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Modelling The Causal Influence of Self-Beliefs Related to Emotional Intelligence on Leadership Using Fuzzy Relational Map Based on Hesitant Fuzzy Sets

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Abstract

Fuzzy Relational Map is a graphical representation of relational structure between two disjoint fuzzy sets. This paper proposes a new approach to construct fuzzy relational map with hesitant fuzzy set, an extension of regular fuzzy set. The fuzzy relational map based on hesitant fuzzy sets is way more effective in capturing the uncertainty of the problem and the hesitancy of the expert than the conventionalone. Effective leadership is the guiding force of a civilised society and high-performance institutions and it is closely linked to emotional intelligence capabilities. In this paper, the influence of beliefs related to emotional intelligence on leadership is studied.

Keywords: Fuzzy Relational Map, hesitant fuzzy set, beliefs, emotional intelligence, leadership

1. Main text

Fuzzy Relational Map (FRM) is a soft computing techniquethat combines fuzzy logic and neural network. FRM can model complex and dynamic systems effectively based on human knowledge and experience [1].The dynamics of FRM is similar to that of FCM except that in FRM the causal relations are between two disjoint fuzzy sets[2]. Since the onset of Fuzzy set theory, several extensions of fuzzy sets have come to existence depending on the need and necessity of the problem. Hesitant fuzzy set is one of the extensions of fuzzy set that takes into account the imprecise and hesitant information of experts [3]. In this paper, a new approach is developed by combining the hesitant fuzzy set (HFS) theory with FRM to deal with a special kind of uncertainty namely, hesitancy. The FRM based on HFS, is effective in dealing with the issues where the experts have varying opinions. The hesitant fuzzy influence of the relationships among the factors can be quantified by this approach.

2. Fuzzy Relational Map Model based on Hesitant Fuzzy Sets

The concept of hesitant fuzzy set (HFS) was introduced by Torra and Narukawa, as one of the extensions of Zadeh (1965)'s fuzzy set, in 2009 [4,5]. The membership degree of an element belonging to a HFS is denoted by several possible values. HFS can express the hesitant information more comprehensively besides the uncertaintythan the other extensions of fuzzy sets. Xu and Xia (2011) defined the concept of hesitant fuzzy element (HFE) which can be considered as the basic unit of an HFS, a simple tool used to express the experts' hesitant opinion [3]. The fuzzy set theory relies on linguistic variables that describes the information qualitatively [6]. The fuzzy linguistic variable that is described by a single term is inadequate to evaluate language variants involving hesitation. In order to resolve this kind of issue Hesitant Fuzzy Linguistic Term Sets (HFLTSs) have been proposed as a solution by Rodriguez et al. in 2012 [7].

Let X be a fixed set, a hesitant fuzzy set (HFS) on X is defined in terms of a function that returns a subset of [0,1], that is, $h_A: X \to \{[0,1]\}$ [4]. A hesitant fuzzy set is denoted by $A = \{x,\ h_A(x) | x \in X\}$ where $h_A(x)$, the hesitant fuzzy element, is a set of some values from $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set A. A hesitant fuzzy linguistic term set (HFLTS), denoted by H_s, is an ordered finite subset of consecutive linguistic terms of the linguistic term set S. The envelope of an HFLTS $env(H_S)$, is a linguistic interval with its upper bound and lower bound as limits. Liu and Rodríguez (2014) proposed a new method to obtain a fuzzy envelope for HFLTS [3]. This is a trapezoidal fuzzy membership function (TFMF) obtained by aggregating the fuzzy membership functions of the linguistic terms of the HFLTS according to their relevance [7,8].

