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Study On Prime Graceful Labeling for Some Special Graphs

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Abstract

In this paper, weextend the satisfaction of Prime Graceful Labeling for the Graphs, Pan Graph, Helm Graph & Triangular Snake Graphand also generalize the cardinality of the edges for Triangular Snake Graphs.

Keywords: PrimeGraceful Labeling,Pan Graph, Helm Graph , Triangular Snake Graph and cardinality of the edges

1. INTRODUCTION

Throughout this paper we consider the graph which is simple, finite and not directed. In this paper, we discuss the ideaof Prime Graceful Labeling introduced by T.M.Selvarajan&R.Subramoniam [5] and we extend the Prime Graceful Labeling for some graphs such as Pan Graph, Helm Graph & Triangular Snake Graphs and also generalize the cardinality of the edges for Triangular Snake Graphs.

2. PRELIMINARIES

The map $\phi : V \to \{1,2,\ldots,k\}$ represents a one to one function from the vertices of G to {1,2,…,k}. Here k denotes the maximum number of vertices. Now the map $\phi^* : E \to \{1,2,\ldots,k-1\}$ represents the induced one to one function from the edges of G to {1,2,…,k-1}. Here k-1 denotes the maximum number of edges.

3.PRIME GRACEFUL LABELING FOR SOME SPECIAL GRAPH

Theorem 3.1

Pan Graph n, where n$\geq$3, is Prime Graceful Labeling.

Proof

Given Graph is Pan Graph n, where n$\geq$3.

Define a map $\varphi : V \to \{1,2,\ldots,k\}$ & $\phi^* : E \to \{1,2,\ldots,k-1\}$ Here K=2(n+1) & $|k| > |V|$. There must be a vertex say v with maximum degree in the Pan Graph n, & label that as1. And the remaining vertices , v_i (i=1 to n) of the Pan Graph are labeled with distinct number from {2,3,…,k} in such a way that labeling of every pair of adjacent vertices has the GCD 1. Let we assign the edge labels (1,2,…,k-1) in the following way, i.e) $\phi^*(v\, v_i) = |\phi\ (v\ ) - \phi\ (v_i)|$ $\phi^*(v_i v_j) = |\phi\ (v_i) - \phi\ (v_j)|$ where i $\neq$ j& i=1,2,…,k such that labeling of edges are distinct.

Hence by using the above condition, the Pan Graph n has Prime Graceful Labeling.

Thus the Pan Graph n , where n$\geq$3 is Prime Graceful Labeling.

