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Approximate Analytical Solution Of Relapsing Remitting Multiple Sclerosis Using Homotopy Perturbation Method

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Abstract

A mathematical model for therelapsing-remitting multiple sclerosisis presented.The concentrations of robust nerve cells, affected brain cells and toxic effectors procured an approximate analytical expression by using "Homotopy Perturbation Method (HPM)". The major intention is to formulate an analytical solution for the Relapsing-Remitting Multiple Sclerosis (RRMS) model. Besides, this work illustrates the Scilab/Matlab program for the numerical stimulation of the non-linear differential problem. The Comparisons between analytical solution and numericalsolution gave a good result of satisfaction. The achieved outcomes are also plausibleto the entire solution.

Keywords: Mathematical Modeling, Nonlinear initial value problem,Homotopy Perturbation Method, Multiple Sclerosis.

Introduction

An inflammatory disease called Multiple sclerosis is situated in the human brain and it is the wide spread cause for the non-traumatic neurological disability in the adolescent. The characterization of this neurotic disorder occurs in the form of episodes over a period of days to weeks. These episodes gave rise to the complete or incomplete remissions occur during numerous occasions. Pre-mature indications of Multiple sclerosis constituteacute optic neuritis, dizziness, double vision, fatigue, gait, limb ataxia, numbness, spasticity and weakness in one or more limbs.The type and harshness of indications vary from one subject to another. The treatment is not necessary for the subject with mild symptoms and for the subject severe symptoms proposes relapse preventive therapies [1]. The majority of Multiple sclerosis identified globally is 100–150 per 100,000 population [2]. The impact can be seen widely among women rather than men. Although it can be recognizedgenerally between the ages of 20 and 40years, but this disorder can also be acknowledged in the early childhood and in later life [3]. Clinically the Multiple sclerosiscomprise various subtypes such as "Relapsing-Remitting Multiple Sclerosis (RRMS), Secondary Progressive Multiple Sclerosis (SPMS), and Primary Progressive Multiple Sclerosis (PPMS)" [4].The subtype called "Relapsing-Remitting Multiple Sclerosis

(RRMS)"identified among 85% of the subject and this distinguished through relapses in the form of episodes over days or weeks, which would worsen the neurotic functions of the brain cells.The recovery periods may varyaccording to the remission, which can be partial or complete recovery from months to years. Apparently there is no further progression of this disorder once the subject is completely recovered[5].In the contemporary years there is a renaissance in the medication of Multiple Sclerosis to lessen the activities of the disorder with the invention of new disease-modifying treatments[6].Inspiteof the accomplishments over the past decades with interest to formulate disease-modifying treatment for relapsing remitting multiple sclerosis, there are numerous of major challenges that still remain to be conquer this neurotic disorder called Multiple Sclerosis [7].An analytical expression is devised for the ratio of x, the robust nerve cells, y, the affected nerve cells and v, the toxic effectors, against time tby employing the method of Homotopy Perturbation

The formulation of the Replasing-Remitting Multiple Sclerosis

In 2020,elettreby et al. [8] evolved the model for Relapsing Remitting Multiple Sclerosismathematically. This detailed non-linear model is independent of time. In this paper we attempt to pioneer in finding the analytical expression for the ratio of x, the robust nerve cells, y, the affected nerve cells and v, toxic effectors against time tfor different values of parameters.

According to elettreby et al.this model of Relapsing-Remitting Multiple Sclerosis can be denoted as

$$\frac{dx}{dt} = r(1-x) + bxv \tag{1}$$

$$\frac{dy}{dt} = bxv - ay \tag{2}$$

$$\frac{dv}{dt} = cy - dxv - ky \tag{3}$$

where $x(t)$ denotes the ratio of the robust nerve cells at time t, y(t)denote the affected nerve cells, and v(t) denote the toxic effectors(either viral or immune).Here, r denotes the growth rate of robust nerve cells $x(t)$. The scientist has perceived that the neurotic disorder is affected through a virus or immune cell.Hence, we presume that the affected nerve cells y(t) secrete the toxic effectors (virus) v(t)which invade the robust nerve cells $x(t)$ at a rate b. The secretion of virus v(t)by the affected nerve cells y(t)is of rate c and their death is of rate a. The robust nerve cells $x(t)$ raids the virus v(t) at a rate d and perish at the rate k.The parameters a, b, c, d, k, r are positive constants.

