Green Inventory Model With The Impact Of Trade Credit And Partial Backordering Under Permissible Delay In Payment

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Abstract:

Green inventory management for reworking the defective item which brings long lasting benefits. In global business, the products are procured from a global supplier in many situations. In this situation, there are chances that the received many items which may contain a fraction of imperfect products. The defective items are also valuable and can be repairable to recover the environment. It is sustainable to repair defective items in a local repair store is affordable when contrasted with sending them back to the retailer. The cost of carbon emissions is also incorporated into the effect on net income for the environment. Meantime, the supplier offers multi-trade-credit-period to the buyer. The proposed model is sustainable and reduces the environmental impact as well as benefits for financing. This paper seeks to maximize the total profit by developing a synergic economic order quantity model by considering multi-trade-credit policy, rework, shortages, green branding and proportion of firms with registered patents in green technologies simultaneously. This model can help in making better decisions to enhance sustainable inventory management efficiency by controlling the cycle time and a fraction of time for a global supply chain.The numerical illustration is given to determine the proposed model.

Keywords: Inventory Model, defective items, Permissible delay in payment, green branding, green patent.

1.Introduction:

Production mangers implement and apply efficient production planning under control systems to achieve 100% perfect items at an economized cost for a sustainable environment. Still, the production system may manufacture with some imperfect items. The imperfect items cut down the income of the buyer and also have a negative effect on the environment by focusing on the extra activity requiring the supplier to exchange these imperfect items. This unintentional supply will cause a loss of goodwill to the customer. Nowadays, in order to purchase sustainable goods or renewable raw material at an economical cost in the supply chain, the customer first finds out the suppliers and then work out the best one. As suppliers and customers are far away from each other, this also makes it difficult for a manufacturer to give the customer all the perfect items. Thus, to ensure good quality and brand reputation, it is an important for a customer to check an entire lot as soon as it reaches an inventory after the goods have been screened. But it may be possible that some percentage of items is detected as defective with minor damage. The exchange of defectiveitems with an immediateshipment is not suitable and it also has negative impact on the environment because the supplier is located miles away from the customer.If the exchange of defective items is going to happen with a global supplier, then it will increase the environmental cost, which in turn also raises the overall cost of the service. These minor damages can be repaired at a local repair shop to maintain sustainable supply of products at an economic cost and to reduce the environmental impact; the sustainable approach is to repair these items at a local repair shop as compared to exchanging them with the supplier.

Green logistics is a measure aimed at reducing the pollution and sustainable policy taken by the logistics industry to minimize the environmental impact on transportation, warehousing and other logistics activities. This policy is helped to create a sustainable company value that balances the economic and environmental efficiency. It is a way that usesnew advanced technology and equipment to reduce environmental damage during productions. Green branding is a method of developing and advertising products based on their environmental sustainability. This practice will create awareness among the public and enhances the sales which also gives more profit to the firms.The total holding cost per unit time is a combination of the holding cost of perfect products that are already in the system and the holding cost of repaired products. Patents of interest for green creation are those for green or environmental advancements. Green technologies which includes technical processes, facilities and goods and services, the purpose of technical nature of which is environmental protection or resource management. These can be classified into two types such as end-of-pipe and integrated technologies. The end-of-pipe (Pollution treatment) technologies which intended for measurement, control, treatment and restoration of pollution, environmental degradation and resource depletion. The integrated (Pollution prevention) technologies used in production processes that are less polluting the atmosphere and less resource-intensive.

Biswajit Sarkar, Waqas Ahmed, Seok-Beom choi and Muhammad Tayyab introduced the partial backordering and Multi-Trade-Credit - Time of sustainable Resource Management for Environmental Impact. Compared the charge for send back to retailer, they suggested the sustainable inventory model to fix defective goods in a nearby repair store. Antonitte Vinoline, Ritha and Merline Vinotha discussed an environmentally sustainable inventory model with the impact of trade credit and partial backordering. Marchi, Zanoni, Zavanella and Jaber developed a Green supply chain vendor-buyer model by considering both a decentralized and a centralized integrated approach. He introduced investing in technology development, and reducing carbon emission from the manufacturing processes. Voros examined models of economic order quantity of production without restriction to the fraction of imperfect products. Kim and Sarkar suggested a multi-stage cleaner production system by eliminating all defective items during the production process.Jaber et al. studied an EOQ model for imperfect quality, reworking and emergency purchases from a local store at a higher cost. Salameh and Jaber proposed the EOQ model for defective items by considering the random fraction of imperfect products in a lot. Wahab et al. extended a sustainable inventory model for a global supply chain by taking into consideration imperfect items and environmental impact.