Example 3.2

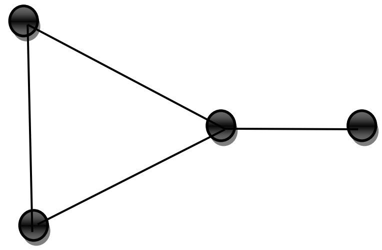
2

1

3 1 2 3

4

5



3-Pan Graph with Prime Graceful Labeling

Example 3.3

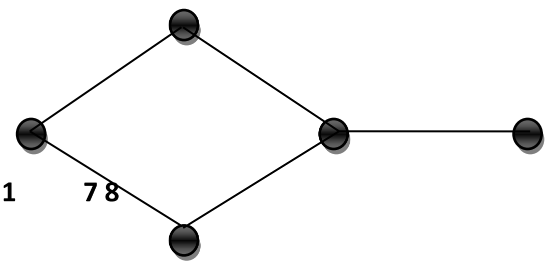
2

5 1

7

4 2 1 7 8

3



4-Pan Graph with Prime Graceful Labeling

Example 3.4

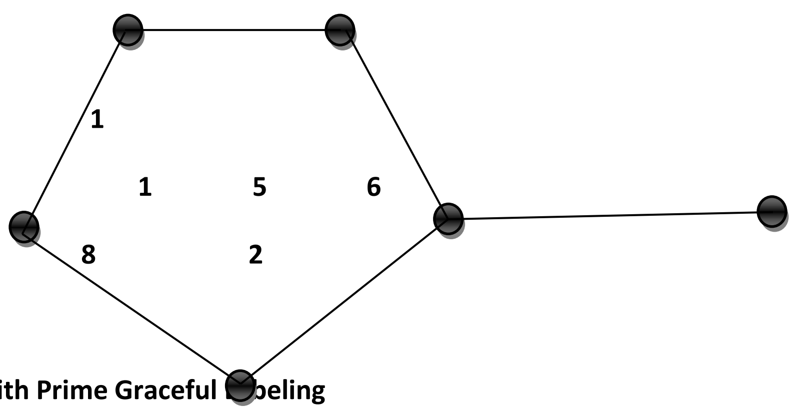
5 3 2

6 1

11 1 5 6

8 2

3



5 -Pan Graph with Prime Graceful Labeling

Alternative Proof for Pan Graph

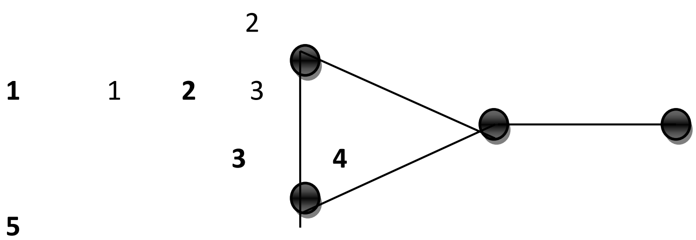
We shall prove that the Pan Graph n is Prime Graceful Labeling by using Mathematical Induction Method.

If n=3 letmaps φ & ϕ^* are defined as in preceding proof, where k=min{2(3+1), 2(3+1)}

k=2(3+1) $\Rightarrow$ k=8

Let us label the vertex with maximum degree as 1.Let v_i (i=1 to 3) be the other vertices of the Pan Graph are labeled with distinct number from 1,2,…,8 in such a way that labeling of every pair of adjacent vertices has the GCD 1 & also assign the edge labels (1,2,…,7) in the following way, i.e) $\phi^*\left(v_x - v_y\right)$= $\left|\phi^*\left(v_x\right) - \phi^*\left(v_y\right)\right|$ where x $\neq$ y & x,y=1,2,…,8 such that labeling of edges are distinct.

n=3



$\therefore$ The Pan Graph n=3 satisfies the above condition.
Hence the Pan Graph n=3 is Prime Graceful Labeling.
Assume that the Pan Graph n=h, where h is some integer is Prime Graceful Labeling.
Next we have to prove that the Pan Graph n=h+1 is Prime Graceful Labeling.
Define similar maps as beforehere k=min{2(h+1+1), 2(h+1+1)} $\Rightarrow$ k=2h+4
Let v be the vertex with maximum degree in the Pan Graph n & label as 1.
Let v_i (i=1 to h) be the other vertices of the Pan Graph are labeled with distinct number from 1,2,…, 2h+4 in such a way that labeling of every pair of adjacent vertices has theGCD 1& also assign the edge labels from 1,2,…, 2h+3 such that labeling of edges are distinct.
$\therefore$ The Pan Graph n=h+1 satisfies the theorem.

