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Green Inventory Model By Controlling Carbon Emissions Under Uncertainty

Dr. S. Rexlin Jeyakumari. , Dr. M. Mary Mejrullo Merlin , M. Fabiana Jacintha Mary

Department of mathematics, Holy Cross College (Autonomous),Tiruchirapalli, Tamil Nadu, India

Abstract Lighting environmental sustainability in the society is possible by promoting green logistics to today's production fields. On observing the drastic effects of carbon emission, few business sectors stepped into introducing green featured products to control GHG emissions. This paper on Green Inventory Model provides an approach to determine the optimum order quantity and optimum total cost with green atmospheric feature. graded mean integration representation method is applied for defuzzification and optimum order quantity is determined by using Kuhn-Tucker method. Numerical example is solved to illustrate the model.

Keywords Fuzzy Arithmetical Operations, Trapezoidal Fuzzy Number, Emission of Carbon, Kuhn-Tucker Method.

Introduction Atmospheric warming at present knocks its attention of the manufacturers towards creating green sustainable environment. a large amount of CO_2 is being emitted due to human activities like burning fossil fuels, tremendous improvement in industry in the states, increased amount of using transportation services, destruction of forests and failure of promoting natural resources, causing global or atmospheric warming. Carbon emission is the major factor in global warming which increases heat-trapping level in earth's atmosphere. Among all the factors of carbon emission into the earth's atmosphere the CO_2 released from various product developing firms and sites are observed as the important contributors causing major threat to our earth. Hence the focus to control carbon emission commence from industrial manufacturing areas.

Many research works on carbon emission examined that in the developing world most of the fossil fuels like coal and petroleum were burned for producing heat and electricity, for functioning of gasoline or diesel powered vehicles, for cement production, for oil and gas production and also for mass production of goods and inventory items. Tapan kumar Datta (2017) says in his findings that our world economy stands firm because of the corporations which promotes fossil fuel for its many product constructing activities. Hence the emission of carbon and other greenhouse gases cannot be deactivated completely instead the rate of carbon emission can be reduced. He concluded that capital investment on green technology for production of goods promotes reduction in emission of carbon dioxide into air.At present manufacturing trade and industries promotes industrial sustainability and they prompt to optimize the usage of energy, resources with eco-friendly criteria. Guowei Hua and Shouyang wang(2011) derived Optimal Order Quantity and examined the results of carbon trade and carbon price so as to reduce CO_2 output and to alleviate global warming. They predicted carbon emission trading to restraint the amount of carbon emission. Maurice Bonney and Mohamed V.Jaber (2011) designed an inventory model reflecting the needs of the environment. Inconveniences faced by the global earth because of industrial activities like disposing of waste inventory materials and packaging of products were explained. And they discussed about implementing eco-friendly inventory factor by including emission costs. Samir Elhedhli and Ryan Merrick proposed the supply chain network design problem including CO_2 costs into the objective function. They developed their model so as to concurrently minimize execution costs and the environmental costs invested to reduce excessive GHG emission providing feasible desirable result.

H.W. Harris was the first person to develop the first inventory model in 1913. And this model was generalized by Wilson in 1934. He introduced Economic Order Quantity (EOQ) in his research work to optimize total cost. The fuzzy set theory was implemented and founded by Lofti Asker Zadeh in 1965 through his published work. Zimmerman (1983) developed his idea by using fuzzy concept in Operational research to remove vagueness, uncertainty and unclearness. M.Vujosevic, D.Petrovic and R.Petrovic proposed their model on EOQ where the costs of inventory are represented as a fuzzy number. In this paper, an inventory fuzzy model is designed and developed to manage carbon emission. Fuzzy parameters are taken as trapezoidal fuzzy numbers and defuzzification is performed by Graded Mean Integration method. Kuhn-Tucker conditions were applied to find the optimum solution. Numerical example is given to illustrate the model.

Definitions and Methodologies

Fuzzy Set:

A fuzzy set $\widetilde{A}$defined on a Universe of discourse X may be written as a collection of ordered pairs, $\widetilde{A} = \left\{\left(x, \mu_{\widetilde{A}}(x)\right) : x \in X, \mu_{\widetilde{A}} \in [0,1]\right\}$, where each pair $\left(x, \mu_{\widetilde{A}}(x)\right)$ is called a singleton and the element $\mu_{\widetilde{A}}(x)$ belongs to the interval $[0,1]$.The function $\mu_{\widetilde{A}}(x)$ is called as membership function.

