

Model And Calculation Algorithm For The Development Of Geotechnological Processes In The Conditions Of A Layered System

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Annotation: In this paper, a mathematical model and an optimal computational algorithm are recommended for numerical solution of the underground mixing process in deposits developed under the conditions of a stratified system.

Keywords: model, algorithm, layered system, mixing, mathematical model, optimal calculation.

Almost all geotechnological methods of mining as objects of research lend themselves to study with great difficulties and assumptions. The reason for this situation is that the parameters of development objects (fields) are practically inaccessible to observation and measurement. In addition, the geological and hydrogeological properties of the deposits are characterized by great variability [1].

The impossibility of direct control of the impact of the in situ leaching (IS) process on the environment determines the need for a number of specific studies to minimize the negative impact of IS. In this regard, the numerical modeling method can act as a powerful research tool capable of solving a number of critical problems for water supply and making a more accurate decision in the design and forecasting of the development of minerals.

In this paper, we propose a mathematical model and an optimal computational algorithm for the numerical solution of the PV process under the conditions of a storey system development. Such processes are considered on a cut field x-z, and their mathematical models differ markedly from the mathematical models of areal fields x-y, since the number of parameters that affect the process is expanding. For example, the specific gravity of the liquid must be taken into account.

It is known that the value of fluid pressure at the top of porous media is less than at the bottom. The reason for this is due to the ratio of the specific gravity of the liquid. Based on this point of view, the calculation of the processes of filtration fluid flows in the x-z section becomes much more complicated.

Let a heterogeneous layer be given, which has ore deposits developed in the conditions of storey systems (Fig. 1). The reservoir contains pumping and injection wells with certain flow rates. It is required to determine the changes in the values of the concentration of the useful component over time, taking into account the specific gravity of the injection fluid.

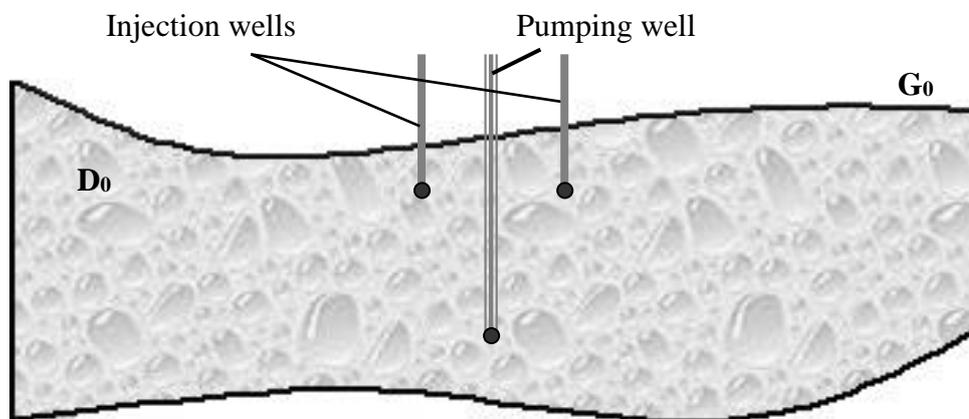


Fig. 1. Model of the ore deposit in the section x-z

Wells under such conditions are installed at different capacities, i.e. the pumping well is opened at a depth of 10-20 m below the pumping well.

The mathematical model of this problem is expressed by the following partial differential equations in general form.

