

Amenable Carbon Emissions Harnessing Optimum Sustainable Inventory Management Amidst Backorder And Deterioration Esteeming Geometric Programming

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ABSTRACT:

Greenhouse gases absorb infrared radiation thereby trapping and increasing the heat in the atmosphere. Growth targets for emission of greenhouse gases should be a part of the effort to mitigate climate change by reducing carbon emission. It's so pivotal to reduce carbon emission and develop a green supply chain to achieve sustainable supply chain. Many researchers have contributed to check carbon emissions on the lot sizing sustainable inventory management.

To rim over the existing vent, this paper promotes a carbon cap and tax regulated sustainable inventory management for a buyer utilizing a linear and non-linear price-dependent demand. Numerical problems are solved theoretically and analytically to arrive at a justifiable amount of profit by applying partial, full and null backordering in a controllable sustainable inventory management. This paper investigates on how the greenhouse firms can manage carbon footprints, derive the optimal order quantity and analyse the impact of carbon cap, carbon emissions and total cost.

KEYWORDS: Sustainable EOQ inventory model, carbon emission, Green technology investment, Geometric programming

1. INTRODUCTION:

Global warming has become a susceptible problem for many countries whose effects increase sea levels, frequency of floods, drought and storms. As global economy is directly dependent on greenhouse gases, reducing emissions in a greenhouse farm and investing in green technology is strenuous. It is evident that a sustainable price-reliant demand inventory model with controllable carbon emission is to be proposed to reduce CO₂ emissions from farm warehousing. The rise of global warming has intigated the producers and consumers to implement various ways to control carbon emission by investing in green technology to maximise profit and save the environment.

Rate deterioration is the prime issue in a deteriorating inventory system as for an instance, plants and flowers are very good examples of deteriorating products in greenhouse gases. By selling

plants and flowers quickly, carbon emission in the warehouse can be maintained at the lowest possible level.

This paper defines the research gap and describes the formulation of the controllable SEOQ model with backordering case. It also solves numerical examples to find globally optimal solution.

Many researchers have proved that the total profit maximized during deterioration can be minimised by prevention technology through reduction of defective items in multi-stage production system. They also laid emphasis on the maximum investment to reduce the setup cost to provide a constant setup cost.

So, through this paper, controllable deterioration and carbon emission with several backorder cases in a sustainable inventory model is attained in relevant to geometric programming.

2. MATHEMATICAL MODEL:

2.1 ASSUMPTIONS:

The following assumptions were scrutinized to evolve the prospective template:

1. A sustainable inventory management for a single type of product was reviewed for which the ultimatum product was presumed to be price- dependent. To procure this, both linear and non-linear types of demand were viewed.
2. The demand function, being a linear and non-linear function was dependent on selling price. The demand function $D(P) = a - bP$, a linear function of selling price in a greenhouse farm was prominent than the fixed demand. The minimum selling price increased the outlay of the selling price when the maximum selling price has a backorder effect. So, scaling parameter is $D(P) = aP^{-b}$ where $a > 0$ and price elasticity index is $b > 1$.
3. This model focusses on initial deterioration θ where $0 < \theta < 1$ and disposal cost is neglected for the deteriorating items.
4. This model operates unique cases with and without shortages inclusive to both partial and full backorder allowing lost sales.
5. To reduce deterioration rate $n(P) = 1 - e^{-\phi P}$, a continuous investment was made accountable with the application of preservation technology.
6. To reduce carbon emissions green technology investment was considered thereby protecting the environment.
7. On the whole, the time horizon of this model was examined infinite with negligible lead time.

2.2 NOTATIONS:

S_p - Cost of selling per unit

α - Length of the period in positive inventory level $0 < \alpha < 1$

T - Inventorytakt time

C_p - Amount endowed in preservation technology

G - Venture capital in Green technology

β - Backorder share

P - Purchasing cost per unit

H - Cost of holding per unit per unit time

D - Deterioration cost per unit

B - Backordering cost per unit

G_L - Goodwill lost sales cost per unit

S - Scrap price per unit

O - Ordering cost per unit

ρ - Carbon tax in kg

ω - Emission of carbon in kg per cycle

C_e - Measure of carbon emission analogous with order start off

g - Emission associated with production or purchase per capita

h - Average inventory obsolescence rate

l - Weight of obsolete units in kg per unit

f - Weight of obsolete inventory in kg

v - Rate of inventory obsolescence in percentage

n - Standard cost of inventory obsolescence in the course of carbon emission

C_p - Cost of preservation technology to reduce the deterioration, $0 \leq P \leq \bar{P}$ where \bar{P}