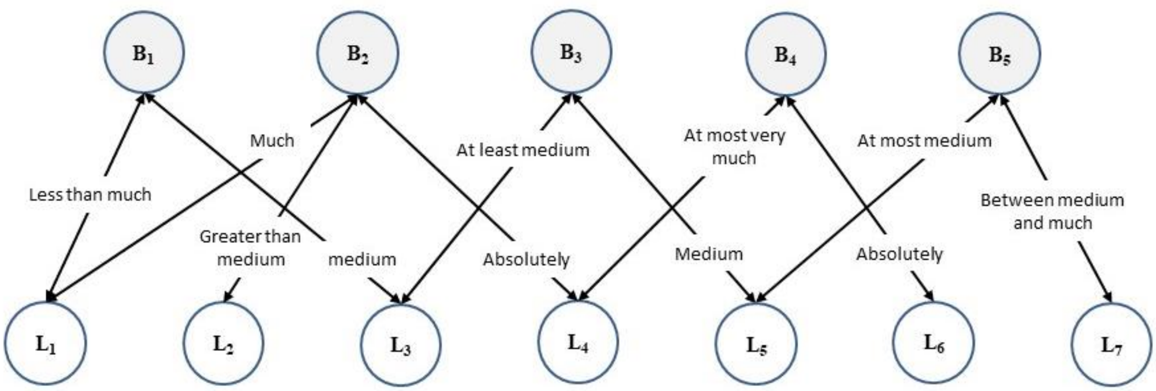

Figure 1: An illustrative model of Fuzzy Relational Map based on HFS

FRM is an efficient computing tool to model knowledge in decision-making processes. FRM is a graphical structure constructed between two disjoint sets of concepts. The concepts are represented by the nodes and the causal relations by the edges between the concepts. The causal connections represent the interactive causality among concepts that it reflects the dynamic relationships in knowledge-based complex systems [9].The causal relations in the FRM can be observed and predicted by making an inference for a long-term circumstance. The concepts $(C_i)$and weighted causal relations $(w_{ij})$are generally characterised by fuzzy sets. FRM based on hesitant fuzzy sets (HFS)represent the unstructured knowledge through causalities expressed in Hesitant Fuzzy Elements (HFEs). The experts provide information represented by HFEs to describe the influence of one variable on another. An illustrative example of an FRM based on HFS is shown in Figure1.

3. Description of the Problem

The greatest challenge of leadership of our generation is to overcome the misconceptions of control and result-oriented approach and move to a new understanding to include values and spiritual qualities [10]. The era where people believed that capacity, intelligence and power are enough to lead an organisation has gone. Recent researches have studied the idea that emotional intelligence is the central theme of successful performance in any sphere of life. Successful and effective leadership must focus on emotional and spiritual intelligence and the beliefs that influence such intelligences. In particular, leadership is a powerful role that demands a high-level intelligence in handling one's own emotions and that of others [11]. The emotional impact of a leader is the least discussed fact but the most influential at every level of performance inan organisation. Emotional leadership is the spark that ignites a company's performance, creating a bonfire of success or a landscape of ashes.

Every individual's action, feelings, behaviours and abilities are results of one's core beliefs [12]. The beliefs make up the cognitive component of human emotions. The dormant beliefs that are stored in the rational brain contribute to emotions and the actions related to it. Emotions are functional tools that are helpful in identifying one's beliefs that operate them. Studying the relationship between emotional intelligence and principles of leadership gives the criteria of successful leadership. In this paper, an attempt is made to study the intrinsic relationship between the belief cluster that underlie emotional intelligence and its influence on leadership. David Ryback, Ph.D. in his book, "Putting Emotional Intelligence to Work: Successful Leadership Is More Than IQ" describes Seven Core Qualities of Leadership [10] that characterise an efficient leader. The domains of beliefs pertaining to emotional intelligence are taken from Chapter 6 of Lynn's "The EQ Difference" [12].

Notation	Factors of Domain Space	Notation	Factors of Range Space
B_1	Self-awareness and Self-control	L_1	Strategic Planning
B_2	Empathy	L_2	Communication and alignment
B_3	Social Expertness	L_3	Team building
B_4	Personal Influence	L_4	Continuous Learning
B_5	Mastery of Purpose and Vision	L_5	Dynamic accountability
		L_6	Systemic Results
		L_7	Actualised Integrity

Table 1: Factors of FRM

4. Construction and Analysis of the Problem using FRM based on HFS

Step 1: The variables are chosen as the nodes of the FRM that describes the complex structure that is to be modelled.

Step 2: The relationships between the nodes are identified based on the experts' domain knowledge. Using the linguistic evaluations that define the causal relations among concepts graph-based FRM model is constructed.

Step 3: The linguistic estimations are transformed to HFLTS according to the linguistic term set such as at least medium, between very low and medium, lower than very much and so on.

Step 4: The membership functions of HFLTS are aggregated using Ordered Weighted Aggregation methodand a trapezoidal fuzzy membership function (TFMF) $\hat{A} = (a, b, c, d)$ is obtained [13].

Step 5: The TFMFs are converted into crisp values using a defuzzification method.

Step 6: The values obtained in the above step is taken as the strength of causal relation between the variables. These values of the causal connection (edges) between the concepts constitute the adjacency matrix of the FRM.

Step 7: Initial state vector of different concepts is passed through the relational matrix in an iterative manner until the fixed point is reached.