1. The distributions of the pressure field H are determined using the equation of the elastic filtration mode:

$$m\beta \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k}{\mu} \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{k}{\mu} \left(\frac{\partial H}{\partial z} - \gamma \right) \right) + \frac{\mu}{k} \sum_{i=1}^N \delta(x - x_i, z - z_i) q_i(t) \quad (1)$$

$(x, z) \in D_0$

with initial and boundary conditions

$$H(x, z, 0) = H_0(x, z), \quad \left(\alpha \frac{\partial H}{\partial n} + (1 - \alpha)H \right) \Big|_{G_0} = \varphi(x, z, t). \quad (2)$$

Let us introduce the notation:

$$f(x, z, t) = \frac{\mu}{k} \sum_{i=1}^N \delta(x - x_i, z - z_i) q_i(t),$$

$$K = \frac{k}{\mu}, \quad M = \beta \cdot m.$$

Then equation (1) takes the form

$$M \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial z} \left(K \left(\frac{\partial H}{\partial z} - \gamma \right) \right) + f(x, z, t) \quad (x, z) \in D_0, \quad t > t_0.$$

2. The filtration rate is determined by Darcy's law:

$$V_x = -K \frac{\partial H}{\partial x}, \quad V_z = -K \frac{\partial H}{\partial z} \quad (3)$$

3. To determine the concentration value, the equations of the following form are solved:

$$\frac{\partial}{\partial x} \left(D_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial z} \left(D_z \frac{\partial C}{\partial z} \right) - \frac{\partial(V_x C)}{\partial x} - \frac{\partial(V_z C)}{\partial z} - \frac{\partial N}{\partial t} = m \frac{\partial C}{\partial t} \quad (4)$$

$$(x, z) \in D_0, \quad t > 0$$

with initial $C(x, z, 0) = 0$, borderline $C(x, z, t)|_{G_0} = 0$ and internal well conditions

$$C(x, z, t)|_{\Gamma_i} = C_i, \quad \frac{\partial C}{\partial n} \Big|_{\Gamma_2} = 0. \quad (5)$$

The equation of kinetics of mass transfer, which determines the rate of transition of a substance from one phase to another, generally has the form

$$\frac{\partial N}{\partial t} = \gamma_2(C) F(C, N, t), \quad N|_{t=0} = N_0. \quad (6)$$

Specific kind of function for each object using the PV process, it is determined separately:

1) in the case of dissolution and precipitation of solutes

$$F(C, N, t) = -(C_m - C) N^\alpha;$$

2) nonequilibrium reversible adsorption and desorption of dissolved and emulsified substances in the Henry isotherm -

$$F(C, N, t) = (C_{n_0} + \alpha N);$$

3) parallel process of dissolution and desorption -

$$F(C, N, t) = \xi(C_m - c) + \xi(C_{n_0} + \alpha N).$$

The boundary of the filtration region G_0 is arbitrary; therefore, for the numerical solution of problem (1) - (5), we construct a numerical algorithm based on the method of fictitious regions.

Difficulties in compiling an algorithm for the numerical solution of (1) - (5) by difference methods are largely related to the geometry of the reservoir. Therefore, it is advisable to develop

algorithms not for specific areas, but for areas of arbitrary type. One of the possible ways to solve this problem is to replace an arbitrary region with a rectangular one. This method is called the dummy region method.

Applying the idea of the method of fictitious areas [2], the given area D_0 replace with a standard rectangular area $D = D_0 + D_1$ with a border (Fig. 2).

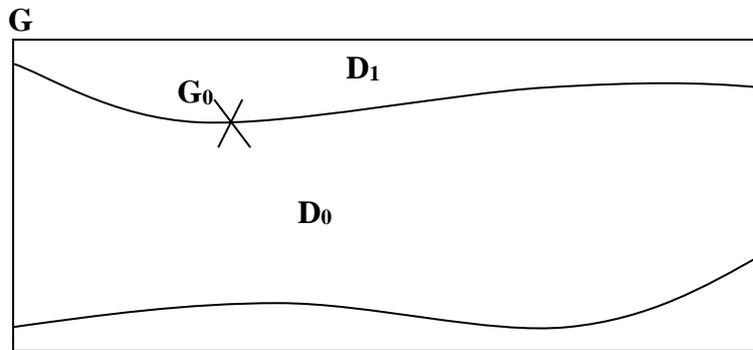


Рис.2.