The initial conditions are

as $t = 0, x = x_i; y = y_i; v = v_i$ (4)

where $x(t)$ representsrobust nerve cells, y(t)represents affected nerve cells, v(t)represents toxic effectors.

Analytical solutions of the concentration of robust nerve cells, affected brain cells and toxic effectors.

In the discipline of applied Mathematics Non-linear equations exhibits a pivotal function. The basic implication of the non-linear equations is to obtain an explicit solution.This can also achieved through several asymptotic methods. Illustration of Homotopy perturbation method vitally present in diverse fields especially in solving non-linear problems in and engineering and physics [9-11].J.H. He, adopts "HomotopyPerturbation Method" and solved the titular equations like the"Lighthill equation" [12], "the Diffusing equation" [13] and "the Blasius equation" [14] etc. As p(imbedding parameter) act as a small parameter in Homotopy Perturbation Method we need only very few iterations to have an accurate solution.Appendix A illustrates the solving procedure for the non-linear differential equations, eqn. (1) -eqn. (3), by using the method of Homoyopy Perturbation in which we acquire the ratio of robust nerve cells, affected brain cells and toxic effectors. The obtained results are as follows;

$$x(t) = \frac{bx_i v_i}{k} e^{-kt} - x_i^2 e^{2rt} + \frac{bx_i v_i}{k} e^{rt} + x_i^2 e^{rt} + x_i e^{rt} \quad (5)$$

$$y(t) = \frac{bx_i v_i}{r+a-k} e^{(r-k)t} - \frac{bx_i v_i}{r+a-k} e^{-at} + y_i e^{at} \quad (6)$$

$$v(t) = \frac{cy_i}{k-a} e^{-at} \frac{dy_i x_i}{r+a-k} e^{(r-a)t} + \left(\frac{dy_i x_i}{r+a-k} - \frac{cy_i}{k-a} \right) e^{-kt} + v_i e^{-kt} \quad (7)$$

eqn. (5)-eqn. (7) constitutethe analytical solutionwhich also satisfies the initial conditionfor the ratio of x, the robust nerve cells, y, the affected nerve cells and v, toxic effectors against time t

Numerical solutions of the concentration of robust nerve cells, affected brain cells and toxic effectors.

The non-linear differential equations eqn. (1)-eqn.(3) for the given initial conditions are solved by using homotopy perturbation method. The above said equations are solved numerically by using MATLAB software -The function pdex 4 which is a function of solving boundary value problems. Our analytical results obtained in this work are compared with the numerical results in Figures 1- 7 and found to be satisfactory for the experimental values of the parameter $r = 0.03, b = 0.025, a = 1.03, c = 0.1, d = 0.01, k = 1.2$ and time t.

Results and Discussion

The nature of spread of any disease can be determined by using the epidemiologic parameter reproduction number R, which is the number of secondary infected cases produced by an infected case in a susceptible population domain over the course of infection. If R< 1, then the average disease spreads is less than one individual from an infected individual which indicates the reduction in the growth of the infection. If R>1, the average production of infection from an infected individual increasesand leads to the birth of new cases which indicates the raise in the growth of the infection [18].

In Figs.1-7, we have plotted theratios of robust nerve cells, affected brain cells, toxic effectorsagainst timetfor some diverse fixed parameters values. The analytical expression thus obtained are then compared with that of the numerical and experimental values [8]. These values give us a satisfied agreement.

Fig 1 illustrates the plot of the robust nerve cells, affected brain cells and toxic effectors against time t for various experimental parameter estimates obtained from [8] to study their qualitative behaviour.

Fig-2 and Fig-3 illustrates the plot of robust nerve cells against time t for diverse fixed parameters values. FromFig-2 and Fig-3, it is observed that the ratio of robust nerve cells increases when time increases. This is the result of the reproduction number, R <1. The amount of robust nerve cells slowly increases and attains the steady state value (1,0,0) as portrayed in [8].