Trapezoidal Fuzzy Number:

The fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$,where $a_1 < a_2 < a_3 < a_4$and defined on R is called the trapezoidal fuzzy number, if the membership function is given by, $\mu_{\widetilde{A}}(\mathbf{x}) = \begin{cases} \mathbf{0} : \mathbf{x} < \mathbf{a_1} \text{ or } \mathbf{x} > \mathbf{a_4} \\ \frac{(x-a_1)}{(a_2-a_1)} : \mathbf{a_1} \leq \mathbf{x} \leq \mathbf{a_2} \\ \mathbf{1} : \mathbf{a_2} \leq \mathbf{x} \leq \mathbf{a_3} \\ \frac{(x-a_4)}{(a_3-a_4)} : \mathbf{a_3} \leq \mathbf{x} \leq \mathbf{a_4} \end{cases}$

Fuzzy Arithmetical Operations

Some of the fuzzy arithmetical operations for trapezoidal fuzzy numbers under function principle are as follows, Let us assume $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4)$ as two trapezoidal fuzzy numbers. Then

(i) The addition of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$, where $a_1, a_2, a_3, a_4,$

b_1, b_2, b_3 and b_4 are any real numbers.

(ii) The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \otimes \widetilde{B} = (c_1, c_2, c_3, c_4)$, where $Z_1 = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $Z_2 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, , $c_1 = \min Z_1, c_2 = \min Z_2, c_3 = \max Z_2, c_4 = \max Z_2$.

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all zero positive real numbers, then $\widetilde{A} \otimes \widetilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$.

(iii) The subtraction of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \ominus \widetilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$, where $-\widetilde{B} = (-b_4, -b_3, -b_2, -b_1)$, also $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(iv) The division of $\widetilde{A}$ and $\widetilde{B}$ is

$$\widetilde{A} \oslash \widetilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right), \text{where} \frac{1}{\widetilde{B}} = \widetilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$$

And b_1, b_2, b_3, b_4 are positive real numbers. Also $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are non-zero positive real numbers.

(v) For any $\alpha \in R$,

a) If $\alpha \geq 0$, then $\alpha \otimes \widetilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$.

b) If $\alpha < 0$, then $\alpha \otimes \widetilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$.

Graded Mean Integration Representation Method

Graded Mean Integration Representation Method is one of the methods used for defuzzifying fuzzy numbers. In this present work of Green Inventory model, trapezoidal fuzzy number is used as the type of all fuzzy parameters. Let $\tilde{C}$ be a trapezoidal fuzzy number, and be denoted as $\tilde{C} = (c_1, c_2, c_3, c_4)$. Then we can get the formula for Graded Mean Integration Representation of $\tilde{C}$ as

$$P(\tilde{C}) = \frac{c_1 + 2c_2 + 2c_3 + c_4}{6}$$

Kuhn-Tucker Method

The Kuhn-Tucker method is a method for finding optimal solutions for non-linear programming problems containing differentiable functions. The Kuhn-tucker conditions are based on the extension of Lagrangian method.

Suppose we consider an optimization problem,

Minimize $Y = f(x)$ subject to the constraints $g_i(x) \geq 0$, i=1,2,..........m.

The non-negativity constraints may be converted into equations by using non negative surplus variables.

Let $\lambda = (\lambda_1, \lambda_2, \lambda_3, \dots \dots . \lambda_m)$,

$g(x) = (g_1(x), g_2(x), g_3(x), \dots \dots ., g_m(x))$ and

$S^2 = (S_1^2, S_2^2, S_3^2, \dots\dots S_m^2)$.

The Kuhn-Tucker conditions need X and λto be a stationary point of this problem of minimization, which can be expressed as follows,

$\lambda_i \geq 0,$

$\nabla f(x) - \boldsymbol{\lambda}\nabla g(x) = 0,$

$\lambda_i g_i(x) = 0, i = 1,2, \dots., m,$

$g_i(x) \geq 0, i = 1,2, \dots, m.$

Assumptions and Notations:

Assumptions:

- ✓ The Lead time is taken as constant.
- ✓ Demand is known and it is fixed as constant.
- ✓ The cost of emissions complies with the assumptions of Economic Order Quantity.
- ✓ Analysation of single product with green features is done for the given period of time.

