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Independent Function on Intuitionistic Fuzzy Graph

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Abstract

Let $G = \langle V, E\rangle$ be an intuitionistic fuzzy graph. A function $f: V \to [0,1]$ is called an independent function if $d_{N_\mu}[v] = 1$, $d_{N_\nu}[v] \neq 1$ for every $v \in V$, where $\mu_1(v) > 0$, $\nu_1(v) \neq 1$. An independent function f is maximal if $d_{N_\mu}[v] \geq 1$, $d_{N_\nu}[v] \neq 1$ for every $v \in V$, where $\mu_1(v) = 0$, $\nu_1(v) \neq 1$. The parameters of an intuitionistic fuzzy graph, such as the independent domination number (i_{if}) and the independence number ($\beta_{0_{if}}$), are also defined.

Keywords: Independent function; independent domination number; independence number.

1. Introduction

Let a graph $G = (V, E)$ and $f: V \to [0,1]$ be a function which weights are assigns to each vertex of a graph in the interval [0,1]. We say f is dominating function(DF) if $f(N[v]) = \sum_{u\in N[v]} f(u) \geq 1$ for every $v \in V$. We say that a dominating function f is a minimal dominating function(MDF) if for all function $g: V \to [0, 1]$ with $g < f$, g is not a dominating function of G. Then the fractional domination number and upper fractional dominating number of G can be defined as $\gamma_f(G) = \min\{|f| : f\ is\ an\ MDF\ of\ G\}$ and$\Gamma_f(G) = \max\{|f| : f\ is\ an\ MDF\ of\ G\}$ in [2,3,4]. Motivated by this concept M. G. Karunambigai et al.[5] introduced the concept of dominating function, minimal dominating function, intuitionistic fractional domination number and upper intuitionistic fractional domination number of an intuitionistic fractional graph. The concept of fractional independent function was introduced by S. Arumugam et al. [1]. Let $G = (V, E)$ be a graph. A function $f: V \to [0,1]$ is called a independent function if for every v with $f(v) > 0$, we have $\sum_{u\in N[v]} f(u) = 1$. An independent function f is called a maximal independent function(MIF) if for every v with $f(v) = 0$, we have $\sum_{u\in N[v]} f(u) \geq 1$. Then the fractional independence number and fractional independent domination number of G can be defined as $\beta_{0_f}(G) = \max\{|f| : f\ is\ an\ MIF\ of\ G\}$ and $i_f(G) = \min\{|f| : f\ is\ an\ MIF\ of\ G\}$. Articles [1,5] motivated us to analyze the intuitionistic fuzzy independent function parameters of intuitionistic fuzzy graphs. Section 2 contains preliminaries and in section 3, we introduce the concept of independent function, maximal independent function, boundary set and positive set of an intuitionistic fuzzy graph(IFG). Also it deals with several results on independent function in IFGs and in Section 4, concludes the paper.

2. Preliminaries

An intuitionistic fuzzy graph(IFG)[7] is of the form $\boldsymbol{G = \langle V, E\rangle}$ where $\boldsymbol{V = \{v_1, v_2 \dots, v_n\}}$ such that $\boldsymbol{\mu_1: V \to [0, 1]}$ and $\boldsymbol{\nu_1: V \to [0, 1]}$ denote the degree of membership function and nonmembership function of $\boldsymbol{v_i \in V}$, respectively, and $\boldsymbol{0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1}$, for every $\boldsymbol{v_i \in V}$, $\boldsymbol{(i = 1, 2, \dots n)}$ and $\mathbf{E \subseteq V \times V}$ where $\boldsymbol{\mu_2: V \times V \to [0, 1]}$ and $\boldsymbol{\nu_2: V \times V \to [0, 1]}$ denote the degree of membership function and nonmembership function of $\boldsymbol{(v_i, v_j) \in E}$,such that $\boldsymbol{\mu_2(v_i, v_j) \leq min[\mu_1(v_i), \mu_1(v_j)]}$, $\boldsymbol{\nu_2(v_i, v_j) \leq max[\nu_1(v_i), \nu_1(v_j)]}$ and $\boldsymbol{0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1}$ for every $\boldsymbol{(v_i, v_j) \in E}$, $\boldsymbol{(i, j = 1, 2, \dots n)}$. An intuitionistic fractional graph $\mathbf{G = (V, E)}$ is a pair in which is a finite set or fuzzy set or intuitionistic fuzzy set and $\boldsymbol{E}$ is a finite set or fuzzy set or intuitionistic fuzzy set of 2-element subsets of $\boldsymbol{V}$. An intuitionistic fuzzy graph is a generalisation of the concept of an intuitionistic fractional graph. The closed neighbourhood degree of the vertex $\boldsymbol{v} \in \boldsymbol{V}$ of an IFG $\boldsymbol{G} = \langle \boldsymbol{V}, \boldsymbol{E} \rangle$, is defined as $\boldsymbol{d_N}[\boldsymbol{v}] = (\boldsymbol{d_{N_\mu}}[\boldsymbol{v}], \boldsymbol{d_{N_\nu}}[\boldsymbol{v}])$ where $\boldsymbol{d_{N_\mu}}[\boldsymbol{v}] = [\boldsymbol{\Sigma_{w \in N(v)} \mu_1(w)}] + \boldsymbol{\mu_1(v)}$ and $\boldsymbol{d_{N_\nu}}[\boldsymbol{v}] = [\boldsymbol{\Sigma_{w \in N(v)} \nu_1(w)}] + \boldsymbol{\nu_1(v)}$ [8]. In this section, we present some basic definitions.