Theorem 3.5

Helm Graph H_n where n $\geq$ 3, is Prime Graceful Labeling

Proof

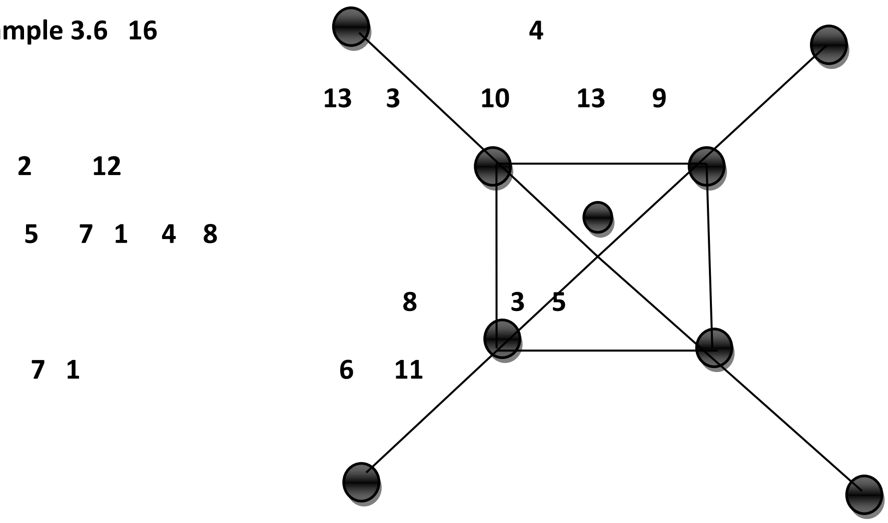
The Helm Graph $H_n$has 2n+1 vertices and n(2+1) edges.
Define a map as in the previous theorem here k=2(2n+1)
There must be a vertex say v with maximum degree in the Helm Graph H_n& label that vertex as 1. And the remaining vertices, v_i (i=1 to 2n) of the Helm Graph H_n are labeled with distinct number from {2,3,…,k} in such a way that labeling of every pair of adjacent vertices has the GCD 1& also assign the edge labels (1,2,…,k-1) in the following way,

i.e) $\phi^*(v\ v_i)=|\phi\ (v\ )-\phi\ (v_i)|$ & $\phi^*(v_iv_j)=|\phi\ (v_i)-\phi\ (v_j)|$ where $i\neq j$ & $i=1,2,\ldots,k-1$ such that labeling of edges are distinct.

Hence by using the above condition, the Helm Graph H_n has Prime Graceful Labeling.

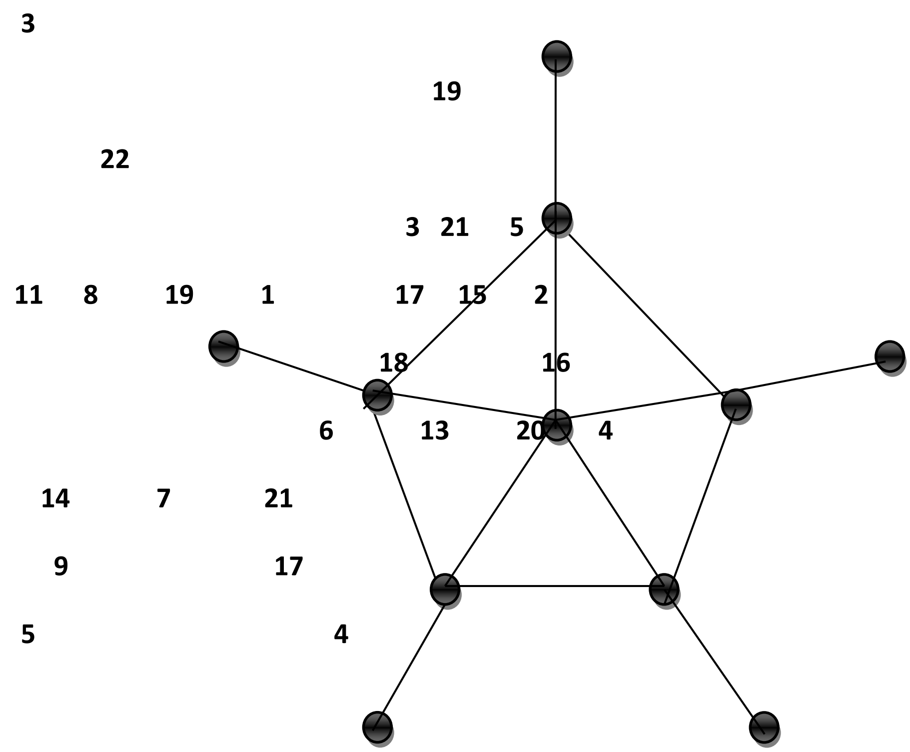
$\therefore$ Helm Graph H_n , where $n\geq 3$, is Prime Graceful Labeling.