The rest of the paper is structured as follows: Section 2 provides fundamental assumptions and notations. Section 3 describes the Mathematical Model in detail. Section 4 illustrates a numerical example. Section 5 concludes the paper.

2. Mathematical Formulation:

To develop the proposed model, the following notations and assumptions are defined throughout this paper.

Notations

Decision Variables

T Cycle Time

F Fraction of time that has a positive inventory level

Dependent Variable

Q Order size per cycle

Parameters

D Demand rate per unit time

X Screening rate

t_s Screening time of products

C_s Screening cost per unit

t_R Rework and return time for imperfect products

R Rework rate

β	Percentage of imperfect items
O	Ordering cost of buyer
S_r	Setup cost of repair store
H	Holding cost of perfect items
H'	Carbon emission cost per item on holding perfect items
H_r	Holding cost of rework products
H_r'	Carbon emission cost per item on holding rework items
H_s	Holding cost at repair store
H_s'	Carbon emission cost per item on holding item at repair store
C_u	Purchasing cost of one unit
C_{lm}	Labor and material cost required to repair a unit product
l	Cost incurred due to a loss of sales
G	Penalty cost incurred due to goodwill loss
W	Percentage of imperfect items passed to customers
u	unit return cost of the imperfect product
π	Backordered cost
P	Selling Price of one unit
γ	Percentage of backordered demand
m	Markup percentage by rework store
M_1	First Permissible delay period for payment
M_2	Second Permissible delay period for payment
I_e	Interest earned
I_{c1}	Interest charged for period M_1

I_{c2} Interest charged for period M_2

a Fixed cost per trip

b Social cost from vehicle emission

d distance travelled

v Average velocity

B_c Green branding cost

P_c Proportion of firms with registered patents in green technologies

Assumptions

1. The inventory model has a single type of product
2. Emission of carbon from warehouse is due to the consumption of electricity per unit item. Thus, according to carbon tax policy, carbon emission cost per unit items $H^{'}$,$H_r^{'}$ and $H_s^{'}$ is considered for holding perfect items, holding repair items and holding items at the repair store respectively.
3. Shortages are permitted and these are partially backordered.
4. Demand and inspection rates are considered as known and constant.
5. The screening techniques and demand occur at the same time, but the screening cost is higher than the price of production (X>D).
6. Imperfect products have minor damage which can be repairable in a controlled system and all defective items are reworked.
7. The percentage of defective items are given and known.
8. The relationship between the purchasing cost and selling price of buyer is $P \geq C_u$.
9. The holding cost of reworked produces exceeds the initial holding cost of perfect items $(H_r > H)$.
10. The dealer grants the buyer M_1 and M_2 multi-trade credit period. At some point in this time, the buyer sell the commodity and utilizes its income to earn interest with the rate of I_e.
11. When the inventory level of the system becomes zero then the reworked items are returned.
12. If the buyer fails to pay the supplier during the first credit period M_1, then interest I_{c1} is charged and later if the purchaser again fails to pay the supplier during second allowable time M_2, then interest I_{c2} is also charged.

13. The percentage of defective item is sent to customers, which are returned back to the buyer in the next cycle. The purchaser pays a cost per unit for these returned products and a cost per unit as a penalty cost incurred due to goodwill loss.

3. Mathematical Model:

This section describes and develops a green inventory model of total profit inventory, with multiple delays in payments, partial back ordering, repair of imperfect products, green branding, green logistics and green patent.

(i) Ordering cost = $\frac{O}{T}$

(ii) Inspection cost = $C_s FD$

(iii) Holding cost = $\left(H+H'\right)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right]+\left(H_r+H_r'\right)\left[\frac{(\beta F)^2TD}{2}\right]$

(iv) Rework cost = $(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\left(H_s+H_s'\right)\left(\frac{\beta FTD}{R}\right)\right]$, $t_R=\left(\frac{\beta FTD}{R}\right)$

The defective items come back into the inventory when the initial inventory level becomes zero, therefore the level of inventory becomes βFTD units.As the cycle ends, $(1-F)TD$ becomes the shortage level of the system. The order quantity for a given cycle is considered to be $Q=FTD+\gamma(1-F)TD$.