Definition 2.1 [5]

A function $\boldsymbol{f_{\mu_1}}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ is called a $\boldsymbol{\mu}$ −dominating function of $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ if the closed neighborhood degree of a vertex $\boldsymbol{v} \in \boldsymbol{V}$ such that $\boldsymbol{f}\left(\boldsymbol{d_{N_{\mu_1}[v]}}\right) = \sum_{\boldsymbol{u} \in N_{\mu_1}[w]} \boldsymbol{\mu_1(u)} \geq \boldsymbol{1}$ for every $\boldsymbol{v} \in \boldsymbol{V}$.

Definition 2.2 [5]

A function $\boldsymbol{f_{\nu_1}}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ is called a $\boldsymbol{\nu}$ −dominating function of $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ if the closed neighborhood degree of a vertex $\boldsymbol{v} \in \boldsymbol{V}$ where $\boldsymbol{\nu_1}(\mathbf{v}) \neq \boldsymbol{1}$ such that $\boldsymbol{f}\left(\boldsymbol{d_{N_{\nu_1}[v]}}\right) = \sum_{\boldsymbol{u} \in N_{\nu_1}[w]} \boldsymbol{\nu_1(u)} < \boldsymbol{1}$ for every $\boldsymbol{v} \in \boldsymbol{V}$.

Definition 2.3 [5]

A function $\boldsymbol{f_{\mu_1,\nu_1}}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ is called a dominating function if it is $\boldsymbol{\mu}$ −dominating and $\boldsymbol{\nu}$ −dominating function of $\mathbf{G}$ with $\boldsymbol{0} \leq \boldsymbol{f_{\mu_1}(v)} + \boldsymbol{f_{\nu_1}(v)} \leq \boldsymbol{1}$ for each $\boldsymbol{v} \in \boldsymbol{V}$ or A function $\boldsymbol{f} = \boldsymbol{f_{\mu_1,\nu_1}}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ is called a dominating function of $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ in which $\mathbf{V}$ is a intuitionistic fuzzy set and $\mathbf{E}$ is a 2-element subsets of $\mathbf{V}$ if the closed neighborhood degree of a vertex $\boldsymbol{v} \in \boldsymbol{V}$ where $\boldsymbol{\mu_1(v)} \geq \boldsymbol{0}, \boldsymbol{\nu_1(v)} \neq \boldsymbol{1}$ such that $\sum_{\boldsymbol{u} \in N_{\mu_1}[v]} \boldsymbol{\mu_1(u)} \geq \boldsymbol{1}$, $\sum_{\boldsymbol{u} \in N_{\nu_1}[v]} \boldsymbol{\nu_1(u)} < \boldsymbol{1}$ for every $\boldsymbol{v} \in \boldsymbol{V}$ with $\boldsymbol{0} \leq \boldsymbol{f_{\mu_1}(v)} + \boldsymbol{f_{\nu_1}(v)} \leq \boldsymbol{1}$ for each $\boldsymbol{v} \in \boldsymbol{V}$.

Definition 2.4 [5]

An dominating function $\boldsymbol{f} = \boldsymbol{f_{\mu_1,\nu_1}}$ of $\mathbf{G}$ is called a minimal dominating function(MDF) if for every $\boldsymbol{v} \in \boldsymbol{V}$, where $\boldsymbol{\nu_1(v)} \neq \boldsymbol{1}$ such that $\sum_{\boldsymbol{u} \in N_{\mu_1}[v]} \boldsymbol{\mu_1(u)} = \boldsymbol{1}$, $\sum_{\boldsymbol{u} \in N_{\nu_1}[v]} \boldsymbol{\nu_1(u)} < \boldsymbol{1}$ for any $\boldsymbol{u} \in \boldsymbol{N}[\boldsymbol{v}]$.

Definition 2.5 [5]

The intuitionistic fractional domination number of $\mathbf{G}$, denoted by $\boldsymbol{\gamma_{if}(G)}$ is defined as, $\boldsymbol{\gamma_{if}(G)} = \boldsymbol{min}\{|\boldsymbol{f}| : \boldsymbol{f\ is\ an\ MDF\ of\ IFG}\}$ where $|\boldsymbol{f}| = \sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f(v)} = \left(\sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f_{\mu_1}(v)}, \sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f_{\nu_1}(v)}\right)$ or $\boldsymbol{\gamma_{if}(G)} = \left(\boldsymbol{\gamma_{if_{\mu_1}}(G)}, \boldsymbol{\gamma_{if_{\nu_1}}(G)}\right)$ where $\boldsymbol{\gamma_{if_{\mu_1}}}$ is a $\boldsymbol{f_{\mu_1}}$ −intuitionistic fractional domination number and $\boldsymbol{\gamma_{if_{\nu_1}}}$ is a $\boldsymbol{f_{\nu_1}}$ −intuitionistic fractional domination number of $\mathbf{G}$.

