is given by

$$C_T^{'}(s_v^{'}) = C_s^{'}S_v^{'} + C_w^{'}L_s^{'}, \quad with \;\; S_v^{'} = 4$$

where $C_T^{'}(s_v^{'})$ is the expected total cost of given model, $C_s^{'}$ is the idle cost of each server, $C_w^{'}$ is the cost related with customer's waiting time during service, $L_s^{'}$ is the average number of passengers spends in the system and $S_v^{'}$ is the total usage of servers.

4. Ranking Method

4.1. Ranking trapezoidal fuzzy numbers using a parametric relation pair:

Let $M_F(A,B)$ be the probability-based preference intensity indexfor the fuzzy sets A andB when they have trapezoidal membership functions. This fuzzy preference intensity consistsof the following steps.

<u>Inputs:</u>

The trapezoidal membership functions of the fuzzy sets A and B having the parameters $\underline{x}_A^L < \overline{x}_A^L \leq \overline{x}_A^R < \underline{x}_A^R$ and $\underline{x}_B^L < \overline{x}_B^L \leq \overline{x}_B^R < \underline{x}_B^R$ respectively.

Step 1: S = ϕ

Step 2:

- If $\overline{x}_A^L - \underline{x}_A^L \neq \overline{x}_B^L - \underline{x}_B^L$, then $\alpha_1 = \dfrac{\underline{x}_A^L - \underline{x}_B^L}{\underline{x}_A^L - \overline{x}_A^L + \overline{x}_B^L - \underline{x}_B^L}$;
- If $\overline{x}_A^L - \underline{x}_A^L \neq \overline{x}_B^R - \underline{x}_B^R$, then $\alpha_2 = \dfrac{\underline{x}_A^L - \underline{x}_B^R}{\underline{x}_A^L - \overline{x}_A^L + \overline{x}_B^R - \underline{x}_B^R}$;
- If $\overline{x}_A^R - \underline{x}_A^R \neq \overline{x}_B^L - \underline{x}_B^L$, then $\alpha_3 = \dfrac{\underline{x}_A^R - \underline{x}_B^L}{\underline{x}_A^R - \overline{x}_A^R + \overline{x}_B^L - \underline{x}_B^L}$;
- If $\overline{x}_A^R - \underline{x}_A^R \neq \overline{x}_B^R - \underline{x}_B^R$, then $\alpha_4 = \dfrac{\underline{x}_A^R - \underline{x}_B^R}{\underline{x}_A^R - \overline{x}_A^R + \overline{x}_B^R - \underline{x}_B^R}$;

Step 3: If $\alpha_i \in (0,1)$ and $\alpha_i \notin S$, then $S := S \cup \{\alpha_i\}$; $\forall i \in \{1,2,3,4\}$

Step 4: $n := |S|$; S contains all the elements of ordered set $\{\alpha_1', \ldots, \alpha_n'\}$ is in increasing order;

$\alpha_0' = 0 \ and \ \alpha_{n+1}' = 1;$

Step 5: Let $\alpha_i^* := \dfrac{\alpha_i' + \alpha_{i+1}'}{2}, \ for \ all \ i \in \{0,1,\ldots,n\}$ and $M_F(A,B) := 0$

$a_A(\alpha_i^*) := \alpha_i^* \overline{x}_A^L + (1-\alpha_i^*)\underline{x}_A^L, \ b_A(\alpha_i^*) := \alpha_i^* \overline{x}_A^R + (1-\alpha_i^*)\underline{x}_A^R,$

$a_B(\alpha_i^*) := \alpha_i^* \overline{x}_B^L + (1-\alpha_i^*)\underline{x}_B^L, \ b_B(\alpha_i^*) := \alpha_i^* \overline{x}_B^R + (1-\alpha_i^*)\underline{x}_B^R,$

