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Integrated Inventory Model With Freight Costs And Two Types Of Quantity Discounts Along With The Carbon Regulations Parameters Such As Carbon Tax And Cap And Trade

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Abstract:

Sustainable environment is the dream for every human being. Today's competitive business world the carbonemission is one of the main sources for the global warming, so every mankind have theresponsibility for reducing the carbon emission to mitigate global warming. It is generallyaccepted that carbon emission trading is one of the most effective market based mechanismto prohibit the amount of carbon emission. The main objective of this proposed model is to reduce the total cost of the system and also analyses the carbon emission regulations parameters such as carbon tax and cap and trade.

Keywords: Supply Chain, two types of quantity discount, Carbon tax, Cap and Trade

Introduction:

Amoreefficientmanagementofinventoriesacrosstheentiresupplychaincanbeachievedthroughbettercoordinationbetweenthesupplierandthebuyer.Inmanypractical conditions, the freight cost is often the major component of the logistic cost.SwensethandGodfrey(2002)showedthatabout50%ofthetotallogisticcostisattributed to transportationactivity.

It is a common practice for supplier to offer lower unit prices on orders for largerquantities as an economic incentive to buyer to purchase in larger lot size. The supplierbenefits from sales of larger quantities by reducing per unit order processing and setupcosts and by increasing volume. The buyer benefits both by the reduced per unit orderingcosts and the lower unit price, at the cost of having to hold more inventory. There are twoquantitydiscountplansareofferedbysupplier:theall-unitsdiscount modelandtheincremental discountmodel. Reducing pollution is also one of the essential need for the today's business world. Carbon emission regulations parameters are also widely used by the industrialist to curb the pollution from their production and transportation activities.

This paper is organized as follows. Section 2 reviews the related literature. Section 3provides a problem description. Numerical example is presented in Section 4.Finally, theconclusionof the paper is describedinSection5.

2. Literature View:

Inventory means it is knowledge of knowing what we have in our storehouse and where the item to be stored. An integratedinventory model is first described by Goyal (1977). Goyal (1988) developed Banerjee's (1986) model by relaxing lot-for-lot assumption. Goyal (1995) developed a model where successive shipment sizes increase by a ratio equal to the production rate divided by the demand rate. Hill (1999) developed a general optimal policy model. Goyal (2003) suggested a simple procedure for determining the optimal operating policy. Baumol and Vinod (1970) were the first authors introduced the inventory-theoretic models which involving of transportation and inventory costs. Langley (1980) considered actual motor carrier freight rates function into lot sizing decision by using enumeration technique. He et al. (2010) explained an algorithm for finding the optimal purchase quantity using actual freight rates. Darwish (2008) extended the model by considering freight rate discount. Abad and Aggarwal (2005) developed a model for determining the buyer's lot sizing and pricing that there are freight and all-unit quantity discount.

Gurtu et al. (2015) modified EOQ models to include transport cost and emission cost. Bazan et al. (2015a, 2015b, 2017) developed an integrated inventory model with greenhouse gases (GHG) emissions and different coordination mechanism. Wangsa (2017) considered stochastic demand, GHG emissions, and penalty and incentive polices for a joint economic lot size (JELS).Jauhari et al. (2016) extended the model by considering freight cost in the supply chain system with two discount schemes, namely: all-units quantity discount and incremental quantity discount.

Based on the literature review this proposed model is extends the work of Wangsa,I.D and Wee,H.M(2020)" Integrated inventory system with freight costs and two types of quantity discounts" with the carbon emission regulations parameters such as carbon tax and cap and trade. The following notations are used to develop the mathematical model

2.1. Notations:

D demandrateinunitsperunittimeonthebuyer

P productionrateinunitsperunittimeofthesupplier

A fixedorderingcostincurredbythebuyer

S fixedsetupcostincurredbysupplier

C_S supplier'sproductioncost

C_{bj} buyer'spurchasingcostatdiscountleveljC$_{b0}$>C$_{b1}$>…>C$_{bJ}$>C$_s$

qj asequenceofinteger quantitieswithbreakpointj,wherej=0toJ,$q_0$istheminimumorder quantityand $q_0<q_1<q_2<q_3<\ldots q_J$

Zj extracostofbuyer'spurchasingcostinanincrementalquantitydiscount

r annualinventoryholdingcostrateperunittime

L lengthoftheleadtimesinweeks

Q orderquantity(decisionvariable)

R reorderpoint

k safetyfactor

F_0 supplier'stransportationcostpertrip

w weightofaunitpartinlbperunit

d transportationdistanceinmiles

α discountfactorforLTLshipments,$0 \le \alpha \le 1$

F_x thefreightrateindollarperpoundforagivenpermileforfulltruckload(FTL)