Definition 2.6 [5]

The upper intuitionistic fractional domination number of $\mathbf{G}$, denoted by $\boldsymbol{\Gamma_{if}(G)}$ is defined as, $\boldsymbol{\Gamma_{if}(G)} = \boldsymbol{max}\{|\boldsymbol{f}| : \boldsymbol{f\ is\ an\ MDF\ of\ IFG}\}$ where $|\boldsymbol{f}| = \sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f(v)} = \left(\sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f_{\mu_1}(v)}, \sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{f_{\nu_1}(v)}\right)$ or $\boldsymbol{\Gamma_{if}(G)} = \left(\boldsymbol{\Gamma_{if_{\mu_1}}(G)}, \boldsymbol{\Gamma_{if_{\nu_1}}(G)}\right)$ where $\boldsymbol{\Gamma_{if_{\mu_1}}}$ is a $\boldsymbol{f_{\mu_1}}$ −upper intuitionistic fractional domination number and $\boldsymbol{\Gamma_{if_{\nu_1}}}$ is a $\boldsymbol{f_{\nu_1}}$ −upper intuitionistic fractional domination number of $\mathbf{G}$.

Definition 2.7 [6]

A function $\boldsymbol{f} = \boldsymbol{f_{\mu_1,\nu_1}}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ is called an irredundant function of $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ in which $\boldsymbol{V}$ is a intuitionistic fuzzy set and $\boldsymbol{E}$ is a 2-element subsets of $\boldsymbol{V}$ if for every $\boldsymbol{v} \in \boldsymbol{V}$ where $\boldsymbol{\mu_1(v)} \geq \boldsymbol{0}, \boldsymbol{\nu_1(v)} \neq \boldsymbol{1}$ such that $\sum_{\boldsymbol{u} \in N_{\mu_1}[w]} \boldsymbol{\mu_1(u)} = \boldsymbol{1}$, $\sum_{\boldsymbol{u} \in N_{\nu_1}[w]} \boldsymbol{\nu_1(u)} < \boldsymbol{1}$ for any $\boldsymbol{w} \in \boldsymbol{N}[\boldsymbol{v}]$ with $\boldsymbol{0} \leq \boldsymbol{f_{\mu_1}(v)} + \boldsymbol{f_{\nu_1}(v)} \leq \boldsymbol{1}$ for each $\boldsymbol{v} \in \boldsymbol{V}$. An irredundant function $\boldsymbol{f} = \boldsymbol{f_{\mu_1,\nu_1}}$ of $\boldsymbol{G}$ is called a maximal irredundant function if for all function $\boldsymbol{g}: \boldsymbol{V(G)} \to [\boldsymbol{0}, \boldsymbol{1}]$ with $\boldsymbol{g} < \boldsymbol{f}$, $\boldsymbol{g}$ is not an irredundant function. Then the intuitionistic fractional irredundance number and upper intuitionistic fractional irredundance number of $\boldsymbol{G}$is defined as, $\boldsymbol{ir_{if}(G)} = \mathbf{min}\{|\boldsymbol{f}| : \boldsymbol{f}$ is an maximal irredundant function of G } or $\boldsymbol{ir_{if}(G)} = \left(\boldsymbol{ir_{if_{\mu_1}}(G), ir_{if_{\nu_1}}(G)}\right)$ and $\boldsymbol{IR_{if}(G)} = \boldsymbol{max}\{|\boldsymbol{f}| : \boldsymbol{f}$ *is* an maximal irredundant function of G} where or $\boldsymbol{IR_{if}(G)} = \left(\boldsymbol{IR_{if_{\mu_1}}(G), IR_{if_{\nu_1}}(G)}\right)$ where $|\boldsymbol{f}| = \sum_{\boldsymbol{v}\in \boldsymbol{V}} \boldsymbol{f(v)} = \left(\sum_{\boldsymbol{v}\in \boldsymbol{V}} \boldsymbol{f_{\mu_1}(v)}, \sum_{\boldsymbol{v}\in \boldsymbol{V}} \boldsymbol{f_{\nu_1}(v)}\right)$, $\boldsymbol{ir_{if_{\mu_1}}}$ is a $\boldsymbol{f_{\mu_1}}-$ intuitionistic fractional irredundance number, $\boldsymbol{ir_{if_{\nu_1}}}$ is a $\boldsymbol{f_{\nu_1}}-$ intuitionistic fractional irredundance number of $\mathbf{G}$, $\boldsymbol{IR_{if_{\mu_1}}}$ is a $\boldsymbol{f_{\mu_1}}$ −upper intuitionistic fractional irredundance number and $\boldsymbol{IR_{if_{\nu_1}}}$ is a $\boldsymbol{f_{\nu_1}}$ −upper intuitionistic fractional irredundance number of $\boldsymbol{G}$.

In the above definitions, we denote the function name $\boldsymbol{f_{\mu_1}}$ by $\boldsymbol{\mu_1}$, $\boldsymbol{f_{\nu_1}}$ by $\boldsymbol{\nu_1}$, $\boldsymbol{f_{\mu_1,\nu_1}}$ by $\boldsymbol{f}$ and the closed neighborhood degree of a vertex $\boldsymbol{v} \in \boldsymbol{V}$ denote $\boldsymbol{f}\left(\boldsymbol{d_{N_{\mu_1}[v]}}\right) = \sum_{\boldsymbol{u}\in \boldsymbol{N_{\mu_1}[v]}} \boldsymbol{\mu_1(u)}$ by $\boldsymbol{d_{N_\mu}[v]}$, $\boldsymbol{f}\left(\boldsymbol{d_{N_{\nu_1}[v]}}\right) = \sum_{\boldsymbol{u}\in \boldsymbol{N_{\nu_1}[v]}} \boldsymbol{\nu_1(u)}$ by $\boldsymbol{d_{N_\mu}[v]}$ by notational convenience for using the forthcoming theorems and results.