$$M_i := \begin{cases} I^p_{aA\geq bB}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_B(\alpha_i^*) < b_B(\alpha_i^*) \leq a_A(\alpha_i^*) < b_A(\alpha_i^*) \\ I^p_{aB\geq bA}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_A(\alpha_i^*) < b_A(\alpha_i^*) \leq a_B(\alpha_i^*) < b_B(\alpha_i^*) \\ I^o_{aA<aB}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_A(\alpha_i^*) < a_B(\alpha_i^*) < b_A(\alpha_i^*) < b_B(\alpha_i^*) \\ I^o_{aA>aB}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_B(\alpha_i^*) < a_A(\alpha_i^*) < b_B(\alpha_i^*) < b_A(\alpha_i^*) \\ I^i_{IA\subseteq IB}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_B(\alpha_i^*) \leq a_A(\alpha_i^*) < b_A(\alpha_i^*) \leq b_B(\alpha_i^*) \\ I^i_{IB\subseteq IA}(\alpha_i^{'},\alpha_{i+1}^{'}), & if\, a_A(\alpha_i^*) \leq a_B(\alpha_i^*) < b_B(\alpha_i^*) \leq b_A(\alpha_i^*) \end{cases}$$

(i) Computing $I^p_{aA\geq bB}(\alpha_i^{'},\alpha_{i+1}^{'}), \quad if\;\; a_B(\alpha_i^*) < b_B(\alpha_i^*) \leq a_A(\alpha_i^*) < b_A(\alpha_i^*)$ and

$I^p_{aB\geq bA}(\alpha_i^{'},\alpha_{i+1}^{'}), \quad if\, a_A(\alpha_i^*) < b_A(\alpha_i^*) \leq a_B(\alpha_i^*) < b_B(\alpha_i^*)$

Obviously we get, $I^P_{aA\geq bB}(\alpha', \alpha'') = 0$

$I^P_{aB\geq bA}(\alpha', \alpha'') = \alpha'' - \alpha'$. ....(1)

(ii)Computing $I^o_{aA<aB}(\alpha_i^{'},\alpha_{i+1}^{'}), \quad if\, a_A(\alpha_i^*) < a_B(\alpha_i^*) < b_A(\alpha_i^*) < b_B(\alpha_i^*)$

$$I^o_{aA<aB}(\alpha', \alpha'') = \left(1 - \frac{A_0^{\,2}}{8A_1A_2}\right)(\alpha'' - \alpha') + \frac{(A_0B_2 - A_2B_0)^2}{8A_2^{\,2}(A_1B_2 - A_2B_1)} \ln\left|\frac{A_2\alpha'' + B_2}{A_2\alpha' + B_2}\right| - \frac{(A_0B_1 - A_1B_0)^2}{8A_1^{\,2}(A_1B_2 - A_2B_1)} \ln\left|\frac{A_1\alpha'' + B_1}{A_1\alpha' + B_1}\right|$$

$$where \;\; A_0 = (\overline{x}_A^R - \underline{x}_A^R) - (\overline{x}_B^L - \underline{x}_B^L), \qquad B_0 = (\underline{x}_A^R - \underline{x}_B^L), \qquad \text{.......(2)}$$

$$A_1 = \frac{1}{2}((\overline{x}_A^R - \underline{x}_A^R) - (\overline{x}_A^L - \underline{x}_A^L)), \quad B_1 = \frac{1}{2}(\underline{x}_A^R - \underline{x}_A^L),$$

$$A_2 = \frac{1}{2}((\overline{x}_B^R - \underline{x}_B^R) - (\overline{x}_B^L - \underline{x}_B^L)), \quad B_2 = \frac{1}{2}(\underline{x}_B^R - \underline{x}_B^L),$$

(iii) Computing $I^o_{aA>aB}(\alpha_i^{'},\alpha_{i+1}^{'}), \quad if\, a_B(\alpha_i^*) < a_A(\alpha_i^*) < b_B(\alpha_i^*) < b_A(\alpha_i^*)$

$$I^{o}_{aA>aB}(\alpha',\alpha'') = \frac{A_0^{\,2}}{8A_1A_2}(\alpha''-\alpha') + \frac{(A_0B_1 - A_1B_0)^2}{8A_1^{\,2}(A_1B_2 - A_2B_1)}\ln\left|\frac{A_1\alpha''+B_1}{A_1\alpha'+B_1}\right|$$
$$-\frac{(A_0B_2 - A_2B_0)^2}{8A_2^{\,2}(A_1B_2 - A_2B_1)}\ln\left|\frac{A_2\alpha''+B_2}{A_2\alpha'+B_2}\right|$$

where $A_0 = (\overline{x}_B^R - \underline{x}_B^R) - (\overline{x}_A^L - \underline{x}_A^L)$, $B_0 = (\underline{x}_B^R - \underline{x}_A^L)$, ...(3)