is the maximum cost invested in preservation technology

$n(C_p)$ - Amount of reduced deterioration rate, $0 < n(C_p) < 1$

G - Cost of green technology to reduce the emission rate per unit of time, $0 < G < \bar{G}$

where \bar{G} is the maximum cost invested in Green technology

λ - Carbon emission fragments after Green technology investment, $0 < \lambda < 1$

m - Susceptible limit of investment to carbon emission tariff, $m > 0$

I - Maximum inventory level

I' - Average inventory level

Q - Order quantity per unit

Z - Total profit obtained during the cycle

3. CRISP MODEL:

SEQ model with partial backordering with PRT and GRT is contemplated in this fragment. Here the sustainable inventory system under the carbon cap and tax policy is analysed where the retailer is consecrated to an initial cap for emissions and is allowed to sell or buy rights to emit in the cap.

The retailer acquires an extra allowance of carbon in concurrence with the carbon trading market if the carbon emission exceeds the carbon cap.

The retailer can sell his extra carbon allowance to other industries in occurrence with the carbon trading market if the carbon emission is lower than the carbon cap.

The total profit obtained during the cycle is given by:

$$Z = S_p D(P)(\alpha + \beta(1 - \alpha)) - \frac{O}{T} - PD(P)(\alpha + \beta(1 - \alpha)) - \frac{HD(P)\alpha^2 T}{2} - \frac{D\theta(1 - (1 - e^{-\eta p}))D(P)\alpha^2 T}{2} - \frac{BD(P)(1 - \alpha)^2 T\beta}{2} - G_L(1 - \beta)(1 - \alpha)D(P) - \frac{(S_p - S)l\alpha D(P)}{2} - C_p - G + \rho \left\{ W - \left(\frac{C_e}{T} + gD(P)(\alpha + \beta(1 - \alpha)) + \frac{(h + fn)D(P)\alpha^2 T}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\}$$

4. SOLUTION OF THE INVENTORY MODEL BY FUZZY GEOMETRIC PROGRAMMING TECHNIQUE:

Let $F = (a, b, c, d)$ be a trapezoidal fuzzy number and the objective function is

$$Z = S_p D(P)(\alpha + \beta(1 - \alpha)) - \frac{O}{T} - PD(P)(\alpha + \beta(1 - \alpha)) - \frac{HD(P)\alpha^2 T}{2} - \frac{D\theta(1 - (1 - e^{-\eta p}))D(P)\alpha^2 T}{2} - \frac{BD(P)(1 - \alpha)^2 T\beta}{2} - G_L(1 - \beta)(1 - \alpha)D(P) - \frac{(S_p - S)l\alpha D(P)}{2} - C_p - G + \rho \left\{ W - \left(\frac{C_e}{T} + gD(P)(\alpha + \beta(1 - \alpha)) + \frac{(h + fn)D(P)\alpha^2 T}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\}$$

The accuracy function of the fuzzy number is

$$F(A) = \frac{a + 2(b + c) + d}{6}$$

The objective function becomes,

$$F(Z) = S_p D(P)(\alpha + \beta(1 - \alpha)) - \frac{F(O)}{T} - F(P)D(P)(\alpha + \beta(1 - \alpha)) - \frac{F(H)D(P)\alpha^2 T}{2} - \frac{F(D)\theta(1 - (1 - e^{-\eta p}))D(P)\alpha^2 T}{2} - \frac{F(B)D(P)(1 - \alpha)^2 T\beta}{2} - F(G_L)(1 - \beta)(1 - \alpha)D(P) - \frac{(S_p - F(S))l\alpha D(P)}{2} - C_p - G + \rho \left\{ W - \left(\frac{C_e}{T} + gD(P)(\alpha + \beta(1 - \alpha)) + \frac{(h + fn)D(P)\alpha^2 T}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\}$$

Applying Geometric programming technique,

$$G(Z) = \prod_{i=1}^n \left\{ (S_p(\alpha + \beta(1-\alpha)) - F(P)(\alpha + \beta(1-\alpha)) - F(G_L)(1-\beta)(1-\alpha) - \frac{(S_p - F(S))l\alpha}{2})D(P) \times \frac{\omega_{1r}}{\omega_{1r}} * \right. \\ \left. \left(\frac{-O}{T^2} - \frac{F(H)D(P)\alpha^2}{2} - \frac{F(D)\theta(1-(1-e^{-\eta p}))D(P)\alpha^2}{2} - \frac{F(B)D(P)(1-\alpha)^2\beta}{2} + \rho \left\{ W - \left(\frac{C_e}{T^2} + \right. \right. \right. \right. \\ \left. \left. \left. gD(P)(\alpha + \beta(1-\alpha)) + \frac{(h + fvn)D(P)\alpha^2}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\} T \right) \times \frac{\omega_{2r}}{\omega_{2r}} * \left(\frac{-C_p}{T} \right) \times \frac{\omega_{3r}}{\omega_{3r}} * \left(\frac{-G}{\omega_{4r}} \right) \times \omega_{4r} \right\}$$