Notations:

D_A –Total Demand

r – Rate of Production

O – Cost of ordering

S_C – Setup cost.

H_t – Cost of holding inventory items per unit time.

C_q – Emission quantity of carbon from order per cycle.

H_q – Emission quantity of carbon from holding items.

S_u – Emission of carbon from setup of inventory.

v – Variable cost.

E_C – Cost of carbon emission.

C_S – Social cost per trip.

a – number of orders taken.

T – travelling distance.

Q – Economic order quantity.

TC – Total cost.

$\widetilde{D_A}$ – Fuzzy total demand.

$\tilde{O}$ – Fuzzy cost of ordering.

$\widetilde{S_C}$ – Fuzzy setup cost.

$\widetilde{H_t}$ – Fuzzy cost of holding items per unit time.

$\widetilde{C_q}$ – Fuzzy emission quantity of carbon from order per cycle.

$\widetilde{S_u}$ – Fuzzy emission of carbon from setup of inventory.

$\widetilde{H_q}$ – Fuzzy emission quantity of carbon from holding items.

$\tilde{v}$ – Fuzzy variable cost.

$\widetilde{C_S}$ – Fuzzy social cost per trip.

Green Inventory Model:

Let us consider a green inventory model with the above prescribed assumptions.

The total cost of the considered model is the sum of the costs associated with ordering, setting and holding items. Also variable and social cost per trip is added to it. It is given as,

$$\mathrm{TC} = \frac{D_A}{Q}\left[\left(O + E_C C_q\right) + \left(\frac{S_C + E_C S_u}{a}\right) + 2C_S\right] + \frac{Q}{2}\left[\left(H_t + E_C H_q\right) + \frac{aE_C D_A}{r} + \frac{vT}{D_A}\right]$$

Differentiating the above equation partially with respect to Q the order quantity is derived as,

$$\Rightarrow Q = \sqrt{\frac{2D_A\left[\left(O + E_C C_q\right) + \left(\frac{S_C + E_C S_u}{a}\right) + 2C_S\right]}{\left[\left(H_t + E_C H_q\right) + \frac{aE_C D_A}{r} + \frac{vT}{D_A}\right]}}$$

Fuzzy Model

The above given green inventory model is now considered in fuzzy sense to remove uncertain, unclearness and ambiguous situations in several costs considerations. Hence the crisp parameters such as total demand, cost of ordering, setup cost, cost of holding items, several emissions cost, variable and social cost are taken as fuzzy parameters.

Suppose,

$\tilde{Q} = (q_1, q_2, q_3, q_4)$

$\widetilde{D_A} = \left(d_{a_1}, d_{a_2}, d_{a_3}, d_{a_4}\right)$

$\tilde{O} = (o_1, o_2, o_3, o_4)$

$\widetilde{S_C} = (s_{c_1}, s_{c_2}, s_{c_3}, s_{c_4})$

$\widetilde{H_t} = (h_{t_1}, h_{t_2}, h_{t_3}, h_{t_4})$

$\widetilde{C_q} = (c_{q_1}, c_{q_2}, c_{q_3}, c_{q_4})$

$\widetilde{S_u} = (s_{u_1}, s_{u_2}, s_{u_3}, s_{u_4})$

$\widetilde{H_q} = (h_{q_1}, h_{q_2}, h_{q_3}, h_{q_4})$

$\tilde{v} = (v_1, v_2, v_3, v_4)$

$\widetilde{C_S} = (c_{s_1}, c_{s_2}, c_{s_3}, c_{s_4})$

are the non-negative trapezoidal fuzzy numbers, then the fuzzy total cost of the given green inventory model is given by,