$A_1 = \frac{1}{2}((\overline{x}_A^R - \underline{x}_A^R) - (\overline{x}_A^L - \underline{x}_A^L))$, $B_1 = \frac{1}{2}(\underline{x}_A^R - \underline{x}_A^L)$,

$A_2 = \frac{1}{2}((\overline{x}_B^R - \underline{x}_B^R) - (\overline{x}_B^L - \underline{x}_B^L))$, $B_2 = \frac{1}{2}(\underline{x}_B^R - \underline{x}_B^L)$,

(iv)Computing $I^{i}_{IA\subseteq IB}(\alpha_i^{'},\alpha_{i+1}^{'})$, $if\, a_B(\alpha_i^*) \le a_A(\alpha_i^*) < b_A(\alpha_i^*) \le b_B(\alpha_i^*)$

$$I^{i}_{IA\subseteq IB}(\alpha',\alpha'') = \frac{1}{2}(\alpha''-\alpha')\left(1+\frac{A_0}{A_1}\right) - \frac{(A_0B_1 - A_1B_0)}{2A_1^{\,2}}\ln\left|\frac{A_1\alpha''+B_1}{A_1\alpha'+B_1}\right|$$

where $A_0 = \frac{1}{2}((\overline{x}_B^R - \underline{x}_B^R) + (\overline{x}_B^L - \underline{x}_B^L) - (\overline{x}_A^L - \underline{x}_A^L) - (\overline{x}_A^R - \underline{x}_A^R))$, ... (4)

$B_0 = \frac{1}{2}(\underline{x}_B^L + \underline{x}_B^R - \underline{x}_A^L - \underline{x}_A^R)$,

$A_1 = \frac{1}{2}((\overline{x}_B^R - \underline{x}_B^R) - (\overline{x}_B^L - \underline{x}_B^L))$, $B_1 = \frac{1}{2}(\underline{x}_B^R - \underline{x}_B^L)$.

(v) Computing $I^{i}_{IB\subseteq IA}(\alpha_i^{'},\alpha_{i+1}^{'})$, $if\;\, a_A(\alpha_i^*) \le a_B(\alpha_i^*) < b_B(\alpha_i^*) \le b_A(\alpha_i^*)$

$$I^{i}_{IB\subseteq IA}(\alpha',\alpha'') = \frac{1}{2}(\alpha''-\alpha')\left(1+\frac{A_0}{A_1}\right) - \frac{(A_0B_1 - A_1B_0)}{2A_1^{\,2}}\ln\left|\frac{A_1\alpha''+B_1}{A_1\alpha'+B_1}\right| \quad \text{......... (5)}$$

where $A_0 = \frac{1}{2}((\overline{x}_B^R - \underline{x}_B^R) + (\overline{x}_B^L - \underline{x}_B^L) - (\overline{x}_A^L - \underline{x}_A^L) - (\overline{x}_A^R - \underline{x}_A^R))$,

$B_0 = \frac{1}{2}(\underline{x}_B^L + \underline{x}_B^R - \underline{x}_A^L - \underline{x}_A^R)$,

$A_1 = \frac{1}{2}((\overline{x}_A^R - \underline{x}_A^R) - (\overline{x}_A^L - \underline{x}_A^L))$, $B_1 = \frac{1}{2}(\underline{x}_A^R - \underline{x}_A^L)$.

Utilizing the above cases, we calculate the values of $M_i, where\; i \in \{1,2,3,4\}$

Hence $M_F(A,B) := M_F(A,B) + M_i$

Output: $M_F(A,B)$

5. Numerical Approximation

5.1. Three stage fuzzy tandem queuing Model:

Let the arrival rate and service rate are trapezoidal fuzzy numbers. Let $\tilde{\lambda} = [1,\ 2,3,4]$; $\tilde{\mu}_1 = [10,11,12,13]$; $\tilde{\mu}_2 = [15,16,17,18]$ *and* $\tilde{\mu}_3 = [20,21,22,23]$. The characteristic measures are being estimated by α -cut approach and interval arithmetic.