Example 3.6



Helm Graph H_4 with Prime Graceful Labeling

Example 3.7



Helm Graph H_5 with Prime Graceful Labeling

Alternative Proof by Induction

We shall prove that the Helm Graph H_n is Prime Graceful Labeling by using Mathematical Induction Method.

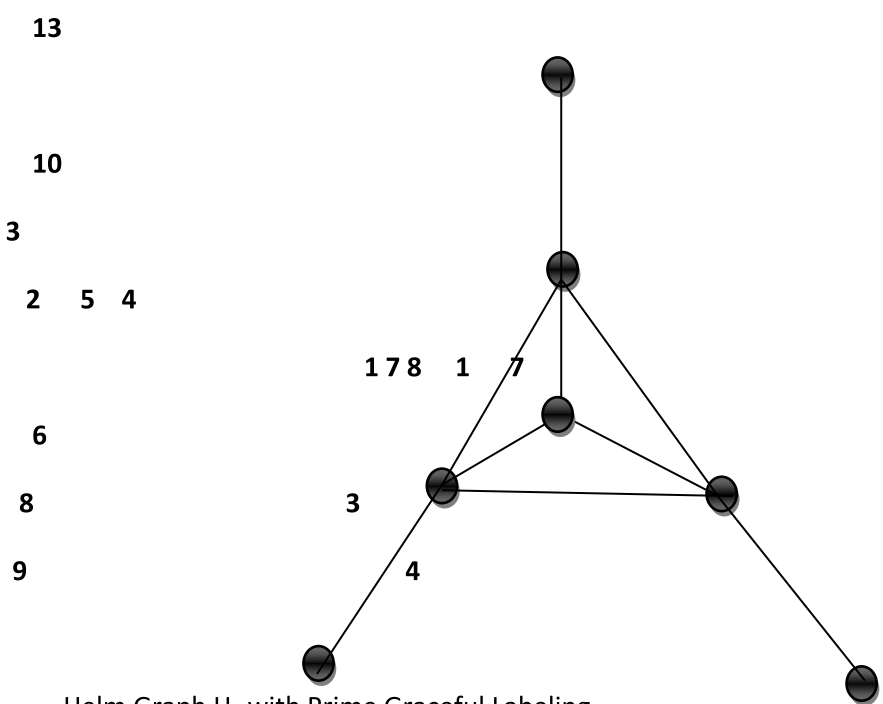
If n=3, label the vertices from 1,2,...k and edges from 1,2,3,.....k-1.

here k=min{2(2(3)+1), 2(3)(2+1)}, k=min{14,18} k=14

label the vertex with maximum degree in Helm Graph H_3as 1.

Let v_i (i=1 to 6) be the other vertices of the Helm Graph H_3 are labeled with distinct number from 1,2,…,14 in such a way that labeling of every pair of adjacent vertices has the GCD1& also assign the edge labels (1,2,…,13) in the following way,

i.e) $\phi^*(v\ v_i)=\left|\phi\ (v\ )-\phi\ (v_i)\right|$ & $\phi^*(v_iv_i)=\left|\phi\ (v_i)-\phi\ (v_j)\right|$ where $i\neq j$& i=1,2,…,13 such that labeling of edges are distinct.



Helm Graph H_3 with Prime Graceful Labeling

∴ The Helm Graph H_3 satisfies the above condition.

Hence the Helm Graph H_3 is Prime Graceful Labeling.

Assume that the Helm Graph H_n, where n=h, where h is some integer, is Prime Graceful Labeling.

Next we have to prove that the Helm Graph H_n, where n=h+1 is Prime Graceful Labeling.

If n=h+1

Let us consider the maps φ and ϕ^* defined as before

where k=min{2(2(h+1)+1), 2(h+1)(2+1)}

k=min{4h+6,6h+6}

k=4h+6

The vertex whose degree with maximum in Helm Graph H_{h+1}, is labelled as 1.