$$\widetilde{TC(Q)} = \frac{\widetilde{D_A}}{Q} \otimes \left[\left(\tilde{O} \oplus \left(E_C \otimes \widetilde{C_q} \right) \right) \oplus \left(\frac{\widetilde{S_C} \oplus \left(E_C \otimes \widetilde{S_u} \right)}{a} \right) \oplus \left(2\widetilde{C_S} \right) \right]$$
$$\oplus \frac{\tilde{Q}}{2} \left[\left(\widetilde{H_t} \oplus \left(E_C \otimes \widetilde{H_q} \right) \right) \oplus \left(\frac{a \otimes E_C \,\widetilde{\otimes D_A}}{r} \right) \oplus \left(\frac{\tilde{v} \otimes T}{\widetilde{D_A}} \right) \right]$$

$$\widetilde{TC(Q)} = \begin{bmatrix} \frac{d_{a_1}}{q_4}\left[\left(o_1 + E_C c_{q_1}\right) + \left(\frac{s_{c_1} + E_C s_{u_1}}{a}\right) + 2c_{s_1}\right] + \frac{q_1}{2}\left[\left(h_{t_1} + E_C h_{q_1}\right) + \left(\frac{aE_C d_{a_1}}{r}\right) + \frac{v_1 T}{d_{a_4}}\right], \\ \frac{d_{a_2}}{q_3}\left[\left(o_2 + E_C c_{q_2}\right) + \left(\frac{s_{c_2} + E_C s_{u_2}}{a}\right) + 2c_{s_2}\right] + \frac{q_2}{2}\left[\left(h_{t_2} + E_C h_{q_2}\right) + \left(\frac{aE_C d_{a_2}}{r}\right) + \frac{v_2 T}{d_{a_3}}\right], \\ \frac{d_{a_3}}{q_2}\left[\left(o_3 + E_C c_{q_3}\right) + \left(\frac{s_{c_3} + E_C s_{u_3}}{a}\right) + 2c_{s_3}\right] + \frac{q_3}{2}\left[\left(h_{t_3} + E_C h_{q_3}\right) + \left(\frac{aE_C d_{a_3}}{r}\right) + \frac{v_3 T}{d_{a_2}}\right], \\ \frac{d_{a_4}}{q_1}\left[\left(o_4 + E_C c_{q_4}\right) + \left(\frac{s_{c_4} + E_C s_{u_4}}{a}\right) + 2c_{s_4}\right] + \frac{q_4}{2}\left[\left(h_{t_4} + E_C h_{q_4}\right) + \left(\frac{aE_C d_{a_4}}{r}\right) + \frac{v_4 T}{d_{a_1}}\right] \end{bmatrix}$$

$\widetilde{TC(Q)} = \left(TC_1(q_1), TC_2(q_2), TC_3(q_3), TC_4(q_4)\right)$

Where $TC_i(q_i) = \frac{d_{a_i}}{q_j}\left[\left(o_i + E_C c_{q_i}\right) + \left(\frac{s_{c_i} + E_C s_{u_i}}{a}\right) + 2c_{s_i}\right] + \frac{q_i}{2}\left[\left(h_{t_i} + E_C h_{q_i}\right) + \left(\frac{aE_C d_{a_i}}{r}\right) + \frac{v_i T}{d_{aj}}\right],$

for $i = 1,2,3,4$ and $j = 4,3,2,1$.

Now for the defuzzification of the fuzzy total cost, graded mean integration representation method is used and it is given by,

$$P\left(\widetilde{TC(Q)}\right) = \frac{1}{6}\left[TC_1(q_1) + 2TC_2(q_2) + 2TC_3(q_3) + TC_4(q_4)\right]$$