(v) Shortage cost = $\frac{\gamma(1-F)^2TD}{2}+l(1-\gamma)(1-F)D$

(vi) Goodwill penalty cost = $(u+G)wFD$

(vii) Green logistics cost = $\frac{1}{T}(2a+2b\frac{d}{v})$

(viii) Green branding cost = $\frac{B_c}{T}$

(ix) Green Patent cost = $\frac{P_c}{T}$

Interest Charged and Interest Earned

If the allowable payment duration is larger than the lead time, it will carry interest income to the purchaser according to the trade credit policy. If this allowed time is smaller than lead time, then it would bring more opportunity cost and less interest income to the buyer; at the same time, the supplier can earn interest income and pay less opportunity cost. Because of this model, the supplier's model has the following cases, based on the permissible time of payment X and length of lead time, the different cost between two likely cases are as follows.

Condition 1:

If the lead time T is less than or equal to the supplier's permissible payment $\mathrm{M_1}$ duration, then only interest income is received as interest paid under such a condition is zero.

Interest income = $\mathrm{PI_e}\left[\mathrm{DM_1} - \frac{\mathrm{TD}}{2}\right]$

Condition 2:

If the lead time T is greater than the supplier's first allowable payment time $\mathrm{M_1}$ and less than or equal to the second allowable payment time $\mathrm{M_2}$ provided to the buyer, than all interest costs will be paid and received.

Interest income = $\mathrm{PI_e}\frac{(\mathrm{DM_1})^2}{2\mathrm{TD}}$

Interest charged = $\mathrm{C_u I_{c1}}\frac{(\mathrm{TD}-\mathrm{DM_1})^2}{2\mathrm{TD}}$

Condition 3:

There is a special case where the buyer will be charged more interest if they fail to give the required payment in the first permitted time. In this case the lead time T is greater than the supplier's second permissible payment time $\mathrm{M_2}$.

Interest income = $\mathrm{PI_e}\frac{(\mathrm{DM_1})^2}{2\mathrm{TD}}$

Interest charged = $\mathrm{C_u I_{c2}}\frac{(\mathrm{TD}-\mathrm{DM_2})^2}{2\mathrm{TD}} - \mathrm{C_u I_{c1}}\frac{\mathrm{D}}{\mathrm{T}}\left(\mathrm{M_2 T} - \mathrm{M_2}^2 - \mathrm{M_1 T} + \mathrm{M_1 M_2}\right) - \mathrm{C_u I_{c1}}\frac{\mathrm{D}(\mathrm{M_2}-\mathrm{M_1})^2}{2\mathrm{T}}$

Total Profit Function

Total Profit = Selling price – [ordering cost + Production cost + Inspection cost + Holding cost + Rework cost + shortage cost + Goodwill penalty cost + Green logistics cost + Green branding cost + Green patent cost + Interest earned – Interest charged]

According to these different conditions of multi-delay-in-payments, three cases are developed and the total profit function for all cases can be given as:

Case 1: Total Profit $(\mathrm{TP_1})$ if $\mathrm{T} \le \mathrm{M_1}$

$$\begin{aligned}TP(F,T) = {} & PD\big(F+\gamma(1-F)\big)\\ & -\Bigg(\frac{O}{T}+C_uD\big(F+\gamma(1-F)\big)+C_sFD+\left(H+H^{'}\right)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right]\\ & +\left(H_r+H_r^{'}\right)\left[\frac{(\beta F)^2TD}{2}\right]+\pi\frac{(1-F)^2\gamma TD}{2}\\ & +\beta FD(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\left(H_s+H_s^{'}\right)\left(\frac{\beta FTD}{R}\right)\right]+l(1-\gamma)(1-F)D\\ & +(u+G)wFD+\frac{1}{T}\left(2a+2b\frac{d}{v}\right)+\frac{B_c}{T}+\frac{P_c}{T}+PI_e\left[DM_1-\frac{TD}{2}\right]\Bigg)\end{aligned}$$