We assume that problem (1) - (5) is also valid in $D \times (0, T]$:

$$K = \begin{cases} K, (x, z) \in D_0, \\ \varepsilon K, (x, z) \in D_1, \end{cases} \quad M = \begin{cases} M, (x, z) \in D_0, \\ \varepsilon M, (x, z) \in D_1, \end{cases} \quad f = \begin{cases} f, (x, z) \in D_0, \\ 0, (x, z) \in D_1, \end{cases}$$

where D_0 - specified area with border G_0 , D_1 - dummy region, ε - small order number 10^{-3} .

Equation coefficients on the boundary G_0 also have gaps. To avoid the introduction of additional stitching conditions on G_0 for the two-dimensional problem (1) - (5), we construct a conservative difference scheme. In what follows, we will not distinguish $H_\varepsilon(x, z, t)$ and $H(x, z, t)$, assuming that the original area D is a unit rectangle in dimensionless form, i.e. $D = \{(x, z) : 0 \leq x \leq 1; 0 \leq z \leq 1\}$.

When solving problem (1), we first use the finite-difference method, namely, an implicit alternating direction scheme with iteration of nonlinear terms. In D we construct a grid uniform in x and z

$$\bar{w}_{h_1 h_2} = \{(x_i, z_j) : x_i = ih_1, h_1 = 1/N_1, i = \overline{0, N_1}; z_j = jh_2, h_2 = 1/N_2, j = \overline{0, N_2}\}. \quad (7)$$

We approximate the two-dimensional problem (1) as follows, taking into account the use of the stream version of the run. The corresponding differences of the equation have the form

$$\frac{W_{i+1/2, j}^{n+1/2} - W_{i-1/2, j}^{n+1/2}}{\Delta x} + \frac{W_{i, j+1/2}^n - W_{i, j-1/2}^n}{\Delta z} = M(H_{i, j}^{n+1/2}) \cdot \frac{H_{i, j}^{n+1/2} - H_{i, j}^n}{0.5 \cdot \tau} + f_{i, j}^{n+1/2}, \quad (8)$$

$$\frac{W_{i+1/2,j}^{n+1/2} - W_{i-1/2,j}^{n+1/2}}{\Delta x} + \frac{W_{i,j+1/2}^{n+1} - W_{i,j-1/2}^{n+1}}{\Delta z} = M(H_{i,j}^{n+1}) \cdot \frac{H_{i,j}^{n+1} - H_{i,j}^{n+1/2}}{0.5 \cdot \tau} + f_{i,j}^{n+1} \quad (9)$$

$$H_{i,j}^0 = \varphi_{i,j}, \quad lH|_G = 0 \quad \lambda K_{i,j} lH|_G + \theta H_{i,j}|_G = \gamma \quad (10)$$

Here $\Delta x, \Delta z$ and τ - coordinate steps OX, OZ and time.

After determining the pressure field H, the filtration rate is determined from relation (3). At the last stage, to determine the concentration value, equation (4) is solved with the corresponding conditions (5).

Equation (4) can be solved in various ways. Consider the monotonic scheme of A.A. Samara, which gives a more effective result.

In order to obtain a monotonic second-order scheme, it is sufficient to write it as one-sided difference derivatives for an equation with perturbed coefficients [3]:

$$m \frac{\partial C}{\partial t} = \tilde{L}C + f, \quad (11)$$

where

$$\begin{aligned} \tilde{L} &= \tilde{L}_1 + \tilde{L}_2, \\ \tilde{L}_1 &= \chi_1 \frac{\partial}{\partial x} \left(k_1 \frac{\partial C}{\partial x} \right) + r_1 \frac{\partial C}{\partial x} - q_1 C, \\ \tilde{L}_2 &= \chi_2 \frac{\partial}{\partial z} \left(k_2 \frac{\partial C}{\partial z} \right) + r_2 \frac{\partial C}{\partial z} - q_2 C. \end{aligned}$$