(i) Average number of passengers in the system (L_s)

$$L_s = \left[\frac{1+\alpha}{12-2\alpha}, \frac{-\alpha+4}{6+2\alpha}\right] + \left[\frac{1+\alpha}{17-2\alpha}, \frac{-\alpha+4}{11+2\alpha}\right] + \left[\frac{1+\alpha}{22-2\alpha}, \frac{-\alpha+4}{16+2\alpha}\right]$$
$$= [0.1876,\ 0.4333,\ 0.7725,\ 1.2803]$$

(ii) Average waiting time of passengers in the system (W_S)

$$W_s = \frac{1}{[6+2\alpha,\ 12-2\alpha]} + \frac{1}{[11+2\alpha,\ 17-2\alpha]} + \frac{1}{[16+2\alpha,\ 22-2\alpha]}$$
$$= [0.1876,\ 0.2167,\ 0.2575,\ 0.3201]$$

(iii) Average number of passengers in the queue (L_q)

$$L_q = \left[\frac{\alpha^2+2\alpha+1}{2\alpha^2-38\alpha+156}, \frac{\alpha^2-8\alpha+16}{2\alpha^2+26\alpha+60}\right] + \left[\frac{\alpha^2+2\alpha+1}{2\alpha^2-53\alpha+306}, \frac{\alpha^2-8\alpha+16}{2\alpha^2+41\alpha+165}\right]$$
$$+ \left[\frac{\alpha^2+2\alpha+1}{2\alpha^2-68\alpha+506}, \frac{\alpha^2-8\alpha+16}{2\alpha^2+56\alpha+320}\right]$$
$$= [0.0117,\ 0.0581,\ 0.1694,\ 0.4137]$$

(iv) Average waiting time of passengers in the queue (W_q)

$$W_q = \left[\frac{1+\alpha}{2\alpha^2-38\alpha+156}, \frac{4-\alpha}{2\alpha^2+26\alpha+60}\right] + \left[\frac{1+\alpha}{2\alpha^2-53\alpha+306}, \frac{4-\alpha}{2\alpha^2+41\alpha+165}\right] +$$
$$+ \left[\frac{1+\alpha}{2\alpha^2-68\alpha+506}, \frac{4-\alpha}{2\alpha^2+56\alpha+320}\right]$$
$$W_q = [0.0117,\ 0.0290,\ 0.0564,\ 0.1034]$$

(v) Total Expected cost

Let $C_S = [100,\ 200, 300, 400]$ bethe fuzzy cost of each operator and

$C_W = [150, 200, 250, 300]$ be the fuzzy cost related with passengers waiting time during service with three service stations.

$$\begin{aligned} C_T(s_v) &= C_s S_v + C_w L_s\,, \quad with \;\; S_v = 3 \\ &= [328.1400,\ 686.6600, 1093.1250, 1584.0900] \\ C_{\alpha T}(s_v) &= [328.14 + 358.52\alpha,\ 1584.09 - 490.9650\alpha] \end{aligned}$$

5.2. Parallel four-state fuzzy tandem open queuing network:

Let the arrival rates and service rates are trapezoidal fuzzy numbers. Let $\tilde{\lambda}_1 = [1,\ 2, 3, 4]$; $\tilde{\lambda}_2 = [5,\ 6, 7, 8]$; $\tilde{\mu}_1 = [10, 11, 12, 13]$; $\tilde{\mu}_2 = [15, 16, 17, 18]$; $\tilde{\mu}_3 = [20, 21, 22, 23]$ and $\tilde{\mu}_4 = [25,\ 26,\ 27,\ 28]$. The characteristic measures are being estimated by α -cut approach and interval arithmetic.

(i) Average number of passengers in the system (L_s')

$$\begin{aligned} L_s' &= \left[\frac{1+\alpha}{12-2\alpha}, \frac{-\alpha+4}{6+2\alpha}\right] + \left[\frac{1+\alpha}{17-2\alpha}, \frac{-\alpha+4}{11+2\alpha}\right] + \left[\frac{5+\alpha}{18-2\alpha}, \frac{-\alpha+8}{12+2\alpha}\right] + \left[\frac{5+\alpha}{23-2\alpha}, \frac{-\alpha+8}{17+2\alpha}\right] \\ &= [0.6373, 0.9940,\ 1.4742,\ 2.1676] \end{aligned}$$

(ii) Average waiting time of passengers in the system (W_S')

$$\begin{aligned} W_s' &= \frac{1}{[11+3\alpha,\ 20-3\alpha]} + \frac{1}{[16+3\alpha,\ 25-3\alpha]} + \frac{1}{[13+3\alpha,\ 22-3\alpha]} + \frac{1}{[18+3\alpha,\ 27-3\alpha]} \\ &= [0.1725,\ 0.1986,\ 0.2341,\ 0.2859] \end{aligned}$$