The profit function for case 1 in the above equation, found by adding +1 and -1 to the order quantity is given as:

$$\begin{aligned}TP(F,T) = {} & PD(1-(1-F)(1-\gamma))\\ & -\Bigg(\frac{O}{T}+C_uD(1-(1-F)(1-\gamma))+C_sFD+\left(H+H^{'}\right)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right]\\ & +\left(H_r+H_r^{'}\right)\left[\frac{(\beta F)^2TD}{2}\right]+\pi\frac{(1-F)^2\gamma TD}{2}\\ & +\beta FD(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\left(H_s+H_s^{'}\right)\left(\frac{\beta FTD}{R}\right)\right]+l(1-\gamma)(1-F)D\\ & +(u+G)wFD+\frac{1}{T}\left(2a+2b\frac{d}{v}\right)+\frac{B_c}{T}+\frac{P_c}{T}+PI_e\left[DM_1-\frac{TD}{2}\right]\Bigg)\end{aligned}$$

Rearranging the terms, this can be expressed as:

$$\begin{aligned}TP(F,T) = {} & PD-C_uD\\ & -\Bigg(\frac{O}{T}+C_sFD+\left(H+H^{'}\right)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right]+\left(H_r+H_r^{'}\right)\left[\frac{(\beta F)^2TD}{2}\right]\\ & +\pi\frac{(1-F)^2\gamma TD}{2}+\beta FD(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\left(H_s+H_s^{'}\right)\left(\frac{\beta FTD}{R}\right)\right]\\ & +l(1-\gamma)(1-F)D+(u+G)wFD+\frac{1}{T}\left(2a+2b\frac{d}{v}\right)+\frac{B_c}{T}+\frac{P_c}{T}+PI_e\left[DM_1-\frac{TD}{2}\right]\\ & +PD\big(1-(1-F)(1-\gamma)\big)-C_uD(1-(1-F)(1-\gamma))\Bigg)\end{aligned}$$

By substituting in $C_z = (P+l-C_u)$, the total profit function becomes

$$
\begin{aligned}
\mathrm{TP}(F,T) = {} & D(P-C_u) \\
& -\Bigg(\frac{O}{T} + C_s FD + (H+H') \left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T(FD)^2}{X}\right] + (H_r + H_r')\left[\frac{(\beta F)^2TD}{2}\right] \\
& + \pi\frac{(1-F)^2\gamma TD}{2} + \beta FD(1+m)\left[\frac{S_r}{\beta FTD} + C_{lm} + (H_s + H_s')\left(\frac{\beta FTD}{R}\right)\right] \\
& + (u+G)wFD + \frac{1}{T}\left(2a + 2b\frac{d}{v}\right) + \frac{B_c}{T} + \frac{P_c}{T} + PI_e\left[DM_1 - \frac{TD}{2}\right] + C_z D(1-(1-F)(1 \\
& -\gamma))\Bigg)
\end{aligned}
$$

The profit function can be again simplified to

$$
\begin{aligned}
\mathrm{TP}(F,T) = {} & D(P-C_u) - PI_e DM_1 - C_z D(1-\gamma) \\
& -\Bigg(\frac{1}{T}\left(O + (1+m)(S_r) + \left(2a + 2b\frac{d}{v}\right) + B_c + P_c\right) + F(C_s D + \beta D(1+m)(C_{lm}) \\
& - C_z D(1-\gamma)) + T(\frac{\pi\gamma}{2} + \frac{PI_e}{2}) + FT(\pi\gamma D) + F^2T(\frac{(1+m)(H_s+H_s')\beta^2D^2}{R} \\
& + \frac{(1-\beta)^2(H+H')D}{2} + \frac{\beta(H+H')D^2}{X} + \frac{(H_r+H_r')\beta^2 D}{2} + \frac{\pi\gamma D}{2}\Bigg)
\end{aligned}
$$

Where $D(P-C_u) - PI_e DM_1 - C_z D(1-\gamma)$ terms are constant. The total profit per year is maximized if the total cost per year is minimized. Therefore the $Y(F,T)$ is

$$
\begin{aligned}
Y(F,T) = {} & \Bigg(\frac{1}{T}\left(O + (1+m)(S_r) + \left(2a + 2b\frac{d}{v}\right) + B_c + P_c\right) + F(C_s D + \beta D(1+m)(C_{lm}) - C_z D(1 \\
& -\gamma)) + T(\frac{\pi\gamma}{2} + \frac{PI_e}{2}) + FT(\pi\gamma D) + F^2T(\frac{(1+m)(H_s+H_s')\beta^2D^2}{R} \\
& + \frac{(1-\beta)^2(H+H')D}{2} + \frac{\beta(H+H')D^2}{X} + \frac{(H_r+H_r')\beta^2 D}{2} + \frac{\pi\gamma D}{2})\Bigg)
\end{aligned}
$$