here,

$$\begin{aligned} \chi_i &= (1 + R_i)^{-1}, \quad R_i = 0,5h_i|r_i|/k_i, \\ k_i &= D_i, \quad i = 1,2, \\ r_1 &= -v_x, \quad r_2 = -v_z, \\ q_1 &= \frac{\gamma}{2} + \frac{\partial v_x}{\partial x}, \quad q_2 = \frac{\gamma}{2} + \frac{\partial v_z}{\partial z}. \end{aligned}$$

r_i is presented as a sum:

$$r_i = r_i^+ + r_i^-, \quad r_i^+ = 0,5 \cdot (r_i + |r_i|) \geq 0, \quad r_i^- = 0,5 \cdot (r_i - |r_i|) \leq 0.$$

Then, using this, we approximate the expression

$$(r_1 C_x) = \left(\frac{r_1}{k_1} (k_1 C_x) \right)_i \approx b_{1,i}^+ a_{i+1} C_{x,i} + b_{1,i}^- a_i C_{\bar{x},i},$$

where

$$b_i^\pm = \frac{r_i^\pm}{k_1}, \quad a_i = k_i, \quad i = 1, 2.$$

Based on this, we write the difference analogue of the operators \tilde{L}_1 and \tilde{L}_2 :

$$\tilde{L}_1 C \approx \Lambda_1 C = \chi_1 (a_i C_{\bar{x},i})_{x,i} + b_{1,i}^+ a_{i+1} C_{x,i} + b_{1,i}^- a_i C_{\bar{x},i} - q_1 C, \tag{12}$$

$$\tilde{L}_2 C \approx \Lambda_2 C = \chi_2 (a_j C_{\bar{z},j})_{z,j} + b_{2,j}^+ a_{j+1} C_{z,j} + b_{2,j}^- a_j C_{\bar{z},j} - q_2 C. \tag{13}$$

Replacing C_t by right-hand difference approximation, we will make the transition from layer n to layer $n + 1$ in two stages based on the application of the alternating direction method:

$$m \frac{C^{n+\frac{1}{2}} - C^n}{0,5\tau} = \tilde{\Lambda}_1 C^{n+\frac{1}{2}} + \tilde{\Lambda}_2 C^n + f^n,$$

$$m \frac{C^{n+1} - C^{n+\frac{1}{2}}}{0,5\tau} = \tilde{\Lambda}_2 C^{n+1} + \tilde{\Lambda}_1 C^{n+\frac{1}{2}} + f^n.$$

By supplying instead $\tilde{\Lambda}_1$ (12) and instead of $\tilde{\Lambda}_2$ (13), making some transformations we get the following systems of algebraic equations:

$$A_i^1 C_{i-1,j}^{n+\frac{1}{2}} - \bar{C}_i^1 C_{i,j}^{n+\frac{1}{2}} + B_i^1 C_{i+1,j}^{n+\frac{1}{2}} = -F_{i,j}^1 \tag{14}$$

$$A_i^2 C_{i,j-1}^{n+1} - \bar{C}_i^2 C_{i,j}^{n+1} + B_i^2 C_{i,j+1}^{n+1} = -F_{i,j}^2 \tag{15}$$

here

$$A_i^1 = \frac{a_i}{h_1^2} (\chi_1 - h_1 b_i^-), \quad A_j^2 = \frac{a_j}{h_2^2} (\chi_2 - h_2 b_j^-),$$

$$B_i^1 = \frac{a_{i+1}}{h_1^2} (\chi_1 + h_1 b_i^+), \quad B_j^2 = \frac{a_{j+1}}{h_2^2} (\chi_2 + h_2 b_j^+),$$

$$\bar{C}_i^1 = A_i^1 + B_i^1 + \frac{m}{0,5\tau} + \gamma, \quad \bar{C}_j^2 = A_j^2 + B_j^2 + \frac{m}{0,5\tau} + \gamma,$$