Fig.-4 and Fig.-5 illustrates the plot of affected brain cells $y(t)$ against time t for diverse fixed parameters values which is fixed.From Fig-4 and Fig-5, it has been noted that the ratio of affected brain cells decreases when time increases. As a result, the graph tends to zero which indicates the extinction of affected brain cells over the course of infection.

Fig.-6 and Fig.-7 illustrates theplot oftoxic effectors $v(t)$ against time t for diverse fixed parameters values .From Fig.-6 and Fig.-7, it has beeninferred that the ratio of toxic effectors decreases with increasing time and gradually tends to zero. This is due to the suppression of toxic effectors which is the result of less than one for the basic reproduction number.

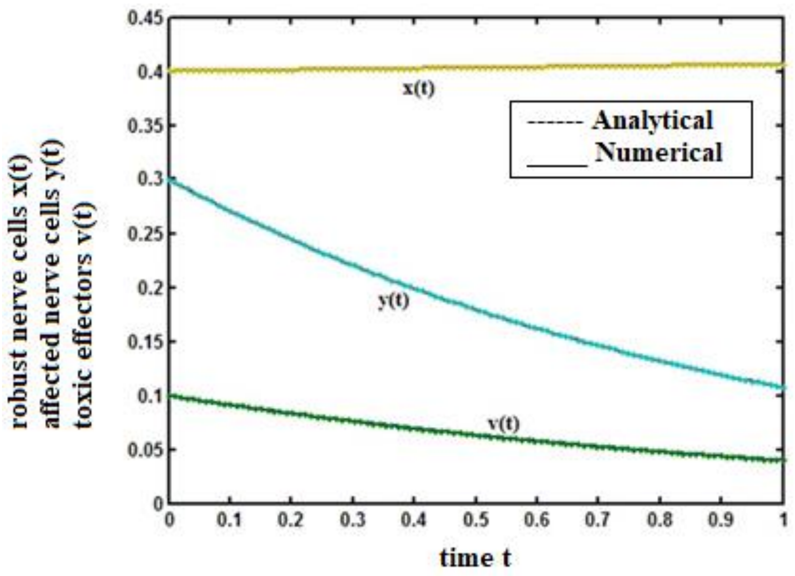

Figure 1: Contrast of robust nerve cells , affected brain cells and toxic effectors against time calculated from eqn. (1) - eqn.(3) for experimental parameter values $r = 0.03, b = 0.025, a = 1.03, c = 0.1, d = 0.01, k = 1.2$ and initial conditions $(0.4,0.3,0.1)$.

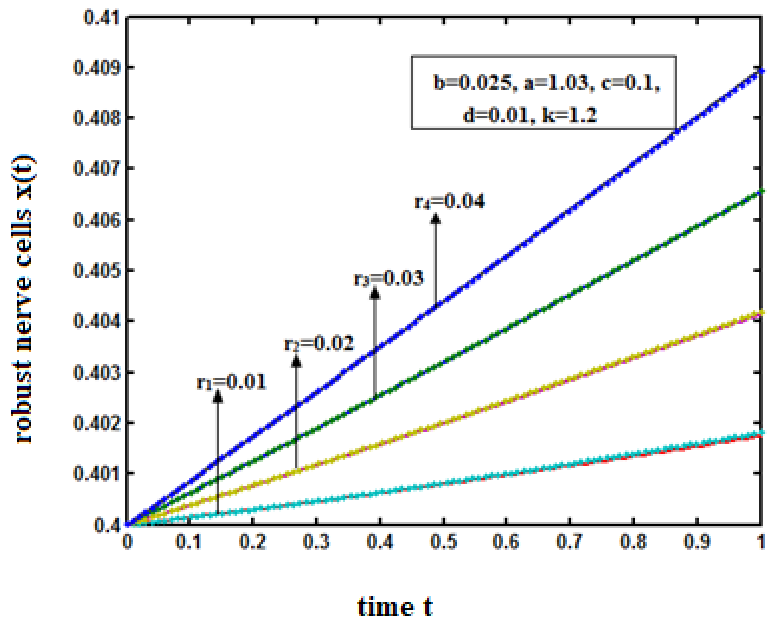

Figure 2: Analytical(dotted line) and numerical (solid line) solutionsfor robust nerve cells against time computed using eqn. (1) forthe experimental parameter values used in $b = 0.025,\ a = 1.03,\ c = 0.1,\ d = 0.01,\ k = 1.2$ with varying values of r.