The linguistic term set, $S = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$is adopted to describe the causal relations between the concepts. An illustrative example of hesitant linguistic terms and their corresponding membership values are given in figure 2. In regular fuzzy set approach, the experts are usually provided with single linguistic terms to express their opinions. There may arise situations where the experts might hesitate among different linguistic terms to express their preferences. In circumstances, where high degree of uncertainty is involved, the experts would like to have a greater flexibility to propose their views in linguistic expressions. This context needs a space for the experts to express their hesitancy with proper linguistic expressions. Hesitant Fuzzy Linguistic Term Sets (HFLTSs) [7] provide rules to generate linguistic expressions that are nearly close to humans' cognitive thinking. The causal relations among the factors are defined with the linguistic evaluations of the experts with the help of HFLTSs. An example of an HFLTS for the linguistic variable $v = At\ least\ medium$ is given as follows. $H_S(v) = \{At\ least\ medium\} = \{Not\ at\ all, Very\ Little, Little, Medium\} = \{S_0, S_1, S_2, S_3\}$. The HFRM model, constructed with the HFLTS provided by the experts, is given in table 2.

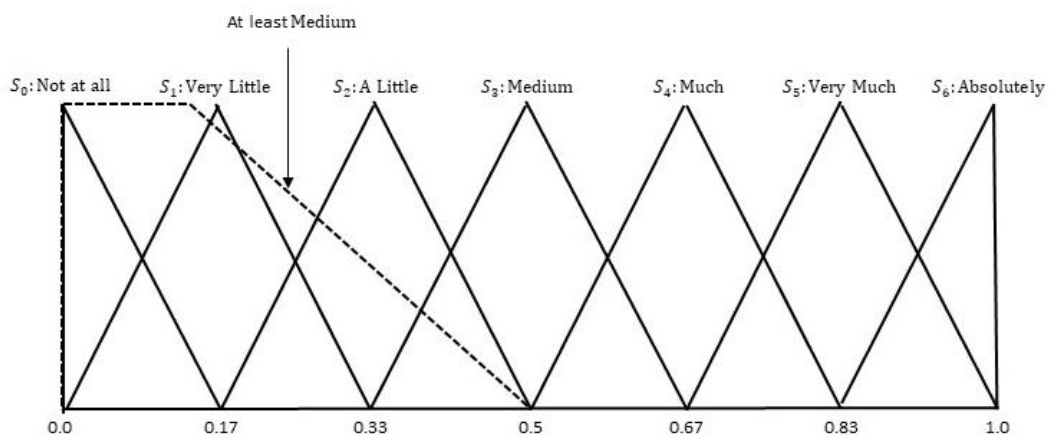

Figure 2: Membership values of HLTS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	$\{s_4, s_5, s_6\}$	$\{s_4, s_5, s_6\}$	-	-	$neg\{s_0, s_1, s_2\}$	-	$\{s_4, s_5, s_6\}$
B_2	-	$\{s_2, s_3\}$	$\{s_3\}$	-	$neg\{s_5, s_6\}$	$neg\{s_5, s_6\}$	-
B_3	-	$\{s_4, s_5\}$	$\{s_3, s_4,\}$	-	-	$neg\{s_5, s_6\}$	-
B_4	-	-	$\{s_4, s_5\}$	$\{s_4, s_5\}$	$\{s_2, s_3\}$	$\{s_5, s_6\}$	$\{s_4, s_5, s_6\}$
B_5	$\{s_4, s_5, s_6\}$	-	$neg\{s_5, s_6\}$	$neg\{s_5, s_6\}$	$\{s_4, s_5, s_6\}$	$\{s_4, s_5, s_6\}$	-

Table 2: Relational Matrix in terms of HLTS

Consider the set of linguistic expressions $S = \{s_0, s_1, \ldots, s_g\}$. Let $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$ be a

HFLTS, where $s_k \in S = \{s_0, s_1, \dots, s_g\}, k \in \{i, \dots, j\}$. Let $T = (a_L^i, a_M^i, a_L^{i+1}, a_R^i, a_M^{i+1}, \dots, a_L^j, a_R^{j-1}, a_M^j, a_R^j)$ be set of points of all membership functions of the linguistic terms in the HFLTS, that is the set of elements to be aggregated. According to the linguistic expressions considered in this study, $a_L^{i+1} = a_R^i = a_M^{i+1}$ for $i = 1,2, \dots, g-1$. The linguistic expressions are converted into fuzzy envelopes using Trapezoid Fuzzy Membership Functions (TFMFs) [5]. The parameters of the TFMFs $A = T\,(a, b, c, d)$ are computed as follows [13]: $a = \min\{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_L^i$ and $d = \max\{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_R^j$.The parameters b and c are computed using OWA operator [13].

If $i + j$ is even then $b = OWA_{W^2}\left(a_M^i, a_M^{i+1}, \dots, a_R^{\frac{i+j-1}{2}}\right)$ and $c = OWA_{W^1}\left(a_M^j, a_M^{j-1}, \dots, a_R^{\frac{i+j+1}{2}}\right)$ and if $i + j$ is odd then $b = OWA_{W^2}\left(a_M^i, a_M^{i+1}, \dots \dots, a_R^{\frac{i+j}{2}}\right)$ and $c = OWA_{W^1}\left(a_M^i, a_M^{i+1}, \dots \dots, a_R^{\frac{i+j}{2}}\right)$. The comparative linguistic expressions of the causal relations of FRM are transformed into TMFMas given below.