Let v_i (i=1 to h) be the other vertices of the Helm Graph H_{h+1} are labeled with distinct number from 1,2,…,4h+6 in such a way that labeling of every pair of adjacent vertices has the GCD1& also assign the edge labels (1,2,…,4h+5) in the following way,

i.e) $\phi^*(v\ v_i) = |\phi\ (v\ ) - \phi\ (v_i)|$ &

$\phi^*(v_i v_j) = |\phi\ (v_i) - \phi\ (v_j)|$ where i $\neq$ i& i=1,2,…,4h+5 such that labeling of edges are distinct.

$\therefore$ The Helm Graph H_n , where n=h+1 satisfies the above condition.

Hence the Helm Graph H_n , where n=h+1 is Prime Graceful Labeling

$\therefore$ Helm Graph H_n , where n$\geq$3, is Prime Graceful Labeling.

Lemma 3.8

The Cardinality of the edges of Triangular Snake Graph TS_n is

$$|E| = \begin{cases} i(2+m) & if \quad n = i + 2j \quad where \quad i = 3, j = 1,2,\ldots \& m = 0,1,2,\ldots \\ 3 & if \quad n = i \quad where \quad i = 3 \end{cases}$$

Proof

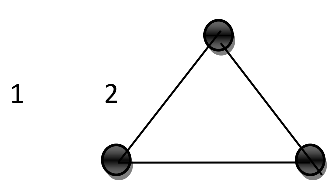
Let $|E|$ denote the number of edges in Triangular Snake Graph TS_n.

Edges in Triangular Snake Graph TS_ndepends on the number of vertices in that graph.

Case(i) If n=i, where i=3, then

letTS_i = i , where i=3 , $\therefore$ TS_3 = 3

$\therefore$ Cardinality of the edges of Triangular Snake Graph for TS_3 is

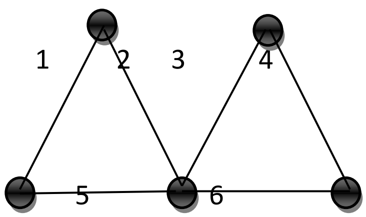


3

Case(ii) If n=i+2j, where i=3, & j=1,thenTS_{i+2j} = 2 i , where i=3 & j=1

TS_{3+2} = 2.3 ∴ TS_5 = 6

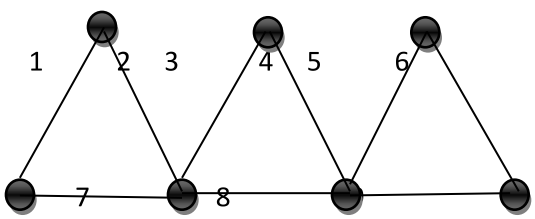
∴ Cardinality of the edges of Triangular Snake Graph for TS_5 is



Case(iii) If n=i+2j, where i=3 & j=2, then TS_{i+2j} = 3 i , where i=3 & j=2

TS_{3+4} = 3.3 , ∴ TS_7 = 9

∴ Cardinality of the edges of Triangular Snake Graph for TS_7 is



Assume the theorem satisfies forn=i+2j where i=3,j=1,2,3,4,5 & m=0,1,2,3,4

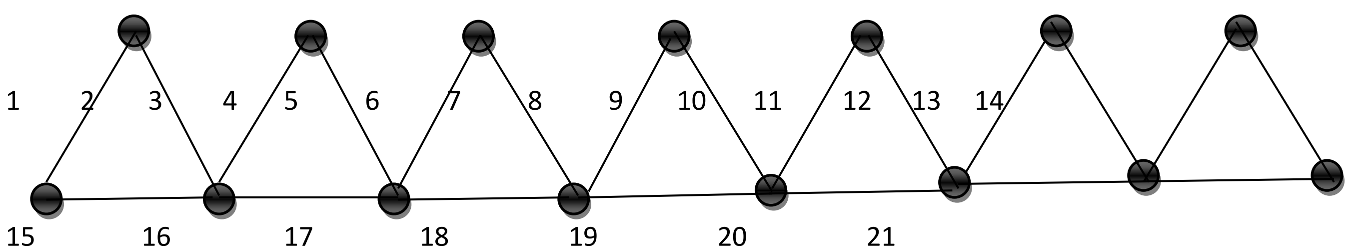
i.e) TS_{i+2j} = i(2+m), TS_{3+10} = 3(2+4) TS_{13} = 18