3. Independent function on intuitionistic fuzzy graph

Definition 3.1

A function $\mu_1: V \to [0,1]$ of an IFG, $G = \langle V, E\rangle$ is called μ_1 −independent function of G if $d_{N_\mu}[v] = 1$ for every $v \in V$, where $\mu_1(v) > 0$.

Definition 3.2

A μ_1 −independent function of an IFG, $G = \langle V, E\rangle$ is called maximal $\mu_1 -$ independent function of G if $d_{N_\mu}[v] \geq 1$ for every $v \in V$, where $\mu_1(v) = 0$.

Definition 3.3

A function $\nu_1: V \to [0,1]$ of an IFG, $G = \langle V, E\rangle$ is called ν_1 −independent function of G if $d_{N_\nu}[v] < 1$ for every $v \in V$, where $\nu_1(v) \neq 1$.

Definition 3.4

A ν_1 −independent function of an IFG, $G = \langle V, E\rangle$ is called maximal $\nu_1 -$ independent function of G if $d_{N_\nu}[v] \neq 1$ for every $v \in V$, where $\nu_1(v) \neq 1$.

Definition 3.5

A independent function $f: V \to [0,1]$ of an IFG, $G = \langle V, E\rangle$ is a μ_1-independent function and a ν_1-independent function on G, where the values of f are of the kind $\langle \mu_1, \nu_1\rangle$ and $\mu_1(v) + \nu_1(v) \leq 1$ for all $v \in V$.

Definition 3.6

Let $G = \langle V, E\rangle$ be an IFG. A independent function $f: V \to [0,1]$ is called a maximal independent function of G if there does not exist a independent function $f \neq g$, for which $g(v) \leq f(v)$ for every $v \in V$. Equivalently a function f is said to be maximal independent function of G if it is a maximal μ_1 −independent function and a maximal ν_1 −independent function of G, that is $d_{N_\mu}[v] \geq 1$, $d_{N_\nu}[v] \neq 1$ for every $v \in V$, where $\mu_1(v) \geq 0$, $\nu_1(v) \neq 1$.

Remark 3.7

Let $f: V \to [0,1]$ is a independent function on an intuitionistic fuzzy graph $G = \langle V, E\rangle$ where $f = \{f_1, f_2, \dots, \}$. Then f is a maximal independent function of G (i.e., $f = \{f_i : f_i \neq f_j, f_i(v) \leq f_j(v)$ for all

$v \in V, i, j = 1,2, \ldots \})$ if and only if whenever $\mu_1(u) > 0, \nu_1(u) \neq 1$ there exists some $v \in N[u]$ such that $d_{N_\mu}[u] = 1, d_{N_\nu}[u] < 1$ and u dominates v.

Definition 3.8

The independent domination number $i_{if}(G)$ and the independence number $\beta_{0_{if}}(G)$ of an IFG G are defined as

(i) $i_{if}(G) = \left(i_{if_{\mu_1}}(G), i_{if_{\nu_1}}(G)\right)$ where $i_{if_{\mu_1}}$ is a $\mu_1 -$ independent domination number and $i_{if_{\nu_1}}$ is a $\nu_1 -$ independent domination number of G. Equivalently $i_{if}(G) = min\{|f_j| : f_j$ is a maximal independent function of IFG $\}$ where $min\{|f_j|\} = min\{|\langle\mu_1, \nu_1\rangle|\} = (\min|\mu_1|, \min|\nu_1|)$, $|\mu_1| = \sum_{v_i \in V} \mu_1(v_i)$, $|\nu_1| = \sum_{v_i \in V} \nu_1(v_i)$, $i = 1,2, \ldots, n$, $j = 1,2, \ldots$.

(ii) $\beta_{0_{if}}(G) = \left(\beta_{0_{if_{\mu_1}}}(G), \beta_{0_{if_{\nu_1}}}(G)\right)$ where $\beta_{0_{if_{\mu_1}}}$ is a $\mu_1 -$ independence number and $\beta_{0_{if_{\nu_1}}}$ is a $\nu_1 -$ independence number of G. Equivalently $\beta_{0_{if}}(G) = max\{|f_j| : f_j$ is a maximal independent function of IFG$\}$ where $max\{|f_j|\} = max\{|\langle\mu_1, \nu_1\rangle|\} = (\max|\mu_1|, \max|\nu_1|)$, $|\mu_1| = \sum_{v_i \in V} \mu_1(v_i)$, $|\nu_1| = \sum_{v_i \in V} \nu_1(v_i)$.

The problem of finding the independent domination number(i_{if}) and independence number($\beta_{0_{if}}$) of an IFG which is equivalent to finding the optimal solution of the following linear programming problem.