$$F_{i,j}^1 = \frac{m}{0,5\tau} C_{i,j}^n + \chi_2 \frac{a_{j+1} C_{z,i}^n - a_j C_{\bar{z},j}^n}{h_2^2} + b_j^+ a_{j+1} C_{z,j}^n + b_j^- a_j C_{\bar{z},j}^n + \gamma c_m,$$

$$F_{i,j}^2 = \frac{m}{0,5\tau} C_{i,j}^{n+\frac{1}{2}} + \chi_1 \frac{a_{i+1} C_{x,i}^n - a_i C_{\bar{x},i}^{n+\frac{1}{2}}}{h_1^2} + b_i^+ a_{i+1} C_{x,i}^{n+\frac{1}{2}} + b_i^- a_i C_{\bar{x},i}^{n+\frac{1}{2}} + \gamma c_m.$$

First, we solve system (14) with the appropriate conditions, applying the sweep method, and obtain the values $C_{i,j}^{n+\frac{1}{2}}$. Then, using the obtained values at the fractional step, we solve system (15)

with the appropriate conditions. As a result, we obtain the values of the desired function at the full time step $C_{i,j}^{n+1}$ for all i, j .

At the initial stage, the direction along X is implicit, and along Z - explicitly, therefore, the sweep is carried out with a fixed direction along Z ($j=1, N_2-1$). And at the next stage, the solution for X is explicit, and for Z it is implicit, so we fix ($i=1, N_1-1$).

To solve problem (1) - (5), a computational algorithm was developed and a software package was created in the modern Delphi language. The results obtained are presented graphically at $T = 30$ days (Fig. 3):

Here, at a depth of 9 m, two injection wells are installed, and at a depth of 22 m - an extraction well in the middle of the field, the reservoir boundary is impermeable.

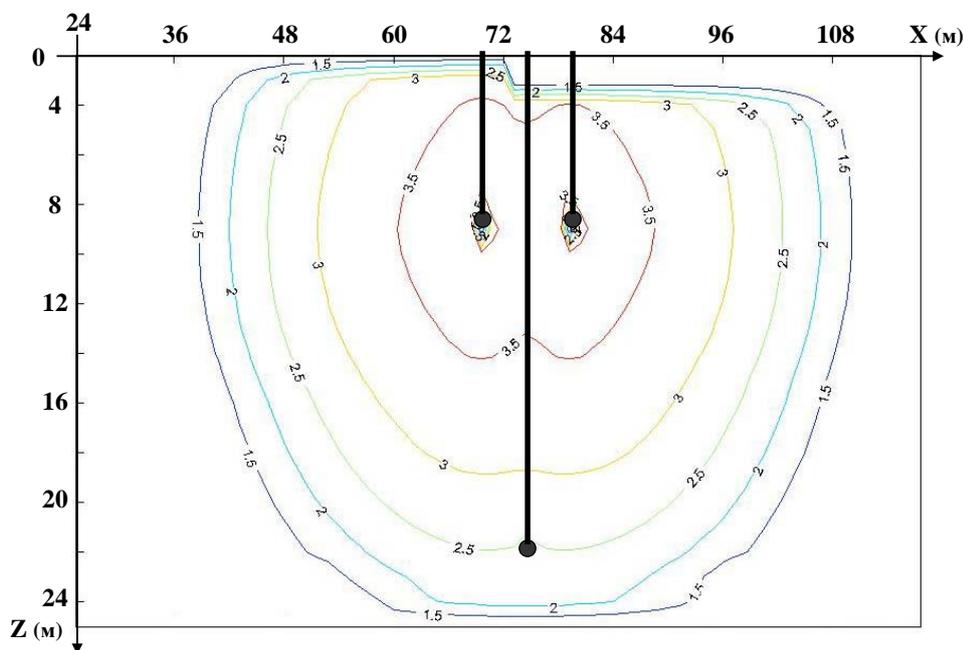


Fig. 3. Isoline of changes in concentration on the section

The results obtained correspond to the physical characteristics of the PV process, whence it follows that the developed computational algorithm and software can be implemented on real PV facilities developed under storey development conditions.

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