Now our claim is the result is true for i=3,j=6 & m=5 in n=i+2j

If n=i+2j, where i=3, j=6 & m=5 then

TS_{3+12} = 7.3, ∴ TS_{15} = 21

∴ Cardinality of the edges of Triangular Snake Graph for TS_{15} is



In general the Cardinality of the edges of Triangular Snake Graph TS_n is

$$|E| = \begin{cases} i(2+m) & if \quad n = i + 2j \quad where \quad i = 3, j = 1,2,\ldots \& m = 0,1,2,\ldots \\ 3 & if \quad n = i \quad where \quad i = 3 \end{cases}$$

Theorem 3.9

If $3 \leq$n<11 & n is odd then TS_n is prime graceful labeling.

Proof

Given graph is Triangular Snake Graph TS_n , where n is a odd number.

Claim: Triangular Snake Graph TS_n is prime graceful labeling for $3 \leq$n<11.

Triangular Snake Graph TS_n has n vertices. Edges in the Triangular Snake Graph depends on the number of vertices in that graph.

From Lemma 3.8, the cardinality of the edges of Triangular Snake Graph TS_n is

$$|E| = \begin{cases} i(2+m) & if \quad n = i+2j \quad where \quad i=3, j=1,2,3 \,\&\, m=0,1,2 \\ 3 & if \quad n=i \quad where \quad i=3 \end{cases}$$

Let us consider the maps φ and ϕ^* defined as before

where k=min{2n,2(i(2+m))} if n=i+2j here i=3, j=1,2,3 & m=0,1,2

or k=min{2n,2(3)}

k=2n

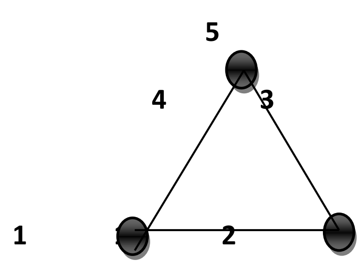
The vertices of the Triangular Snake Graph TS_n are labeled with distinct number from 1,2,…,k in such a way that labeling of every pair of adjacent vertices has the GCD 1& also assign the edge labels (1,2,…,k-1) in the following way,

i.e) $\phi^*(v_x - v_y) = |\phi(v_x) - \phi(v_y)|$ where $x \neq y$ & x,y=1,2,…,k such that labeling of edges are distinct.If the graph satisfies these condition, then graph is said to be prime graceful labeling.Check this condition for Triangular Snake Graph TS_n for $3 \leq$n<11, where n is odd in the following four cases.

Case (i): n=3

In this case, Triangular Snake Graph TS_3 satisfies the above conditions.

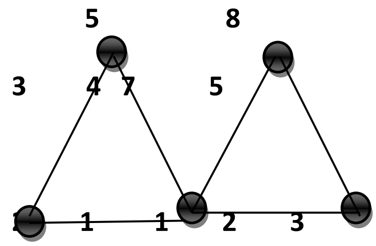
Prime Graceful Labeling for Triangular Snake Graph TS_3 is



Case (ii): n=5

In this case, Triangular Snake Graph TS_5 satisfies the above conditions.

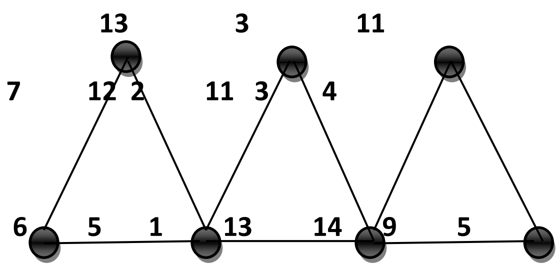
Prime Graceful Labeling for Triangular Snake Graph TS_5 is



Case (iii): n=7

In this case, Triangular Snake Graph TS_7 satisfies the above conditions.