The compact form of $Y(F,T)$ is expressed as

$$Y(F,T) = \frac{1}{T}(K_1) + T(K_2 - K_4F + K_5F^2) + K_3F$$

We can rewrite the above equation as

$$Y(F,T) = \frac{1}{T}(K_1) + T\theta(F) + \phi(F)$$

Where $\theta(F) = K_2 - K_4F + K_5F^2$ and $\phi(F) = K_3F$.

The total cost equation reaches its least value with respect to T, we have

$$T^* = \sqrt{\frac{K_1}{\theta(F)}}$$

The minimum value for the total cost by substituting T^* in the cost equation is

$$Y(F) = 2\sqrt{K_1\theta(F)} + \phi(F)$$

The optimal T^* depends upon F. An algebraic method is used to get the optimal values of F. The optimal value of F is given as

$$F^* = \frac{K_4 T - K_3}{2K_5 T}$$

By putting the values of K_3, K_4 and K_5 in the above equation, we have

$$F^* = \frac{\pi\gamma T - (C_s + \beta(1+m)(C_{lm}) - C_z(1-\gamma))}{\left[2\frac{(1+m)(H_s+H_s')\beta^2 D}{R} + (1-\beta)^2(H+H') + \frac{2\beta D(H+H')}{X} + (H_r+H_r')\beta^2 + \pi\gamma\right]T}$$

Substituting the optimal value of F in T^*

$$T^* = \sqrt{\frac{K_1}{K_2 - K_4 F + K_5 F^2}}$$

$$T^* = \sqrt{\frac{K_1}{K_2 - K_4\left(\frac{K_4 T - K_3}{2K_5 T}\right) + K_5\left(\frac{K_4 T - K_3}{2K_5 T}\right)^2}}$$

By putting the values ofK_1, $K_2 K_3$, K_4 and K_5 in the above equation, the optimal T^* finally becomes

$$T^* = \sqrt{\frac{\left(O + (1+m)(S_r) + \left(2a + 2b\frac{d}{v}\right) + B_c + P_c\right) \left[\frac{(1+m)(H_s+H_s')\beta^2 D}{R} + \frac{(1-\beta)^2(H+H')}{2} + \frac{\beta D(H+H')}{X} + \frac{(H_r+H_r')\beta^2}{2} + \frac{\pi\gamma}{2}\right] - \frac{D}{4}(C_s + \beta(1+m)(C_{lm}) - C_z(1-\gamma))^2}{\left(\frac{\pi\gamma}{2} + \frac{PI_e}{2}\right)\left(\frac{(1+m)(H_s+H_s')\beta^2 D^2}{R} + \frac{(1-\beta)^2(H+H')D}{2} + \frac{\beta(H+H')D^2}{X} + \frac{(H_r+H_r')\beta^2 D}{2} + \frac{\pi\gamma D}{2}\right) - \frac{(\pi\gamma)^2 D}{4}}}$$

Case 2: Total profit (TP_2) if $M_1 < T \le M_2$

$$
\begin{aligned}
TP(F,T) &= PD\big(F+\gamma(1-F)\big) \\
&- \Bigg(\frac{O}{T} + C_u D\big(F+\gamma(1-F)\big) + C_s FD + \big(H+H'\big)\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T (FD)^2}{X}\right] \\
&+ \big(H_r + H_r{}'\big)\left[\frac{(\beta F)^2 TD}{2}\right] + \pi\frac{(1-F)^2\gamma TD}{2} \\
&+ \beta FD(1+m)\left[\frac{S_r}{\beta FTD} + C_{lm} + \big(H_s + H_s{}'\big)\left(\frac{\beta FTD}{R}\right)\right] + l(1-\gamma)(1-F)D \\
&+ (u+G)wFD + \frac{1}{T}\left(2a + 2b\frac{d}{v}\right) + \frac{B_c}{T} + \frac{P_c}{T} + PI_e\frac{(DM_1)^2}{2TD} - C_u I_{c1}\frac{(TD-DM_1)^2}{2TD}\Bigg)
\end{aligned}
$$