(a) $i_{if_{\mu_1}} = min\, cX_{if} = \sum_{i=1}^{n} \mu_1(v_i)$, Subject to $d_{N_\mu}[v_i] \geq \vec{1}$ with $\mu_1(v_i) \in [0,1]\ \forall\ v_i \in V(G)$

(b) $i_{if_{\nu_1}} = min\, cX_{if} = \sum_{i=1}^{n} \nu_1(v_i)$, Subject to $d_{N_\nu}[v_i] \geq \vec{1}$ with $\nu_1(v_i) \in [0,1]\ \forall\ v_i \in V(G)$

(c) $\beta_{0_{if_{\mu_1}}} = max\, cX_{if} = \sum_{i=1}^{n} \mu_1(v_i)$, Subject to $d_{N_\mu}[v_i] \leq \vec{1}$ with $\mu_1(v_i) \in [0,1]\ \forall\ v_i \in V(G)$

(d) $\beta_{0_{if_{\nu_1}}} = max\, cX_{if} = \sum_{i=1}^{n} \nu_1(v_i)$, Subject to $d_{N_\nu}[v_i] \leq \vec{1}$ with $\nu_1(v_i) \in [0,1]\ \forall\ v_i \in V(G)$

Note that $d_{N_\mu}[v_i] = N.X_{if}$ where N is the closed neighborhood of $\mu_1 -$ independent function value of the matrix , $X_{if} = [\mu_1(v_1), \mu_1(v_2), \ldots, \mu_1(v_n)]^t$ & $\vec{1} = [1,1, \ldots, 1]^t$ be the column vector, $c = \overrightarrow{1_i} = [1,1, \ldots, 1]$, $i = 1,2, \ldots, n$ be the row vector. Similarly $d_{N_\nu}[v_i] = N.X_{if}$ where N is the closed neighborhood of $\nu_1 -$ independent function value of the matrix and $X_{if} = [\nu_1(v_1), \nu_1(v_2), \ldots, \nu_1(v_n)]^t$ be the column vector.

The above L.P.P. is to be solved by using Linear Program Solver software(LiPS).

Remark 3.9

Since every maximal independent function is an minimal dominating function, we have

$\gamma_{if_{\mu_1}}(G) \leq i_{if_{\mu_1}}(G) \leq \beta_{0_{if_{\mu_1}}}(G) \leq \Gamma_{if_{\mu_1}}(G)$ and

$\gamma_{if_{\nu_1}}(G) = i_{if_{\nu_1}}(G) < \beta_{0_{if_{\nu_1}}}(G) \leq \Gamma_{if_{\nu_1}}(G)$.

Hence we obtain the following analogue of domination chain for the intuitionistic fuzzy domination:

$ir_{if_{\mu_1}}(G) \leq \gamma_{if_{\mu_1}}(G) \leq i_{if_{\mu_1}}(G) \leq \beta_{0_{if_{\mu_1}}}(G) \leq \Gamma_{if_{\mu_1}}(G) \leq IR_{if_{\mu_1}}(G)$ and

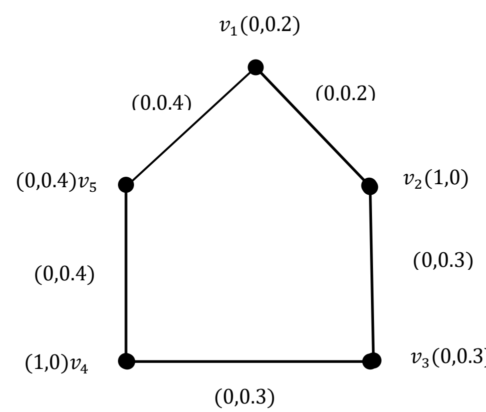

Fig 1

$$ir_{if_{v_1}}(G) = \gamma_{if_{v_1}}(G) = i_{if_{v_1}}(G) < \beta_{0_{if_{v_1}}}(G) \leq \Gamma_{if_{v_1}}(G) \leq IR_{if_{v_1}}(G)$$

Example 3.10

Here $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$.

In Figure 1, $N[v_1] = \{v_1, v_2, v_5\}$, $N[v_2] = \{v_1, v_2, v_3\}$,
$N[v_3] = \{v_2, v_3, v_4\}$, $N[v_4] = \{v_3, v_4, v_5\}$, $N[v_5] = \{v_1, v_4, v_5\}$

Then $d_{N_\mu}[v_i] = \begin{cases} 1 & if \quad v_i = v_1 \\ 1 & if \quad v_i = v_2 \\ 2 & if \quad v_i = v_3 \\ 1 & if \quad v_i = v_4 \\ 1 & if \quad v_i = v_5 \end{cases}$, $d_{N_\nu}[v_i] = \begin{cases} 0.6 & if \quad v_i = v_1 \\ 0.5 & if \quad v_i = v_2 \\ 0.3 & if \quad v_i = v_3 \\ 0.7 & if \quad v_i = v_4 \\ 0.6 & if \quad v_i = v_5 \end{cases}$

Therefore a function $f: V \to [0,1]$ of an IFG, G is a maximal independent function. Since $d_{N_\mu}[v_i] \geq 1$, $d_{N_\nu}[v_i] \neq 1$ for every $v \in V$. In that case,

$i_{if}(G) = \left(i_{if_{\mu_1}}(G), i_{if_{v_1}}(G)\right) = \left(\frac{5}{3}, 0\right)$ and $\beta_{0_{if}}(G) = \left(\beta_{0_{if_{\mu_1}}}(G), \beta_{0_{if_{v_1}}}(G)\right) = \left(2, \frac{5}{3}\right)$

Definition 3.11

For a dominating function (DF) f of an G G, the boundary set B_{if} and positive set P_{if} are defined by

$B_{if} = \{v \in V / d_{N_\mu}[v_i] = 1, d_{N_\nu}[v_i] < 1\}$ and $P_{if} = \left\{v \in V / \begin{array}{c} 0 \leq \mu_1(u) \leq 1 \\ 0 < \nu_1(u) < 1 \end{array}\right\}$.