subject to the conditions

$$\omega_{1r} + \omega_{2r} + \omega_{3r} + \omega_{4r} = 1$$

$$\omega_{1r} + \omega_{2r} = 0$$

$$\omega_{2r} - \omega_{3r} = 0$$

$$\omega_{4r} = 0$$

Solving the equations we get

$$\omega_{1r} = -1; \omega_{2r} = 1; \omega_{3r} = 1; \omega_{4r} = 0$$

By applying Duffin's and Peterson's theorem

$$(S_p(\alpha + \beta(1-\alpha)) - F(P)(\alpha + \beta(1-\alpha)) - F(G_L)(1-\beta)(1-\alpha) - \frac{(S_p - F(S))l\alpha}{2})D(P) = \omega_{1r} g(\omega_{1r}, \omega_{2r}) \\ \left(\frac{-O}{T^2} - \frac{F(H)D(P)\alpha^2}{2} - \frac{F(D)\theta(1-(1-e^{-\eta p}))D(P)\alpha^2}{2} - \frac{F(B)D(P)(1-\alpha)^2\beta}{2} + \rho \left\{ W - \left(\frac{C_e}{T^2} + \right. \right. \right. \\ \left. \left. \left. gD(P)(\alpha + \beta(1-\alpha)) + \frac{(h + fvn)D(P)\alpha^2}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\} T \right) = \omega_{2r} g(\omega_{1r}, \omega_{2r})$$

$$Z^* = S_p D(P)(\alpha + \beta(1-\alpha)) - \frac{F(O)}{T} - F(P)D(P)(\alpha + \beta(1-\alpha)) - \frac{F(H)D(P)\alpha^2 T}{2} - \\ \frac{F(D)\theta(1-(1-e^{-\eta p}))D(P)\alpha^2 T}{2} - \frac{F(B)D(P)(1-\alpha)^2 T \beta}{2} - F(G_L)(1-\beta)(1-\alpha)D(P) - \frac{(S_p - F(S))l\alpha D(P)}{2} - C_p - \\ G + \rho \left\{ W - \left(\frac{C_e}{T} + gD(P)(\alpha + \beta(1-\alpha)) + \frac{(h + fvn)D(P)\alpha^2 T}{2} \right) (1 - \lambda(1 - e^{-\eta G})) \right\}$$

5. NUMERICAL EXAMPLE:

5.1 CRISP MODEL:

$\beta = 0.1$; $O = \$20/\text{order}$; $P = \$0.2/\text{order}$; $H = \$0.4/\text{unit}$; $D = \$0.5/\text{unit}$; $\theta = 0.2$; $\psi = 0.6 \text{ unit}$; $\mu = 0.2 \text{ unit}$; $\eta = 0.6$; $B = \$0.5/\text{unit}$; $G_L = \$0.4/\text{unit}$; $S = \$0.7/\text{unit}$; $l = 2 \text{ kg/unit}$; $\rho = \$0.2/\text{kg}$; $W = 100\text{kg/cycle}$; $g = 0.2 \text{ unit}$; $h = 0.4 \text{ unit}$; $f = 0.4 \text{ kg}$; $n = 0.4 \text{ unit}$; $v = 0.6 \text{ unit}$

5.1.1 OPTIMAL SOLUTION FOR LINEAR PRICE-DEPENDENT DEMAND:

OPTIMUM VALUES	SEOQ WITH PARTIAL BACKORDER	SEOQ WITH FULL BACKORDER	SEOQ WITH LOST SALES	SEOQ WITH NO BACKORDER
S_p	1.389	0.25	34.57	36.69
α	0.95	0.99	0.92	1
C_p	1.52	1.67	0.47	0.49
G	1.89	2.06	0.58	0.55
T	1.00	0.92	1.69	1.68
I	87.23	91.06	48.04	44.62
I'	41.46	45.30	22.16	22.31
Q	87.56	91.52	48.04	44.62
L	4.10	0	4.03	0
B'	0.45	0.45	0	0
$D(P)$	92.21	99.50	30.86	26.63
$DTRC$	1.67	1.67	1.67	1.67
$CERC$	13.85	13.85	13.86	13.85
Z	12.2558	21.6334	9.8294	5.3403