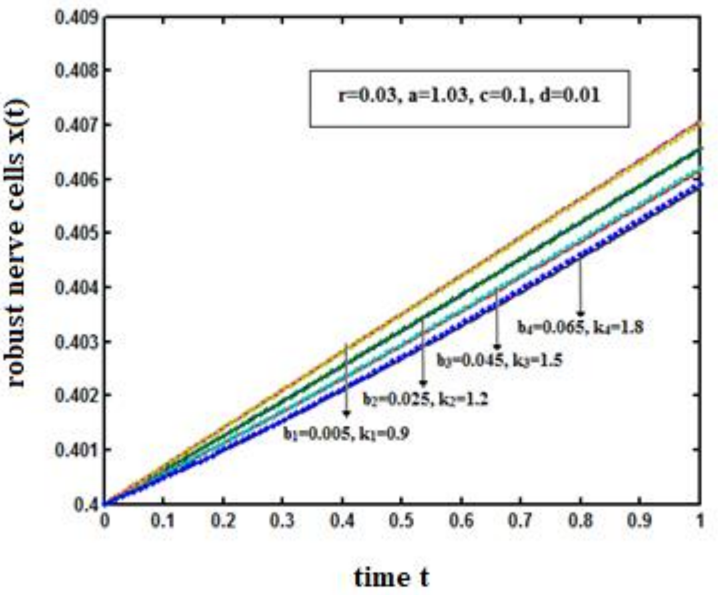

Figure 3: Analytical (dotted line) and numerical (solid line) solutionsfor robust nerve cells against time computed using eqn. (1) for the experimental parameter values $r = 0.03, a = 1.03, c = 0.1, d = 0.01$ with varying values of b and k.

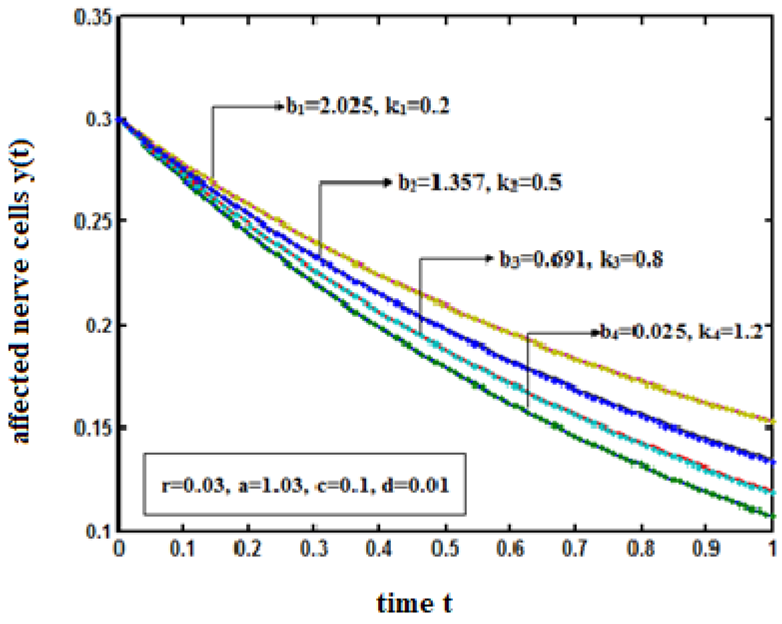

Figure 4: Analytical (dotted line) and numerical (solid line) solutionsfor affected brain cells against time computed using eqn. (2) for the experimental parameter values $r = 0.03, a = 1.03, c = 0.1, d = 0.01$ with varying values of b and k.