Definition 3.12

Let $G = (V, E)$ be an IFG and let $A, B \subseteq V$. A is said to dominate B if each $v \in B - A$ is adjacent to a vertex in A. If $A \; dominates \; B$, we write $(A \to B)$.

Example 3.13

In Fig 1, $B_{if} = \{v_1, v_2, v_4, v_5\}$ and $P_{if} = \{v_1, v_3, v_5\}$. we take $A = \{v_1, v_2, v_4, v_5\}$, $B = \{v_1, v_3, v_5\}$. Then $B - A = \{v_3\}$ is adjacent to a vertex in A. Therefore B_{if} dominates P_{if} denoted by $B_{if} \to P_{if}$

Definition 3.14

Let f, g be two domination functions of an IFG G, and let $0 < \lambda < 1$. Then $h_\lambda = \lambda f + (1 - \lambda)g$ is called a convex combination of f and g.

Theorem 3.15

A domination function f of an IFG G is an minimal dominating function if and only if $B_{if} \to P_{if}$.

Proof:

Let f be an minimal dominating function of an IFG G and take $B_{if}, P_{if} \subseteq V(G)$. Then any vertex $v \in P_{if} - B_{if}$ is adjacent to a vertex in B_{if}. Hence $B_{if} \to P_{if}$. The converse part is trivially true by the definition of B_{if} and P_{if}.

Remark 3.16

If a function $f: V \to [0,1]$ is an independent function, then $P_{if} \subseteq B_{if}$

Theorem 3.17

Every maximal independent function of an IFG G is an minimal dominating function.

Proof:

Let f be a maximal independent function of an IFG G. It follows from the definition that $d_{N_\mu}[v] \geq 1$, $d_{N_\nu}[v] \neq 1$ for all $v \in V$. Hence f is a dominating function.

Further $P_{if} \subseteq B_{if}$ so that $B_{if} \to P_{if}$. Hence by Theorem 3.15 that f is an minimal dominating function.

Lemma 3.18

Any minimal dominating function f of an IFG G with $B_{if} = V$ is an maximal independent function of G.

Proof:

Since $B_{if} = V$, we have $P_{if} \subseteq B_{if}$ and hence f is an independent function. Also since f is an minimal dominating function, $d_{N_\mu}[v] \geq 1, d_{N_\nu}[v] \neq 1$ for all $v \in V$ and hence f is an maximal independent function.

Corollary 3.19

If G is an intuitionistic fuzzy graph with $B_{if} = V$ for every minimal dominating function f of G then $\gamma_{if} = i_{if}$ and $\beta_{0_{if}} = \Gamma_{if}$.

Remark 3.20

The convex combination of two dominating functions of an IFG G is again a dominating function and also the convex combination of two minimal dominating functions of an IFG G again a minimal dominating function.

Theorem 3.21

Let $f \; and \; g$ be an minimal dominating functions of G. Let $h_\lambda = \lambda f + (1-\lambda)g$, where $0 < \lambda < 1$. Then h_λ is an minimal dominating function of G if and only if $B_{if} \cap B_{ig} \rightarrow P_{if} \cup P_{ig}$

Proof:

We prove that $B_{h_\lambda} = B_{if} \cap B_{ig}$ and $P_{h_\lambda} = P_{if} \cup P_{ig}$. The result is then immediate from Theorem 3.15. If $v \in P_{if} \cup P_{ig}$, then $f(v) = g(v) = h_\lambda(v) = (\mu_1(v) = 0, 0 < \nu_1(v) < 1)$. If, say, $v \in P_{if}$, then $(h_\lambda(v) = (\mu_1(v) > 0, 0 < \nu_1(v) < 1)) \geq (\lambda f(v) = (\lambda\mu_1(v) > 0, \lambda\nu_1(v) > 0))$. Thus $P_{h_\lambda} = P_{if} \cup P_{ig}$.

Suppose $v \in B_{if} \cap B_{ig}$. Then

$$h_\lambda\left(d_{N_\mu}[v]\right) = \lambda\left(f\left(d_{N_\mu}[v]\right)\right) + (1-\lambda)\left(g\left(d_{N_\mu}[v]\right)\right) = 1$$

$$h_\lambda\left(d_{N_\nu}[v]\right) = \lambda\left(f\left(d_{N_\nu}[v]\right)\right) + (1-\lambda)\left(g\left(d_{N_\nu}[v]\right)\right) < 1$$

A similar calculation shows $h_\lambda\left(d_{N_\mu}[v]\right) > 1, h_\lambda\left(d_{N_\nu}[v]\right) < 1$

for $v \notin B_{if} \cap B_{ig}$ and hence $B_{h_\lambda} = B_{if} \cap B_{ig}$.

Remark 3.22

The convex combination of two maximal independent functions of an IFG G, need not be an independent function.

Example 3.23

Let f & g be two maximal independent functions of a IFG G.