5.1.2 OPTIMAL SOLUTION FOR NON-LINEAR PRICE-DEPENDENT DEMAND:

OPTIMUM VALUES	SEOQ WITH PARTIAL BACKORDER	SEOQ WITH FULL BACKORDER	SEOQ WITH LOST SALES	SEOQ WITH NO BACKORDER
S_p	1.51	1.52	1.80	1.92
α	0.75	0.76	0.92	1
C_p	0.42	0.45	0.47	0.49
G	0.64	0.62	0.58	0.55

T	1.73	1.68	1.69	1.68
I	57.08	56.67	48.04	44.62
I'	21.38	20.89	22.16	22.31
Q	58.99	54.62	48.01	44.59
L	17.18	0	4.03	0
B'	1.91	1.90	0	0
$D(P)$	44.14	44.11	30.86	26.63
$DTRC$	1.67	1.67	1.67	1.67
$CERC$	13.85	13.85	13.86	13.85
Z	12.1505	10.4656	4.6923	3.2697

5.2 FUZZY MODEL:

$\beta = 0.1$; $O = \$17.5/\text{order}$; $P = \$0.225/\text{order}$; $H = \$0.45/\text{unit}$; $D = \$0.401/\text{unit}$; $\theta = 0.2$; $\psi = 0.6 \text{ unit}$;
 $\mu = 0.2 \text{ unit}$; $\eta = 0.6$; $B = \$0.5/\text{unit}$; $G_L = \$0.375/\text{unit}$; $S = \$0.7/\text{unit}$; $l = 2 \text{ kg/unit}$; $\rho = \$0.2/\text{kg}$; $W = 100\text{kg/cycle}$; $g = 0.2 \text{ unit}$; $h = 0.4 \text{ unit}$; $f = 0.4 \text{ kg}$; $n = 0.4 \text{ unit}$; $v = 0.6 \text{ unit}$

5.2.1 OPTIMAL SOLUTION FOR LINEAR PRICE-DEPENDENT DEMAND:

OPTIMUM VALUES	SEOQ WITH PARTIAL BACKORDER	SEOQ WITH FULL BACKORDER	SEOQ WITH LOST SALES	SEOQ WITH NO BACKORDER
S_p	1.389	0.25	34.57	36.69
α	0.95	0.99	0.92	1
C_p	1.52	1.67	0.47	0.49
G	1.89	2.06	0.58	0.55
T	1.00	0.92	1.69	1.68
I	87.23	91.06	48.04	44.62
I'	41.46	45.30	22.16	22.31
Q	87.56	91.52	48.04	44.62
L	4.10	0	4.03	0
B'	0.45	0.45	0	0
$D(P)$	92.21	99.50	30.86	26.63
$DTRC$	1.67	1.67	1.67	1.67
$CERC$	13.85	13.85	13.86	13.85

Z	10.6574	20.6158	9.6546	5.0932
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5.2.2 OPTIMAL SOLUTION FOR NON-LINEAR PRICE-DEPENDENT DEMAND:

OPTIMUM VALUES	SEOQ WITH PARTIAL BACKORDER	SEOQ WITH FULL BACKORDER	SEOQ WITH LOST SALES	SEOQ WITH NO BACKORDER
S_p	1.51	1.52	1.80	1.92
α	0.75	0.76	0.92	1
C_p	0.42	0.45	0.47	0.49
G	0.64	0.62	0.58	0.55
T	1.73	1.68	1.69	1.68
I	57.08	56.67	48.04	44.62
I'	21.38	20.89	22.16	22.31
Q	58.99	54.62	48.01	44.59
L	17.18	0	4.03	0
B'	1.91	1.90	0	0
$D(P)$	44.14	44.11	30.86	26.63
$DTRC$	1.67	1.67	1.67	1.67
$CERC$	13.85	13.85	13.86	13.85
Z	12.0868	10.4274	4.543	3.1119

CONCLUSION:

This model is acceptable for greenhouses that can employ an inventory management system to benchmark deterioration and carbon emission to preserve plants and flowers that could be sold at a maximum price to attain maximum profit. These greenhouse plants / flowers reduce carbon emissions from the environment.

Greenhouse owners can reduce the temperature in their greenhouses by implementing advanced technology considering both preservation and green technology investment.

So, in this model a sustainable EOQ model is put forward using two types of price-dependent demands for controllable carbon emissions and deterioration rates. To achieve the above, the

greenhouse owner is expected to invest in green technology and preservation technology under backordering and no backordering cases.

The significant upshots working in this model are,

- Implementation of carbon tax to achieve positive effect on total profit.
- Reduction of deterioration of carbon emission rates through Green and preservation technology investment.

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