The profit function for case 2 in the above equation, found by adding +1 and -1 in order quantity is given as

$$
\begin{aligned}
TP(F,T) &= PD\big(1-(1-F)(1-\gamma)\big) \\
&- \Bigg(\frac{O}{T} + C_u D\big(1-(1-F)(1-\gamma)\big) + C_s FD + \big(H+H'\big)\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T (FD)^2}{X}\right] \\
&+ \big(H_r + H_r{}'\big)\left[\frac{(\beta F)^2 TD}{2}\right] + \pi\frac{(1-F)^2\gamma TD}{2} \\
&+ \beta FD(1+m)\left[\frac{S_r}{\beta FTD} + C_{lm} + \big(H_s + H_s{}'\big)\left(\frac{\beta FTD}{R}\right)\right] + l(1-\gamma)(1-F)D \\
&+ (u+G)wFD + \frac{1}{T}\left(2a + 2b\frac{d}{v}\right) + \frac{B_c}{T} + \frac{P_c}{T} + PI_e\frac{(DM_1)^2}{2TD} - C_u I_{c1}\frac{(TD-DM_1)^2}{2TD}\Bigg)
\end{aligned}
$$

The optimum value of F^* and T^* are

$$
F^* = \frac{\pi\gamma T - \big(C_s + \beta(1+m)(C_{lm}) - C_z(1-\gamma)\big)}{\left[2\frac{(1+m)(H_s+H_s{}')\beta^2 D}{R} + (1-\beta)^2(H+H') + \frac{2\beta D(H+H')}{X} + \big(H_r+H_r{}'\big)\beta^2 + \pi\gamma\right]T}
$$

$$
T^* = \sqrt{\frac{\dfrac{\left(O + (1+m)(S_r) + \left(2a+2b\frac{d}{v}\right) + B_c + P_c + \frac{C_u I_{c1} DM_1{}^2}{2} - \frac{PI_e DM_1{}^2}{2}\right)}{\left[\frac{(1+m)(H_s+H_s{}')\beta^2 D}{R} + \frac{(1-\beta)^2(H+H')}{2} + \frac{\beta D(H+H')}{X} + \frac{(H_r+H_r{}')\beta^2}{2} + \frac{\pi\gamma}{2}\right]} - \frac{D}{4}\big(C_s + \beta(1+m)(C_{lm}) - C_z(1-\gamma)\big)^2}{\dfrac{\left(\frac{\pi\gamma}{2} + \frac{C_u I_{c1}}{2}\right)}{\left(\frac{(1+m)(H_s+H_s{}')\beta^2 D^2}{R} + \frac{(1-\beta)^2(H+H')D}{2} + \frac{\beta(H+H')D^2}{X} + \frac{(H_r+H_r{}')\beta^2 D}{2} + \frac{\pi\gamma D}{2}\right)} - \frac{(\pi\gamma)^2 D}{4}}}
$$

Case 3: Total Profit (TP_3) if $T > M_2$

$$\begin{aligned}TP(F,T) = {} & PD\big(F+\gamma(1-F)\big) \\ & -\Bigg(\frac{O}{T}+C_uD\big(F+\gamma(1-F)\big)+C_sFD+\big(H+H'\big)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right] \\ & +\big(H_r+H_r{'}\big)\left[\frac{(\beta F)^2TD}{2}\right]+\pi\frac{(1-F)^2\gamma TD}{2} \\ & +\beta FD(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\big(H_s+H_s{'}\big)\left(\frac{\beta FTD}{R}\right)\right]+l(1-\gamma)(1-F)D \\ & +(u+G)wFD+\frac{1}{T}\left(2a+2b\frac{d}{v}\right)+\frac{B_c}{T}+\frac{P_c}{T}+PI_e\frac{(DM_1)^2}{2TD}-C_uI_{c2}\frac{(TD-DM_2)^2}{2TD} \\ & -C_uI_{c1}\frac{D}{T}\big(M_2T-{M_2}^2-M_1T+M_1M_2\big)-C_uI_{c1}\frac{D(M_2-M_1)^2}{2T}\Bigg)\end{aligned}$$