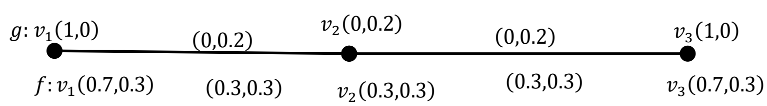

Fig 2

Let $h_\lambda = \lambda f + (1-\lambda)g \;\; where \;\; 0 < \lambda < 1$. Here $\lambda = 0.4$ then

$h_{0.4}(v_1, v_2, v_3) = 0.4f + (1-0.4)g$

$$= 0.4[(0.7,0.3),(0.3,0.3),(0.7,0.3)] + (0.6)[(1,0),(0,0.2),(1,0)]$$
$$= [(0.28,0.12),(0.12,0.12),(0.28,0.12)] + [(0.6,0),(0,0.12),(0.6,0)]$$
$$= [(0.28+0.6,0.12+0),(0.12+0,0.12+0.12),(0.28+0.6,0.12+0)]$$

$h_{0.4}(v_1,v_2,v_3) = (0.88,0.12),(0.12,0.24),(0.88,0.12)$.

$h_{0.4}$ is a maximal independent function of G.

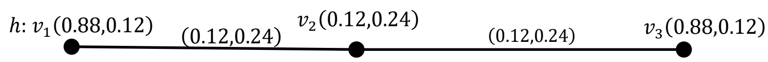

Fig 3

Note 3.24

By Theorem 3.17, Every maximal independent function of an IFG G is an minimal dominating function. In the above example h_λ is maximal independent function and also a minimal dominating function.

In Figure 2, $P_{if} = \{v_1,v_2,v_3\}$, $P_{ig} = \{v_2\}$, $B_{if} = B_{ig} = \{v_1,v_3\}$, $P_{if} \cup P_{ig} = \{v_1,v_2,v_3\}$ and $B_{if} \cap B_{ig} = \{v_1,v_3\}$ then $B_{if} \cap B_{ig} \to P_{if} \cup P_{ig} = \{v_1,v_3\} \to \{v_1,v_2,v_3\}$. That is $\{v_1,v_2,v_3\} - \{v_1,v_3\} = \{v_2\}$ is adjacent to a vertex in v_1 *and* v_3 .

Remark 3.25

The convex combination of two independent functions of an IFG G, need not be an independent function of an IFG.

Remark 3.26

Let f *and* g be two independent functions of an IFG G. Let $h_\lambda = \lambda f + (1-\lambda)g$, where $0 < \lambda < 1$. Then h_λ is an independent function if and only if $P_{if} \cup P_{ig} \subseteq B_{if} \cap B_{ig}$.

Example 3.27

Let f, g and h_λ are independent functions of G in Figure 4 as given below and also take $\lambda = 0.2$ (Refer Table 1)

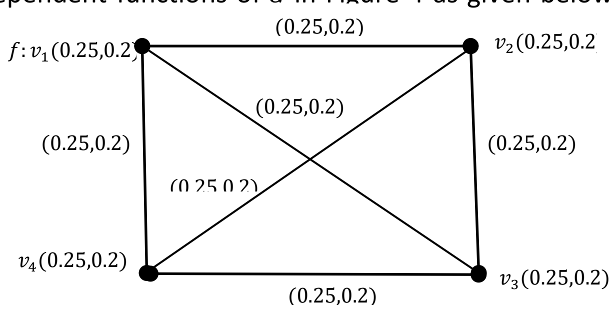

Fig 4

Table 1

	v_1	v_2	v_3	v_4	$d_{N_\mu}[v]$ $\forall\ v \in V$	$d_{N_\nu}[v]$ $\forall\ v \in V$

f	(0.25,0.2)	(0.25,0.2	(0.25,0.2)	(0.25,0.2)	1	0.8
g	(0.75,0.1)	(0.1,0.2)	(0.1,0.3)	(0.05,0.3)	1	0.9
h	(0.65,0.12)	(0.21,0.2)	(0.29,0.28)	(0.09,028)	1	0.88

Lemma 3.28

Let $f\ and\ g$ be two maximal independent functions. Then either all convex combinations of $f\ and\ g$ are maximal independent functions or none of them is an maximal independent functions.

Proof:

Let $h_\lambda = \lambda f + (1-\lambda)g \quad where \ \ 0 < \lambda < 1$. Suppose that h_{λ_1} is an maximal independent function and let $\lambda \neq \lambda_1$. We claim that h_λ is an maximal independent function.

Let $v \in V$. Suppose $h_\lambda(v) = (\mu_1(v) = 0{,}0 < \nu_1(v) < 1)$. Then $f(v) = g(v) = (\mu_1(v) = 0, 0 < \nu_1(v) < 1)$. Since f and g are maximal independent functions, we have $f(\ d_{N_\mu}[v])\ \geq 1, f\big(\ d_{N_\nu}[v]\big) < 1$ and $g(\ d_{N_\mu}[v])\ \geq 1\ , g\big(\ d_{N_\nu}[v]\big) < 1$. Hence it follows that $h_\lambda(d_{N_\mu}[v])\ \geq 1\ \ , h_\lambda\big(d_{N_\nu}[v]\big) < 1$.