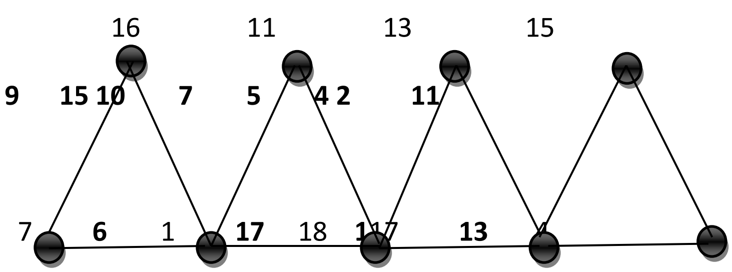
Prime Graceful Labeling for Triangular Snake Graph TS_7 is



Case (iv): n=9

In this case, Triangular Snake Graph TS_9 satisfies the above conditions.

Prime Graceful Labeling for Triangular Snake Graph TS_9 is



Hence the theorem.

Theorem 3.10

If the Triangular Snake Graph TS_n with $n \geq 11$ & k=4n where n is odd number then it is prime graceful labeling.

Proof

Assume that the Triangular Snake Graph TS_n with $n \geq 11$ & k=4n where n is odd number.

The proof byMathematical Induction Method.

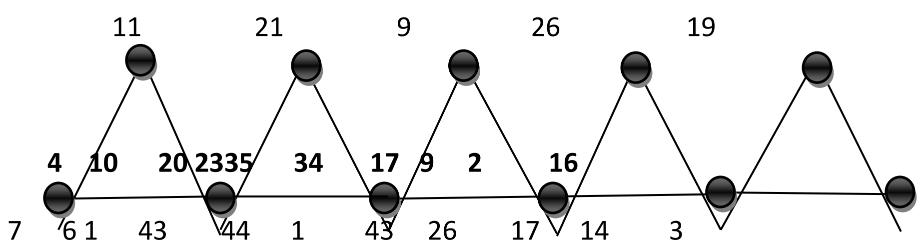
Case (i) n=11

Since k=4n, define a map φ and ϕ^* as before here k=min{4n,4(i(2+m))} if n=i+2j here i=3, j=4,5,… & m=3,4,… . $\therefore$ k=4n

The vertices of theconsidered graph TS_{11} are labeled with distinct number from 1,2,…,44 in such a way that labeling of every pair of adjacent vertices has the GCD 1& also assign the edge labels (1,2,…,43) in the following way,

i.e) $\phi^*\left(v_x - v_y\right) = \left|\phi\left(v_x\right) - \phi\left(v_y\right)\right|$ where $x \neq y$ & x,y=1,2,…,44 such that labeling of edges are distinct.

The Triangular Snake Graph of TS_{11} is



$\therefore$ The Triangular Snake Graph TS_{11} satisfies the above condition.

Hence the Triangular Snake Graph TS_{11} is Prime Graceful Labeling.

Assume the theorem satisfies for n=h , where h is some positive integer and prove this result for n=h+2

Since k=4n, Let us consider the maps φ and ϕ^* defined as before

here k=min{4h+8,4(i(2+m))} if n=i+2j here i=3, j=4,5,… & m=3,4,…. then k=4h+8

The vertices of the Triangular Snake Graph TS_{h+2} are labeled with distinct number from 1,2,…,4h+8 in such a way that labeling of every pair of adjacent vertices has the GCD 1& also assign the edge labels (1,2,…,4h+7) such that labeling of edges are distinct.

$\therefore$ The Triangular Snake Graph TS_{h+2}satisfies the above condition.

Hence the Triangular Snake Graph $TS_{h+2}$is Prime Graceful Labeling.

Thus the Triangular Snake Graph TS_n with $n \geq 11$ & k=4n where n is odd number is prime graceful labeling.

4. Conclusion

The existence of prime graceful labeling for some special graphs such as Pan Graph, Helm Graph & Triangular snake Graph are provedand generalized the cardinality ofedges of Triangular snake Graph.It can also be extend the Prime graceful labeling further moregraphs.

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