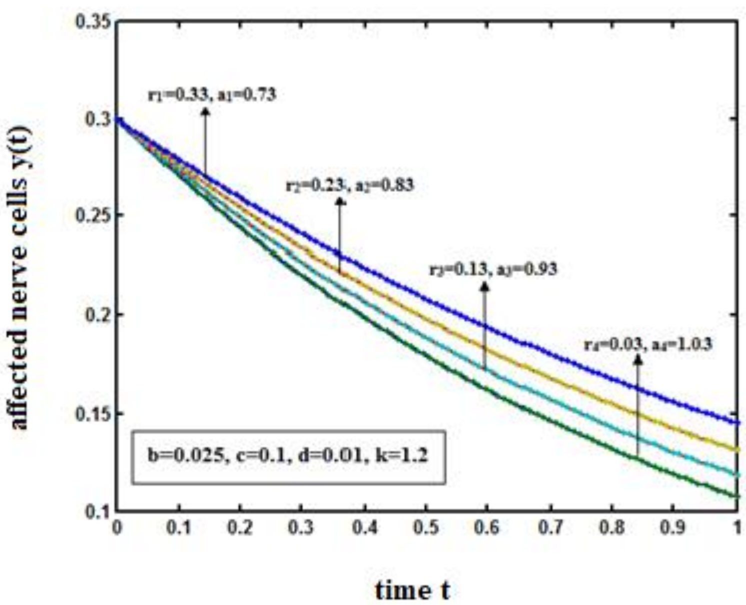

Figure 5: Analytical (dotted line) and numerical (solid line) solutionsfor affected brain cells against time computed using eqn. (1) forthe experimental parameter values $b = 0.025, c = 0.1, d = 0.01, k = 1.2$ with varying values of r and a.

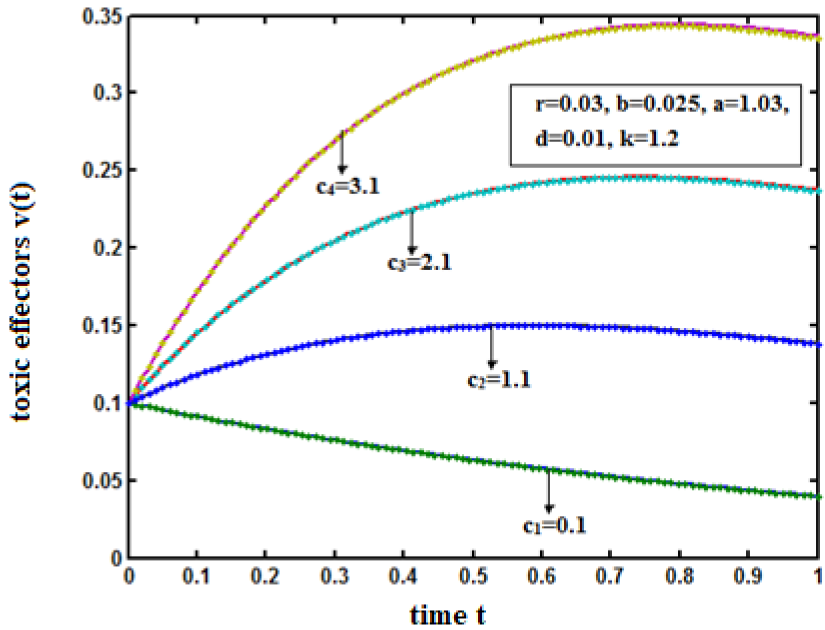

Figure 6: Analytical (dotted line) and numerical (solid line) solutions) for toxic effectors against time computed using eqn. (3) for the experimental parameter values $r=0.03, b=0.025, a=1.03, d=0.01, k=1.2$ with varying values of parameter c.