(iii) Average number of passengers in the queue (L_q')

$$\begin{aligned} L_q' = &\left[\frac{\alpha^2+2\alpha+1}{2\alpha^2-38\alpha+156}, \frac{\alpha^2-8\alpha+16}{2\alpha^2+26\alpha+60}\right] + \left[\frac{\alpha^2+2\alpha+1}{2\alpha^2-53\alpha+306}, \frac{\alpha^2-8\alpha+16}{2\alpha^2+41\alpha+165}\right] \\ &+ \left[\frac{\alpha^2+10\alpha+25}{2\alpha^2-64\alpha+414}, \frac{\alpha^2-16\alpha+64}{2\alpha^2+52\alpha+240}\right] + \left[\frac{\alpha^2+10\alpha+25}{2\alpha^2-79\alpha+644}, \frac{\alpha^2-16\alpha+64}{2\alpha^2+67\alpha+425}\right] \\ = &\ [0.1089,\ 0.2148,\ 0.4115,\ 0.7810] \end{aligned}$$

(iv) Average waiting time of passengers in the queue (W_q')

$$W_q' = \left[\frac{1+\alpha}{(8-\alpha)(2\alpha^2-38\alpha+156)}, \frac{4-\alpha}{(5+\alpha)(2\alpha^2+26\alpha+60)}\right] + \left[\frac{1+\alpha}{(8-\alpha)(2\alpha^2-53\alpha+306)}, \frac{4-\alpha}{(5+\alpha)(2\alpha^2+41\alpha+165)}\right]$$
$$+\left[\frac{5+\alpha}{(4-\alpha)(2\alpha^2-64\alpha+414)}, \frac{8-\alpha}{(1+\alpha)(2\alpha^2+52\alpha+240}\right] + \left[\frac{5+\alpha}{(4-\alpha)(2\alpha^2-79\alpha+644)}, \frac{8-\alpha}{(1+\alpha)(2\alpha^2+67\alpha+425}\right]$$
$$= [0.0061, 0.0127, 0.0271, 0.0702]$$

(v) Total Expected cost

Let $C_S' = [50, 180, 230, 280]$ and $C_W' = [120, 140, 180, 200]$ be the fuzzy cost of each Operator and fuzzy cost related with passengers waiting time during service with four service stations.

$$C_T'(s_v') = C_s' S_v' + C_w' L_s', \quad with \;\; S_v' = 4$$
$$= [276.4760, 859.16, 1185.356, 1553.52]$$
$$C_{\alpha T}'(s_v') = [276.4760 + 582.6840\alpha, 1553.52 - 368.1640\alpha]$$

5.3. Preference Relation of Trapezoidal fuzzy numbers using a parametric relation Pair:

The cost functions of the two tandem queuing models are taken as follows;

$$A_\alpha = [328.14 + 358.52\alpha, 1584.09 - 490.9650\alpha]$$

$$B_\alpha = [276.4760 + 582.6840\alpha, 1553.52 - 368.1640\alpha]$$

<u>Inputs</u>:

The trapezoidal membership functions of the fuzzy sets A and B having the parameters $\underline{x}_A^L < \overline{x}_A^L \le \overline{x}_A^R < \underline{x}_A^R$ and $\underline{x}_B^L < \overline{x}_B^L \le \overline{x}_B^R < \underline{x}_B^R$ respectively. Let the parameter values be

$\underline{x}_A^L = 328.1400,$ $\quad \underline{x}_B^L = 276.4760$

$\overline{x}_A^L = 686.6600,$ $\quad \overline{x}_B^L = 859.1600$

$\overline{x}_A^R = 1093.1250,$ $\quad \overline{x}_B^R = 1185.356$

$\underline{x}_A^R = 1584.0900,$ $\quad \underline{x}_B^R = 1553.52$

<u>Step 1</u>: S = ϕ

<u>Step 2</u>: To find the intersection points of the trapezoids A_α & B_α

- If $\overline{x}_A^L - \underline{x}_A^L \neq \overline{x}_B^L - \underline{x}_B^L$, then $\alpha_1 = 0.2305$
- If $\overline{x}_A^L - \underline{x}_A^L \neq \overline{x}_B^R - \underline{x}_B^R$, then $\alpha_2 = 1.6863$
- If $\overline{x}_A^R - \underline{x}_A^R \neq \overline{x}_B^L - \underline{x}_B^L$, then $\alpha_3 = 1.2179$
- If $\overline{x}_A^R - \underline{x}_A^R \neq \overline{x}_B^R - \underline{x}_B^R$, then $\alpha_4 = 0.2489$