F_y thefreightrateindollarperpoundforagivenpermileforpartialload

W_x FTLshippingweightinlbs

W_y actualshippingweightinlbs

m numberofdeliveriesfromthesuppliertothebuyer

π_x backordercostperunitofthebuyer

π_0 grossmarginalprofitperunitofthebuyer

β thebackorderratio,$0 \le \beta \le 1$

B(R)expecteddemandshortageattheendofcycle

γ Social cost from vehicle emission (mu/h)

v average velocity(km/h)

C Carbon Cap

T_e Carbon Tax

E(Q) emission cost

α' carbon emission trading price \$ co_2lbs

3. Mathematical Model:

In this section, we briefly discuss the all-units quantity discount and

incremental quantitydiscount models along with the carbon regulations parameter such as carbon tax andcap and trade

3.1.1. All units quantity discount with Carbon tax:

The total cost function of the retailer is consist of ordering cost, holding cost, shortage cost and freight cost, it is demonstrated as follows

$$TC_{bj}(Q) = \frac{AD}{Q} + rC_{bj}\left[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] + \frac{D}{Q}\left[\pi_x\beta + \pi_\circ(1-\beta)\right]\sigma\sqrt{L}\psi(k) + \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x \quad (1)$$

The total cost for the supplier is consists of setup cost, transportation cost and holding cost for the supplier, it is demonstrated as follows

$$TCs = \frac{SD}{mQ} + \frac{F_\circ D}{Q} + rC_S\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \quad (2)$$

The integrated total cost per unit time, which is the total cost incurred to the buyer and the supplier along with the emission cost both from the supplier and retailer and considering carbon tax to reduce the carbon emission from the production and transportation activities. So the cost function is shown as below.

$$ITC(Q) = \frac{AD}{Q} + rC_{bj}\left[\frac{Q}{2} + R - D_L + (1-\beta)B(R)\right] + \frac{D}{Q}\left[\pi_x\beta + \pi_\circ(1-\beta)\right]B(R) + \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x \quad (3)$$

$$+ \frac{SD}{mQ} + \frac{F_\circ D}{Q} + rC_S\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{\gamma dD}{vQ} + \frac{T_e D}{Q}$$

Differentiating the above equation with respect to 'Q' we get the optimal order quantity as

$$Q^{*}=\sqrt{\frac{2D\left[A+\frac{s}{m}+F_{\circ}+\alpha F_{x}w_{x}d+\gamma\frac{d}{v}+T_{e}+\left[\pi_{x}\beta+\pi_{\circ}(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right]}{r\left\{c_{bj}+c_{s}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\right\}}}$$

3.1.2. Incremental Quantity Discount with Carbon Tax:

The buyer's cost functions are formulated as follows:

$$TC_{bj}=\frac{AD}{Q}+r\left(\frac{Z_{j}-C_{bj}q_{j}}{Q}+C_{bj}\right)\left[k\sigma\sqrt{L}+(1-\beta)\sigma\sqrt{L}\psi(k)\right] \quad (4)$$

$$+\frac{D}{Q}\alpha F_{x}W_{x}d+\frac{D}{Q}\left[\pi_{x}\beta+\pi_{o}(1-\beta)\sigma\sqrt{L}\psi(k)\right]+\mathrm{Ddw}(1-\alpha)\mathrm{F_x}$$

The vendors cost functions are formulated as follows:

$$TC_{s}=\frac{SD}{mQ}+\frac{F_{\circ}D}{Q}+\frac{r}{2}\left\{Z_{j}-C_{bj}q_{j}+C_{bj}Q+QC_{s}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\right\} \quad (5)$$

The integrated total cost per unit time for incremental quantity discount is shown below along with the emission cost both from the supplier and retailer and considering carbon tax to reduce the carbon emission from the production and transportation activities. So the cost function is

$$ITC(Q)=\frac{AD}{Q}+r\left(\frac{Z_{j}-C_{bj}q_{j}}{Q}+C_{bj}\right)\left[k\sigma\sqrt{L}+(1-\beta)\sigma\sqrt{L}\psi(k)\right]$$

$$+\frac{D}{Q}\alpha F_{x}W_{x}d+\frac{D}{Q}\left[\pi_{x}\beta+\pi_{o}(1-\beta)\sigma\sqrt{L}\psi(k)\right]+\mathrm{Ddw}(1-\alpha)\mathrm{F_x}$$

$$+\frac{SD}{mQ}+\frac{F_{\circ}D}{Q}+\frac{r}{2}\left\{Z_{j}-C_{bj}q_{j}+C_{bj}Q+QC_{s}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\right\}+\frac{\gamma dD}{vQ}+\frac{T_{e}D}{Q} \quad (6)$$