The profit function for case 3 in the above equation, found by adding +1 and -1 in order quantity is given as

$$\begin{aligned}TP(F,T) = {} & PD\big(1-(1-F)(1-\gamma)\big) \\ & -\Bigg(\frac{O}{T}+C_uD\big(1-(1-F)(1-\gamma)\big)+C_sFD+\big(H+H'\big)\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{X}\right] \\ & +\big(H_r+H_r{'}\big)\left[\frac{(\beta F)^2TD}{2}\right]+\pi\frac{(1-F)^2\gamma TD}{2} \\ & +\beta FD(1+m)\left[\frac{S_r}{\beta FTD}+C_{lm}+\big(H_s+H_s{'}\big)\left(\frac{\beta FTD}{R}\right)\right]+l(1-\gamma)(1-F)D \\ & +(u+G)wFD+\frac{1}{T}\left(2a+2b\frac{d}{v}\right)+\frac{B_c}{T}+\frac{P_c}{T}+PI_e\frac{(DM_1)^2}{2TD}-C_uI_{c2}\frac{(TD-DM_2)^2}{2TD} \\ & -C_uI_{c1}\frac{D}{T}\big(M_2T-{M_2}^2-M_1T+M_1M_2\big)-C_uI_{c1}\frac{D(M_2-M_1)^2}{2T}\Bigg)\end{aligned}$$

The optimum value of F^* and T^* are

$$F^*=\frac{\pi\gamma T-(C_s+\beta(1+m)(C_{lm})-C_z(1-\gamma))}{\left[2\frac{(1+m)\left(H_s+H_s{'}\right)\beta^2D}{R}+(1-\beta)^2(H+H')+\frac{2\beta D(H+H')}{X}+\big(H_r+H_r{'}\big)\beta^2+\pi\gamma\right]T}$$

$$T^* = \sqrt{\frac{\begin{gathered}\left(\begin{gathered} O + (1+m)(S_r) + \left(2a + 2b\frac{d}{v}\right) + B_c + P_c + \frac{C_u I_{c2} D {M_2}^2}{2} - \frac{P I_e D {M_1}^2}{2} \\ - C_u I_{c1} D {M_2}^2 + C_u I_{c1} D M_1 M_2 + \frac{C_u I_{c1} D (M_2 - M_1)^2}{2} \end{gathered}\right) \\ \left[\frac{(1+m)(H_s + H_s')\beta^2 D}{R} + \frac{(1-\beta)^2 (H+H')}{2} + \frac{\beta D (H+H')}{X} + \frac{(H_r + H_r')\beta^2}{2} + \frac{\pi\gamma}{2}\right] \\ - \frac{D}{4}(C_s + \beta(1+m)(C_{lm}) - C_z(1-\gamma))^2 \end{gathered}}{\begin{gathered}\left(\frac{\pi\gamma}{2} + \frac{C_u I_{c2}}{2}\right) \\ \left(\frac{(1+m)(H_s + H_s')\beta^2 D^2}{R} + \frac{(1-\beta)^2 (H+H') D}{2} + \frac{\beta (H+H') D^2}{X} + \frac{(H_r + H_r')\beta^2 D}{2} + \frac{\pi\gamma D}{2}\right) \\ - \frac{(\pi\gamma)^2 D}{4} \end{gathered}}}$$

4. Numerical Example

Consider the following data to illustrate the proposed model.

D	50000 units/year	C_{lm}	5 \$/unit
X	175200 units/year	l	0.5 \$/unit/year
C_s	0.5 \$/unit	G	15 \$/unit
R	50000 units/year	W	0.02%
β	0.04%	u	3 \$/unit
O	100 \$/order	π	20 \$/unit/year
S_r	100 \$/setup	P	50 \$/unit
H	4 \$/unit/year	γ	97%
H'	1 \$/unit/year	m	20%
H_r	5 \$/unit/year	M_1	12 days
H_r'	1 \$/unit/year	M_2	35 days
H_s	3 \$/unit/year	I_e	12%
H_s'	1 \$/unit/year	I_{c1}	12%
C_u	25 \$/unit	I_{c2}	15%
		a	5 \$/unit

b	0.5 $/unit	B_c	30 $/unit
d	250 km	P_c	80 $/unit
v	180 km/h		

Optimum values for different cases are given below:

Scenario	T (Year)	F (%)	Q (units)	TP ($)
Case 1	0.0329	1.1223	1650	1,130,419
Case 2	0.0336	1.1155	1685	1,137,012
Case 3	0.0584	0.9794	2920	1,101,067

5. Conclusion:

In this paper, the integration of backordering, green patent, green logistics and multi-delay- in-payment, the green inventory model was studied with synergic effects of reworking of imperfect items for environmental impact. In addition, the cost of carbon emission is also included in the model to expand on the environmental impact in the benefit feature. The multi-delay-in-payment acts as a source of interim financial investment and can be used to boost sales. The optimal solution according to different scenarios of the cycle time and a fraction of time with permissible delay-in-payment is derived. Finally, a numerical example demonstrated the proposed green inventory model.

References:

1. Antonitte Vinoline, Ritha and Merline Vinotha. Environmentally Sustainable Inventory Model with the Impact of Trade Credit and Partial Backordering. European Journal of Molecular & Clinical Medicine, **2020**, vol. 07, Issue 09, 3095-3106.
2. Bazan, E.; Jaber, M.Y.; El Saadany, A.M. Carbon emissions and energy effects on manufacturing–remanufacturing inventory models. Comput. Ind. Eng. **2015**, 88, 307–316.
3. Biswajit Sarkar, Waqas Ahmed, Seok-Beom Choi and Muhammad Tayyab. Sustainable Inventory Management for Environmental Impact through Partial Backordering and Multi-Trade-Credit-Period. Sustainability, **2018**, 10,4761; doi:10.3390/su10124761.
4. Eroglu, A.; Ozdemir, G. An economic order quantity model with defective items and shortages. Int. J.Prod. Econ. **2007**, 106, 544–549.
5. Goyal, S.K. Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. **1985**, 36, 335–338.

6. Hsu, J.-T.; Hsu, L.-F. An EOQ model with imperfect quality items, inspection errors, shortage backordering and sales returns. Int. J. Prod. Econ. **2013**, 143, 162–170.
7. Huang, Y.-F. Optimal retailer’s ordering policies in the EOQ model under trade credit financing. J. Oper. Res. Soc. **2003**, 54, 1011–1015.
8. Kim, M.-S.; Sarkar, B. Multi-stage cleaner production process with quality improvement and lead time dependent ordering cost. J. Clean. Prod. **2017**, 144, 572–590.
9. Li, D.; Zhao, Y.; Zhang, L.; Chen, X.; Cao, C. Impact of quality management on green innovation. J. Clean. Prod. **2018**, 170, 462–470.
10. Marchi, B.; Zanoni, S.; Zavanella, L.; Jaber, M. Green supply chain with learning in production and environmental investments. IFAC-PapersOnLine **2018**, 51, 1738–1743.
11. Salameh, M.; Jaber, M. Economic production quantity model for items with imperfect quality. Int. J.Prod. Econ. **2000**, 64, 59–64.
12. Sarkar, B.; Ahmed,W.; Kim, N. Joint effects of variable carbon emission cost and multi-delay-in-payments under single-setup-multiple-delivery policy in a global sustainable supply chain. J. Clean. Prod. **2018**, 185, 421–445.
13. Sarkar, B.; Cárdenas-Barrón, L.E.; Sarkar, M.; Singgih, M.L. An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. J. Manuf. Syst. **2014**, 33, 423–435.
14. Taleizadeh, A.A.; Khanbaglo, M.P.S.; Cárdenas-Barrón, L.E. An EOQ inventory model with partial backordering and reparation of imperfect products. Int. J. Prod. Econ. **2016**, 182, 418–434.
15. Tiwari, S.; Ahmed,W.; Sarkar, B. Multi-item sustainable green production system under trade-credit and partial backordering. J. Clean. Prod. **2018**, 204, 82–95.
16. Vörös, J. Economic order and production quantity models without constraint on the percentage of defective items. Cent. Eur. J. Oper. Res. **2013**, 21, 867–885.
17. Wahab, M.; Mamun, S.; Ongkunaruk, P. EOQ models for a coordinated two-level international supply chainconsidering imperfect items and environmental impact. Int. J. Prod. Econ. **2011**, 134, 151–158.