Step 3: The set $S=\{\alpha_i:\alpha_i\in(0,1), i\in 1,2,3\}=\{0.2305, 0.2489\}$.

Therefore the trapezoids A_α and B_α have two intersection points.

Step 4: The ordered set $S=\{\alpha_1', \alpha_2'\}=\{0.2305, 0.2489\}$ with n =2

$\alpha_0^{'}=0 \; and \; \alpha_{n+1}^{'}=1;$

Step 5: Let $\alpha_i^* := \dfrac{\alpha_i^{'}+\alpha_{i+1}^{'}}{2}, \; for\, all\, i\in\{0,1,\ldots,n\}$

$\alpha_0^{'}=0,\; \alpha_1^{'}=0.2305,\; \alpha_2^{'}=0.2489 \; and \; \alpha_3^{'}=1;$

To find M_0:

$$\alpha_0^*=\frac{\alpha_0^{'}+\alpha_1'}{2}=0.1153$$

$$a_A(\alpha_0^*)=369.4774, \quad a_B(\alpha_0^*)=343.6595$$
$$b_A(\alpha_0^*)=1527.4817,\; b_B(\alpha_0^*)=1151.0707$$

Since $a_B(\alpha_1^*)<a_A(\alpha_1^*)<b_B(\alpha_1^*)<b_A(\alpha_1^*)$ holds then by using equation (3) $I^o_{aA>aB}(0,\alpha_1^{'})$,

Weget $M_0=0.1111$

To find M_1:

$$\alpha_1^*=\frac{\alpha_1^{'}+\alpha_2'}{2}=0.2397$$

$$a_A(\alpha_1^*)=414.0772, \quad a_B(\alpha_1^*)=416.1454$$
$$b_A(\alpha_1^*)=1466.4059,\; b_B(\alpha_1^*)=1465.2711$$

Since $a_A(\alpha_1^*)<a_B(\alpha_1^*)<b_B(\alpha_1^*)<b_A(\alpha_1^*)$ holds then by using equation (5) $I^i_{IB\subseteq IA}(\alpha_1^{'},\alpha_2^{'})$,

We get $M_1=0.0092$

To find M_2:

$$\alpha_2^*=\frac{\alpha_2^{'}+\alpha_3'}{2}=0.6245$$

$a_A(\alpha_2^*) = 552.0357, \quad a_B(\alpha_2^*) = 640.3622$
$b_A(\alpha_2^*) = 1277.4824, \; b_B(\alpha_2^*) = 1323.6016$

Since $a_A(\alpha_2^*) < a_B(\alpha_2^*) < b_A(\alpha_2^*) < b_B(\alpha_2^*)$ holds then by using equation (2) $I^o_{aA<aB}(\alpha_2^{'}, \alpha_3^{'}),$,

We get $M_2 = 0.4570$

Step 6:

$$M_0 = I^o_{aA>aB}(0, \alpha_1^{'}) \;\; = 0.1111$$
$$M_1 = I^i_{IB \subseteq IA}(\alpha_1^{'}, \alpha_2^{'}) = 0.0092$$
$$M_2 = I^o_{aA<aB}(\alpha_3^{'}, 1) \;\; = 0.4570$$

Hence $M_F(A_\alpha, B_\alpha) = M_0 + M_1 + M_2 = 0.5773 > \frac{1}{2}$ which is denoted as A_α is preferred to B_α

6. Conclusion

When it comes to classifying fuzzy numbers, the preference relationship is preferable to defuzzification. The preference ratio accelerates defuzzification's robustness and the fuzzy preference ratio. Using the trapezoidal fuzzy number, this paper investigates the performance metrics of two different fuzzy tandem queuing systems. We compare the cost measures of two different queuing systems and use a pair of parametric relationships to determine their preference relationship. This fuzzy preference relationship minimises the cost of making a better choice.

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