Differentiating the above equation with respect to 'Q' we get the optimal order quantity as

$$Q^{*}=\sqrt{\frac{2D\left[A+\frac{s}{m}+F_{\circ}+\alpha F_{x}w_{x}d+\gamma\frac{d}{v}+T_{e}+\left[\pi_{x}\beta+\pi_{\circ}(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right]+2r\left(Z_{j}-C_{bj}q_{j}\right)\left[k\sigma\sqrt{L}+(1-\beta)\sigma\sqrt{L}\psi(k)\right]}{r\left\{c_{bj}+c_{s}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\right\}}}$$

3.2.1. Carbon Trade model under all quantity discounts:

The integrated total cost per unit time is TC (Q) + α'X

The supplier and buyer carbon emissions amount to X = E(Q) – C

$$\text{ITC (Q)} = \frac{AD}{Q} + rC_{bj}\left[\frac{Q}{2} + R - D_L + (1-\beta)B(R)\right] + \qquad (7)$$

$$\frac{D}{Q}\left[\pi_x\beta + \pi_\circ(1-\beta)\right]B(R) + \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x$$

$+ \frac{SD}{mQ} + \frac{F_\circ D}{Q} + rC_s\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{\alpha'\gamma dD}{vQ}$ - α'CDifferentiating the above equation with respect to 'Q' we get the optimal order quantity as

$$Q^* = \sqrt{\frac{2D\left[A + \frac{s}{m} + F_\circ + \alpha F_x w_x d + \alpha\gamma\frac{d}{v} + \left[\pi_x\beta + \pi_\circ(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right]}{r\left\{c_{bj} + c_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\}}}$$

3.2.2. Carbon Trade model under incremental quantity discount:

The integrated total cost per unit time is TC (Q) + α'X

The supplier and buyer carbon emissions amount to X = E (Q) – C

$$\text{ITC (Q)} = \frac{AD}{Q} + r\left(\frac{Z_j - C_{bj}q_j}{Q} + C_{bj}\right)\left[k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] (8)$$

$$+ \frac{D}{Q}\alpha F_x W_x d + \frac{D}{Q}\left[\pi_x\beta + \pi_o(1-\beta)\sigma\sqrt{L}\psi(k)\right] + \mathrm{Ddw}(1-\alpha)\mathrm{F_x}$$

$$+\frac{SD}{mQ} + \frac{F_\circ D}{Q} + \frac{r}{2}\left\{Z_j - C_{bj}q_j + C_{bj}Q + QC_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\} + \frac{\alpha'\gamma dD}{vQ} - \alpha'C$$

Differentiating the above equation withrespect to 'Q' we get the optimal order quantity as

$$Q^* = \sqrt{\frac{2D\left[A + \frac{s}{m} + F_\circ + \alpha F_x w_x d + \alpha\gamma\frac{d}{v} + \left[\pi_x\beta + \pi_\circ(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right] + 2r\left(Z_j - C_{bj}q_j\right)\left[k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right]}{r\left\{c_{bj} + c_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\}}}$$

4. Numerical Example:

The following parameters are used for calculating the numerical example

D = 10,000 unit/year, P =40,000 units/year, A = 30 $/order, S = 3600 $/setup,

$r = 0.2$, $C_v = 190$ \$/unit, $L = 56$ days, $F_0 = 50$ \$/trip, $\pi_0 = 300$ \$/unit, $\sigma = 7$ units/week,
$\beta = 0.25$, $\alpha = 0.11246$, $\alpha' = 0.2$, $w = 22$ lbs/unit, $d = 600$ miles, $F_x = 0.000040217$ \$/lb/mile,
$W_x = 46000$ lbs, $C = 10200$, $T_e = 23.20$, $\gamma = 0.5$, $m = 3$, $v = 180$

Carbon Emission Regulations Parameters	**All Quantity Discount**	**Incremental Discount**
Carbon tax	$Q^* = 225.00$ TC = 31000.31	$Q^* = 210.00$ TC = 31565.45
Cap and trade	$Q^* = 225.00$ TC = 28060	$Q^* = 210.00$ TC = 29525

5. Conclusion

In this proposed model we have discussed about the integrated inventory model with two types of quantity discounts along with the analyses of carbon emission regulation parameters such as cap and trade and carbon tax. In the numerical example the total cost is less in the cap and trade system compared to the total cost with the carbon tax. And also the carbon regulations parameters are play a vital role to lead a sustainable environment.

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