Now suppose $h_\lambda(v) = (\mu_1(v) > 0, 0 < \nu_1(v) < 1)$. Then either $f(v) = (\mu_1(v) > 0, 0 < \nu_1(v) < 1)$ or $g(v) = (\mu_1(v) > 0, 0 < \nu_1(v) < 1)$. Hence $h_{\lambda_1}(v) = (\mu_1(v) > 0, 0 < \nu_1(v) < 1)$. Since h_{λ_1} is an maximal independent function, $h_{\lambda_1}\left(\ d_{N_\mu}[v]\right) = 1, h_\lambda\big(\ d_{N_\nu}[v]\big) < 1$ so that $f\left(d_{N_\mu}[v]\right) = g\left(d_{N_\mu}[v]\right) = 1$ & $f\big(d_{N_\nu}[v]\big) = g\big(d_{N_\nu}[v]\big) < 1$. Hence $h_\lambda\left(d_{N_\mu}[v]\right) = 1$ and $h_\lambda\big(d_{N_\nu}[v]\big) < 1$. Thus $P_{h_\lambda} \subseteq B_{h_\lambda}$ and $h_\lambda(d_{N_\mu}[v])\ \geq 1$ and $h_\lambda\big(d_{N_\nu}[v]\big) < 1$ for all $v \in V$. Hence h_λ is an maximal independent function.

Example 3.29

Let f, g, h_λ and h_{λ_1} are maximal independent functions of G in Fig 1 as given below and also take $\lambda(= 0.7) \neq \lambda_1(= 0.2)$ (Refer Table 2 & 3)

Table 2

	v_1	v_2	v_3	v_4	v_5
f:	(0,0.2)	(1,0)	(0,0.3)	(1,0)	(0,0.4)
g:	(0,0.3)	(0.5,0.3)	(0.5,0.2)	(0,0.1)	(1,0)
$h_{0.7}$:	(0,0.23)	(0.85,0.09)	(0.15,0.27)	(0.7,0.03)	(0.3,0.28)
$h_{0.2}$:	(0,0.28)	(0.6,0.24)	(0.4,0.22)	(0.2,0.08)	(0.8,0.08)

Table 3

	$d_{N_\mu}[v] \geq 1$					$d_{N_\nu}[v] < 1$				
	v_1	v_2	v_3	v_4	v_5	v_1	v_2	v_3	v_4	v_5
f:	1	**1**	2	1	**1**	0.6	0.5	0.3	0.7	0.6
g:	1.5	**1**	1	1.5	**1**	0.6	0.8	0.6	0.3	0.4
$h_{0.7}$:	1.15	**1**	1.7	1.15	**1**	0.6	0.59	0.39	0.58	0.54

$h_{0.2}$:	1.4	**1**	1.2	1.4	**1**	0.6	0.74	0.54	0.38	0.44

Observation 3.30:

The intuitionistic fuzzy independent domination number can be computed in a greedy way: Begin by assigning membership and non membership weights of 1 and 0 to each vertex, and then simply decrease membership weights and put zero to non membership weights on vertices (in any order), ensuring that each closed neighbourhood degree of the vertex has a membership weight of equal to one and a non membership weight of less than one. When the last vertex is reached, the sum of all the membership and non membership weights will be $i_{if_{\mu_1}}(\mathrm{G})$ and $i_{if_{\nu_1}}(\mathrm{G})$.

The intuitionistic fuzzy independence number can be computed in a greedy way: Begin by assigning membership and non membership weights of 1 and 0 to each vertex, and then simply decrease membership weights and increase non membership weights that are strictly greater than or equal to zero and not equal to one on vertices (in any order), ensuring that each closed neighbourhood degree of the vertex has a membership weight of at least one and a non membership weight of less than one. When the last vertex is reached, the sum of all the membership and non membership weights will be $\beta_{0_{if_{\mu_1}}}(\mathrm{G})$ and $\beta_{0_{if_{\nu_1}}}(\mathrm{G})$.

4. Conclusion

In this paper, we introduce the concept of independent function, maximal independent function, convex combinations of two independent function, convex combinations of two maximal independent function, intuitionistic fuzzy independent domination number, intuitionistic fuzzy independence number, boundary set and positive set of an intuitionistic fuzzy graph(IFG) have been discussed. The intuitionistic fuzzy dominating set and its function has been commonly used for routing and broadcasting the information to the mobile devices in mobile networks. In Further, we will study the real life problems can be premeditated and solutions can be predicted by using these intuitionistic fuzzy graphs.

References

[1] S. Arumugam and K. Rejikumar, Fractional independence and fractional domination chain in graphs, AKCE J Graphs. Combin.,. 4 (2) (2007) 161–169.

[2] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker Inc., New York, 1998.

[3] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, Domination in Graphs: Advanced topics, Marcel Dekker Inc., New York, 1998.

[4] S.M. Hedetniemi, S.T. Hedetniemi, T.V. Wimer, Linear time resource allocation algorithms for trees, Tech. Rept. URI- 014, Department of Mathematical Sciences, Clemson University, 1987.

[5] M.G. Karunambigai, A.Sathishkumar, Dominating function in intuitionistic fractional graph, Malaya Journal of Matematik, 8 (4) (2020) 1653-1660.

[6] M.G. Karunambigai, A. Sathishkumar, Irredundant function in intuitionistic fractional graph, Journal of the Maharaja Sayajirao University of Baroda(print only) , 54 (2 (IIX)) (2020) 67-72.

[7] M.G. Karunambigai, R. Parvathi, Intuitionistic fuzzy graphs, Computational Intelligence, Theory and Applications, Springer Berlin Heidelberg, 2006, 139-150.

[8] A. Nagoor Gani and S. Shajitha Begum, Degree order and size in intuitionistic fuzzy graphs, International Journal of Algorithms, Computing and Mathematics, 3 (3) (2010) 11-16.