$$P\left(\widetilde{TC(Q)}\right)=\frac{1}{6}\left[\begin{array}{l}\frac{d_{a_1}}{q_4}\left[\left(o_1+E_Cc_{q_1}\right)+\left(\frac{s_{c_1}+E_Cs_{u_1}}{a}\right)+2c_{s_1}\right]+\frac{q_1}{2}\left[\left(h_{t_1}+E_Ch_{q_1}\right)+\left(\frac{aE_Cd_{a_1}}{r}\right)+\frac{v_1T}{d_{a_4}}\right]\\ +2\left[\frac{d_{a_2}}{q_3}\left[\left(o_2+E_Cc_{q_2}\right)+\left(\frac{s_{c_2}+E_Cs_{u_2}}{a}\right)+2c_{s_2}\right]+\frac{q_2}{2}\left[\left(h_{t_2}+E_Ch_{q_2}\right)+\left(\frac{aE_Cd_{a_2}}{r}\right)+\frac{v_2T}{d_{a_3}}\right]\right]\\ +2\left[\frac{d_{a_3}}{q_2}\left[\left(o_3+E_Cc_{q_3}\right)+\left(\frac{s_{c_3}+E_Cs_{u_3}}{a}\right)+2c_{s_3}\right]+\frac{q_3}{2}\left[\left(h_{t_3}+E_Ch_{q_3}\right)+\left(\frac{aE_Cd_{a_3}}{r}\right)+\frac{v_3T}{d_{a_2}}\right]\right]\\ +\frac{d_{a_4}}{q_1}\left[\left(o_4+E_Cc_{q_4}\right)+\left(\frac{s_{c_4}+E_Cs_{u_4}}{a}\right)+2c_{s_4}\right]+\frac{q_4}{2}\left[\left(h_{t_4}+E_Ch_{q_4}\right)+\left(\frac{aE_Cd_{a_4}}{r}\right)+\frac{v_4T}{d_{a_1}}\right]\end{array}\right]$$

with $0<q_1\le q_2\le q_3\le q_4 \qquad ---(1).$

The meaning of the constraints given here will not change, even if we replace the inequality conditions $0< q_1\le q_2\le q_3\le q_4$ into the following inequality constraints,

$q_2-q_1\ge 0, q_3-q_2\ge 0, q_4-q_3\ge 0$ and $q_1>0$.

Now we use Kuhn-tucker conditions for finding the solutions of q_1, q_2, q_3, q_4 and to minimize the fuzzy total cost which is denoted as $P\left(\widetilde{TC(Q)}\right)$ subject to $q_4-q_3\ge 0, q_3-q_2\ge 0, q_2-q_1\ge 0$ and $q_1>0$.

Applying the Kuhn-tucker first condition $\lambda_i\le 0$,

$$\Rightarrow \lambda_1,\lambda_2,\lambda_3,\lambda_4\le 0 \qquad ---(2)$$

Now applying the Kuhn-tucker second condition,

$$\nabla f(x)-\boldsymbol{\lambda}\nabla g(x)=0$$

Applying the above condition with respect to q_1,q_2,q_3 and q_4, we get

$$\frac{1}{6}\left[-\frac{d_{a_4}}{q_1^2}\left[\left(o_4+E_Cc_{q_4}\right)+\left(\frac{s_{c_4}+E_Cs_{u_4}}{a}\right)+2c_{s_4}\right]+\frac{1}{2}\left[\left(h_{t_1}+E_Ch_{q_1}\right)+\left(\frac{aE_Cd_{a_1}}{r}\right)+\frac{v_1T}{d_{a_4}}\right]\right]+\lambda_1-\lambda_4=0$$

$$---(3)$$

$$\frac{1}{6}\left[-\frac{2d_{a_3}}{q_2^2}\left[\left(o_3+E_Cc_{q_3}\right)+\left(\frac{s_{c_3}+E_Cs_{u_3}}{a}\right)+2c_{s_3}\right]+\frac{2}{2}\left[\left(h_{t_2}+E_Ch_{q_2}\right)+\left(\frac{aE_Cd_{a_2}}{r}\right)+\frac{v_2T}{d_{a_3}}\right]\right]-\lambda_1+\lambda_2=0$$

$$---(4)$$

$$\frac{1}{6}\left[-\frac{2d_{a_2}}{q_3^2}\left[\left(o_2+E_Cc_{q_2}\right)+\left(\frac{s_{c_2}+E_Cs_{u_2}}{a}\right)+2c_{s_2}\right]+\frac{2}{2}\left[\left(h_{t_3}+E_Ch_{q_3}\right)+\left(\frac{aE_Cd_{a_3}}{r}\right)+\frac{v_3T}{d_{a_2}}\right]\right]-\lambda_2+\lambda_3=0$$

$$---(5)$$

$$\frac{1}{6}\left[-\frac{d_{a_1}}{q_4^2}\left[\left(o_1+E_Cc_{q_1}\right)+\left(\frac{s_{c_1}+E_Cs_{u_1}}{a}\right)+2c_{s_1}\right]+\frac{1}{2}\left[\left(h_{t_4}+E_Ch_{q_4}\right)+\left(\frac{aE_Cd_{a_4}}{r}\right)+\frac{v_4T}{d_{a_1}}\right]\right]-\lambda_3=0$$

$$---(6)$$

The third Kuhn-tucker condition says,$\lambda_i g_i(x) = 0$ for $i = 1,2,3, \dots \dots m$.