$\tilde{R}_{11} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{12} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{15} = T(0, 0, 0.15, 0.5)$
$\tilde{R}_{17} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{22} = T(0.17\,, 0.33, 0.5, 0.67)$
$\tilde{R}_{23} = T(0.33\,, 0.5, 0.5, 0.67)$
$\tilde{R}_{25} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{26} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{32} = T(0.5\,, 0.67, 0.83, 1)$
$\tilde{R}_{33} = T(0.33\,, 0.5, 0.67, 0.83)$
$\tilde{R}_{36} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{43} = T(0.5\,, 0.67, 0.83, 1)$
$\tilde{R}_{44} = T(0.5\,, 0.67, 0.83, 1)$
$\tilde{R}_{45} = T(0.17\,, 0.33, 0.5, 0.67)$
$\tilde{R}_{46} = T(0.67\,, 0.97, 1, 1)$
$\tilde{R}_{47} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{51} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{53} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{54} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{55} = T(0.5\,, 0.85, 1, 1)$
$\tilde{R}_{56} = T(0.5\,, 0.85, 1, 1)$

The trapezoidal fuzzy membership functions are converted into crisp values by weighted average defuzzification method within the interval [−1, 1]. The crisp values obtained denote the edge weights of the FRM. The adjacency matrix of the concepts is taken to be the relational matrix of the FRM and is given in the table 3. The obtained FRM model is studied for different input vectors and the equilibrium state values of the factors are obtained.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	0.8873	0.8873	0	0	-0.4192	0	0.8873
B_2	0	0.5010	0.5289	0	-0.9313	-0.9313	0
B_3	0	0.7959	0.6424	0	0	-0.9313	0
B_4	0	0	0.7959	0.7959	0.5010	0	0.8873
B_5	0.8873	0	-0.9313	-0.9313	0.8873	0.8873	0

Table 3: Relational Matrix of FRM

5. Results and Discussion

The input vectors are passed through the dynamical system of FRM and the resultant vector is thresholded using Sigmoid activation function and updated until the fixed point is obtained [1-2]. For all possible input vectors, the fixed points obtained are presented in table 4.The concepts B_1(Self-awareness and self-control), B_4(Personal Influence) and B_5(Mastery of purpose and Vision) behave in the same manner and turn ON maximum components in the corresponding resultant state vectors. These belief clusters turn ON all the attributes except continuous learning.The concepts B_2(Empathy) and B_3 (Social Expertness) influence the system in a similar manner and turn ON all the principles of leadership except two factors required for producing results. The concepts B_1(Self-awareness and self-control), B_3 (Social Expertness) and B_4(Personal Influence) are the most influential emotional intelligence related beliefs that endow an individual with leadership characteristics.

Sl. No	**Input Vector**	**Fixed point**
1	$\{1\ \ 0\ \ 0\ \ 0\ \ 0\}$	$\{(1\ \ 0\ \ 1\ \ 1\ \ 1), (1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1)\}$
2	$\{0\ \ 1\ \ 0\ \ 0\ \ 0\}$	$\{(1\ \ 1\ \ 1\ \ 1\ \ 0), (1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1)\}$
3	$\{0\ \ 0\ \ 1\ \ 0\ \ 0\}$	$\{(1\ \ 1\ \ 1\ \ 1\ \ 0), (1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1)\}$
4	$\{0\ \ 0\ \ 0\ \ 1\ \ 0\}$	$\{(1\ \ 0\ \ 1\ \ 1\ \ 1), (1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1)\}$
5	$\{0\ \ 0\ \ 0\ \ 0\ \ 1\}$	$\{(1\ \ 0\ \ 1\ \ 1\ \ 1), (1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1)\}$

Table 4: Fixed points of FRM

6. Conclusion

In this paper, the graphical-relational model of emotional intelligence and leadership based on hesitant fuzzy sets is constructed with the aid of FRM. The causal relationships between the belief domains of emotional intelligence and the principles of leaderships are identified and quantified using hesitant fuzzy sets. The hesitant linguistic term sets enable the experts to express their hesitancy and be flexible in their assessment. In FRM based on hesitant fuzzy sets, the loss of information is reduced as the expert's overall opinion is taken into account and quantified using comprehensive methods of aggregation and mathematical calculations.The study of causal relationship between the belief domains of emotional intelligence and principles of effective leadership using FRM based on hesitant fuzzy sets gives the realistic view of the problem without loss of much information. The causal influence between these two constructs can be further studied by quantifying the uncertainty using the advanced hesitant fuzzy sets.

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