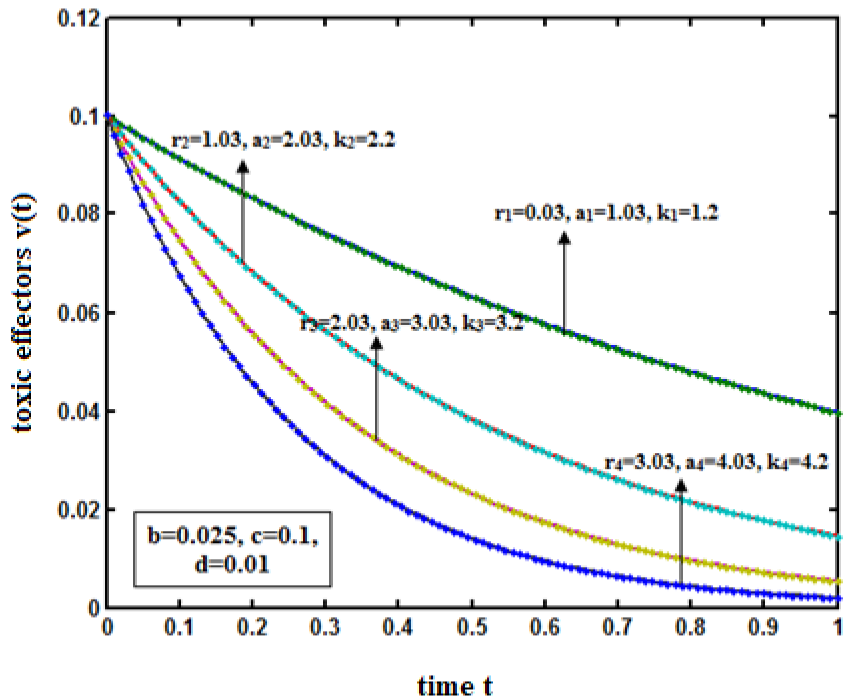

Figure 7: Analytical (dotted line) and numerical (solid line) solutionsfortoxic effectors against time computed using eqn. (3) for the experimental parameter values $b = 0.025, c = 0.1, d = 0.01$ with varying values of parameter r,a and k.

Conclusion

This theoretical model describes a neurotic disorder called Relapsing-Remitting Multiple Sclerosis, which has beenanalyzed in this paper. Analytically the nonlinear time-dependent differential equations is solved by using the method of "Homotopy Perturbation"and an analytical expression pertaining to the robust nerve cells, affected brain cellsand toxic effectorsfor distinct values of the parameter are obtained.The attained results are then analogized with the numerical results and and are found to be in satisfied agreement.

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Symbol	Meaning
x(t)	Robust nerve cells at time t
y(t)	Affected nerve cells at time t
v(t)	Toxic effectors(virus or immune)
T	Time
a, b, c, k, d	Positive constants
r	Growth rate of the robust nerve cells
R	Reproduction Number

Nomenclature

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Appendix A: The construction of Homotopy for finding the solution of equation (1)-(3)

The construction of Homotopy for finding the solution of equation (1)-(3) are

$$(1-p)\left[\frac{dx}{dt}-rx\right]+p\left[\frac{dx}{dt}-rx+rx^{2}+bxv\right]=0 \tag{A.1}$$

$$(1-p)\left[\frac{dy}{dt}+ay\right]+p\left[\frac{dy}{dt}+ay-bxv\right]=0 \tag{A.2}$$

$$(1-p)\left[\frac{dv}{dt}+kv\right]+p\left[\frac{dv}{dt}+kv+dxv-cy\right]=0 \tag{A.3}$$

The approximate solutions of the above three equations are

$$x=x_0+px_1+p^2x_2+\ldots \tag{A.4}$$

$$y=y_0+py_1+p^2y_2+\ldots \tag{A.5}$$

$$v=v_0+pv_1+p^2v_2+\ldots \tag{A.6}$$

Substituting the Eqns. (A.4-A.6) respectively into Eqns. (A.1-A.3) And on obtaining the identical powers of p, we get

$$p^0 : \frac{dx_0}{dt} - rx_0 = 0 \tag{A.7}$$

$$p^0 : \frac{dy_0}{dt} + ay_0 = 0 \tag{A.8}$$

$$p^0 : \frac{dv_0}{dt} + kv_0 = 0 \tag{A.9}$$

$$p^1 : \frac{dx_1}{dt} - rx_1 + rx_0{}^2 + bx_0v_0 = 0 \tag{A.10}$$

$$p^1 : \frac{dy_1}{dt} + ay_1 - bx_0v_0 = 0 \tag{A.11}$$

$$p^1 : \frac{dv_1}{dt} + kv_1 - cy_0 + dx_0v_0 = 0 \tag{A.12}$$