Hence by applying this condition we have,

$$\left.\begin{array}{c} \lambda_1\, g_1(Q) = 0 \quad \Rightarrow \lambda_1(q_2 - q_1) = 0 \\ \lambda_2\, g_2(Q) = 0 \quad \Rightarrow \lambda_2(q_3 - q_2) = 0 \\ \lambda_3\, g_3(Q) = 0 \quad \Rightarrow \lambda_3(q_4 - q_3) = 0 \\ \lambda_4\, g_4(Q) = 0 \quad \Rightarrow \lambda_4(q_1) = 0 \end{array}\right\} \qquad ---(7)$$

The fourth statement of Kuhn-tucker condition says, $g_i(Q) \geq 0$.

Therefore by applying this we have certain inequalities as follows,

$$\left.\begin{array}{c} g_1(Q) \geq 0 \quad \Rightarrow (q_2 - q_1) \geq 0 \\ g_2(Q) \geq 0 \quad \Rightarrow (q_3 - q_2) \geq 0 \\ g_3(Q) \geq 0 \quad \Rightarrow (q_4 - q_3) \geq 0 \\ g_4(Q) \geq 0 \quad \Rightarrow q_1 > 0 \end{array}\right\} \qquad ---(8)$$

From the last statement of equations (7) and (8) we have,

$q_1 > 0$ and $\lambda_4(q_1) = 0$.

$$\Rightarrow \lambda_4 = 0 \qquad ---(9)$$

If $\lambda_1 = \lambda_2 = \lambda_3 = 0$, we have from equation (7) the following relations,

$q_2 - q_1 < 0$, $q_3 - q_2 < 0$,$q_4 - q_3 < 0$.

$\Rightarrow q_2 < q_1,\ q_3 < q_2,\ q_4 < q_3$.

From the above inequalities we observe that, $q_4 < q_3 < q_2 < q_1$.

It does not satisfy the constraints

$0 < q_1 \leq q_2 \leq q_3 \leq q_4$.

Hence $\lambda_1 = \lambda_2 = \lambda_3 \neq 0$.

As $\lambda_1 = \lambda_2 = \lambda_3 \neq 0$ by observing the set of equations in (7) we have,

$q_2 - q_1 = 0$,$q_3 - q_2 = 0$,$q_4 - q_3 = 0$.

$$\Rightarrow q_1 = q_2 = q_3 = q_4 = \widetilde{Q}^* \qquad ---(10)$$

Now adding the equations (3), (4), (5), (6) and using the equations (9) and (10) we get,

$$\frac{1}{6}\begin{bmatrix} -\frac{d_{a_4}}{Q^{*2}}\left[\left(o_4+E_C c_{q_4}\right)+\left(\frac{s_{c_4}+E_C s_{u_4}}{a}\right)+2c_{s_4}\right]+\frac{1}{2}\left[\left(h_{t_1}+E_C h_{q_1}\right)+\left(\frac{aE_C d_{a_1}}{r}\right)+\frac{v_1 T}{d_{a_4}}\right] \\ -\frac{2d_{a_3}}{Q^{*2}}\left[\left(o_3+E_C c_{q_3}\right)+\left(\frac{s_{c_3}+E_C s_{u_3}}{a}\right)+2c_{s_3}\right]+\frac{2}{2}\left[\left(h_{t_2}+E_C h_{q_2}\right)+\left(\frac{aE_C d_{a_2}}{r}\right)+\frac{v_2 T}{d_{a_3}}\right] \\ -\frac{2d_{a_2}}{Q^{*2}}\left[\left(o_2+E_C c_{q_2}\right)+\left(\frac{s_{c_2}+E_C s_{u_2}}{a}\right)+2c_{s_2}\right]+\frac{2}{2}\left[\left(h_{t_3}+E_C h_{q_3}\right)+\left(\frac{aE_C d_{a_3}}{r}\right)+\frac{v_3 T}{d_{a_2}}\right] \\ -\frac{d_{a_1}}{Q^{*2}}\left[\left(o_1+E_C c_{q_1}\right)+\left(\frac{s_{c_1}+E_C s_{u_1}}{a}\right)+2c_{s_1}\right]+\frac{1}{2}\left[\left(h_{t_4}+E_C h_{q_4}\right)+\left(\frac{aE_C d_{a_4}}{r}\right)+\frac{v_4 T}{d_{a_1}}\right] \end{bmatrix}=0$$

$$\Rightarrow \widetilde{Q}^* = \sqrt{\frac{2\begin{bmatrix} d_{a_1}\left[\left(o_1+E_C c_{q_1}\right)+\left(\frac{s_{c_1}+E_C s_{u_1}}{a}\right)+2c_{s_1}\right] \\ +2d_{a_2}\left[\left(o_2+E_C c_{q_2}\right)+\left(\frac{s_{c_2}+E_C s_{u_2}}{a}\right)+2c_{s_2}\right] \\ +2d_{a_3}\left[\left(o_3+E_C c_{q_3}\right)+\left(\frac{s_{c_3}+E_C s_{u_3}}{a}\right)+2c_{s_3}\right] \\ +d_{a_4}\left[\left(o_4+E_C c_{q_4}\right)+\left(\frac{s_{c_4}+E_C s_{u_4}}{a}\right)+2c_{s_4}\right] \end{bmatrix}}{\begin{bmatrix} \left[\left(h_{t_1}+E_C h_{q_1}\right)+\left(\frac{aE_C d_{a_1}}{r}\right)+\frac{v_1 T}{d_{a_4}}\right] \\ +2\left[\left(h_{t_2}+E_C h_{q_2}\right)+\left(\frac{aE_C d_{a_2}}{r}\right)+\frac{v_2 T}{d_{a_3}}\right] \\ +2\left[\left(h_{t_3}+E_C h_{q_3}\right)+\left(\frac{aE_C d_{a_3}}{r}\right)+\frac{v_3 T}{d_{a_2}}\right] \\ +\left[\left(h_{t_4}+E_C h_{q_4}\right)+\left(\frac{aE_C d_{a_4}}{r}\right)+\frac{v_4 T}{d_{a_1}}\right] \end{bmatrix}}}$$

Numerical Example

Crisp Model:

For the considered green inventory model, the values taken for different parameters are as follows,

Total Demand, $D_A = 5000$/units/year, Rate of Production, $r = 7000$/units/year, Cost of ordering ,$O = 10$ per cycle, Setup cost,$S_C = 70$, Cost of holding inventory items ,$H_t = 0.7$/unit time.Emission quantity of carbon from order, $C_q = 30$/per cycle, Emission quantity of carbon from holding items, $H_q = 1$, Emission of carbon from setup of inventory, $S_u = 30$, Variable cost, $v = 4$/unit transported, Cost of carbon emission, $E_C = 0.4$, Social cost, $C_S = 0.6$/per trip, Number of orders taken, $a = 0.9$, Travelling distance, $T = 240$ km.

Order Quantity:

$$Q^* = \sqrt{\frac{2D_A\left[\left(O+E_C C_q\right)+\left(\frac{S_C+E_C S_u}{a}\right)+2C_S\right]}{\left[\left(H_t+E_C H_q\right)+\frac{aE_C D_A}{r}+\frac{vT}{D_A}\right]}}$$

$$Q^* = \sqrt{\frac{(2\times 5000)\left[\left(10+(0.4\times 30)\right)+\left(\frac{70+(0.4\times 30)}{0.9}\right)+2(0.6)\right]}{\left[\left(0.7+(0.4\times 1)\right)+\left(\frac{0.9\times 0.4\times 5000}{7000}\right)+\left(\frac{(4\times 240)}{5000}\right)\right]}} = 859 \text{ units}$$

Total cost:

$$TC^{*}=\frac{D_A}{Q}\left[\left(O+E_C C_q\right)+\left(\frac{S_C+E_C S_u}{a}\right)+2C_S\right]+\frac{Q}{2}\left[\left(H_t+E_C H_q\right)+\frac{aE_C D_A}{r}+\frac{vT}{D_A}\right]=\text{Rs}.\,1330.73$$

Fuzzy Model:

$\widetilde{D_A}=(4999,4999.5,5000.5,5001)$, $\widetilde{O}=(9,9.5,10.5,11)$, $\widetilde{S_C}=(68,69,71,72)$, $\widetilde{H_t}=(0.5,0.6,0.8,0.9)$,

$\widetilde{C_q}=(28,29,31,32)$, $\tilde{v}=(2,3,5,6)$, $\widetilde{S_u}=(28,29,31,32)$, $\widetilde{H_q}=(0.8,0.9,1.1,1.2)$, $\widetilde{C_S}=(0.4,0.5,0.7,0.8)$

$r=7000$, $E_C=0.4$, $a=0.9$, $T=240$.

Optimum Order Quantity:

$$\tilde{Q}^{*}=\sqrt{\frac{2\begin{bmatrix} d_{a_1}\left[\left(o_1+E_C c_{q_1}\right)+\left(\frac{s_{c_1}+E_C s_{u_1}}{a}\right)+2c_{s_1}\right] \\ +2d_{a_2}\left[\left(o_2+E_C c_{q_2}\right)+\left(\frac{s_{c_2}+E_C s_{u_2}}{a}\right)+2c_{s_2}\right] \\ +2d_{a_3}\left[\left(o_3+E_C c_{q_3}\right)+\left(\frac{s_{c_3}+E_C s_{u_3}}{a}\right)+2c_{s_3}\right] \\ +d_{a_4}\left[\left(o_4+E_C c_{q_4}\right)+\left(\frac{s_{c_4}+E_C s_{u_4}}{a}\right)+2c_{s_4}\right]\end{bmatrix}}{\begin{bmatrix}\left[\left(h_{t_1}+E_C h_{q_1}\right)+\left(\frac{aE_C d_{a_1}}{r}\right)+\frac{v_1 T}{d_{a_4}}\right] \\ +2\left[\left(h_{t_2}+E_C h_{q_2}\right)+\left(\frac{aE_C d_{a_2}}{r}\right)+\frac{v_2 T}{d_{a_3}}\right] \\ +2\left[\left(h_{t_3}+E_C h_{q_3}\right)+\left(\frac{aE_C d_{a_3}}{r}\right)+\frac{v_3 T}{d_{a_2}}\right] \\ +\left[\left(h_{t_4}+E_C h_{q_4}\right)+\left(\frac{aE_C d_{a_4}}{r}\right)+\frac{v_4 T}{d_{a_1}}\right]\end{bmatrix}}}=(859,859,859,859)$$

Fuzzy Total Cost:

The fuzzy total cost is given by $\widetilde{TC(Q)}^{*}=\left(TC_1(Q^{*}),TC_2(Q^{*}),TC_3(Q^{*}),TC_4(Q^{*})\right)$

$$(\text{i.e}),\ P\left(\widetilde{TC(Q)}\right)=\frac{1}{6}\left[\widetilde{TC}_1(Q^{*})+2\widetilde{TC}_2(Q^{*})+2\widetilde{TC}_3(Q^{*})+\widetilde{TC}_4(Q^{*})\right]$$

Where $TC_1(Q^{*})=1138.164806, TC_2(Q^{*})=1234.443811.$

$TC_3(Q^{*})=1427.023466,\ TC_4(Q^{*})=1523.324119.$

$$\widetilde{TC(Q)}^{*}=(1138.16,1234.44,1427.02,1523.32)$$

Conclusion:

As there is a necessity to transform business entities and production interests to implement green atmospheric features thereby reducing high emission of carbon many industries admitted themselves to make green sustainable environment. But optimization of the business is also the ultimate goal of any concern. So referring to certain green inventory models helps them to optimize their business. Hence in this present work, a green inventory model by controlling carbon emissions have been discussed by using fuzzy techniques in order to remove the uncertain situations. graded mean representation method for trapezoidal fuzzy number is used for defuzzifying fuzzy parameters and Kuhn-Tucker method is applied to find optimum order quantity and optimum total cost. So this developed model helps the respective traders to accomplish in their business and also they can safeguard the atmosphere and earth from destructions. This also can be extended by implementing various criterias.

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