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Design And Analysis Of Lqr Controller Using Bees Colony And Particle Swarm Algorithm

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Abstract-

The inverted pendulum is a standard benchmark control problem and numerous control algorithms have evolved over ages. LQR is an optimal control method which is used to control the system. One of the important challenges in the design of LQR in real time applications is the optimal choice of error and control weighting matrices (Q and R), which play a vital role in determining the performance and optimality of the controller. Commonly, trial and error approach are employed for selecting the weighting matrices, which not only burdens the design but also results in non-optimal response. Hence, to choose the elements of Q and R matrices optimally, optimization algorithms are used for selecting the most optimal Q and R matrices which also reduces the performance index of the system. However, stability is only a bare minimum requirement in the system design. Ensuring optimality guarantees the stability of the nonlinear system. The main objective of this project is to design a linear quadratic regulator (LQR) using various optimization algorithms like Artificial Bees Colony (ABC) and Particle Swarm Optimization (PSO) for the inverted pendulum system. The results show that Particle swarm optimization algorithm is efficient in tuning the parameters to give the optimum response.

Keywords- *Inverted Pendulum, optimal control, Linear Quadratic Regulator, Artificial bees' colony (ABC), Particle Swarm Optimization (PSO)*

I. INTRODUCTION

In the real time all the systems are affected by various uncertainties due to modelling errors, external disturbances and parameter variations. Controlling such a dynamical system is difficult and there arises the need for optimal controllers. These controllers will achieve the desired performance of system despite uncertainties. The inverted pendulum is a standard benchmark control problem and for the control of which numerous control algorithms have evolved over the ages. Linear quadratic regulator (LQR) is one among the control algorithm. One of the challenging problems in the design of LQR is the choice of Q and R matrices. Conventionally, the weights of an LQR controller are chosen based on a trial and error approach to determine the optimum state feedback controller gains. However, it is often time consuming and tedious to tune the controller gains via a trial and error method. An Artificial Bee Colony (ABC) algorithm and particle swarm optimization algorithm (PSO) was used to minimize the performance index or cost function by selecting the optimal weighting matrices to overcome LQR design difficulties for the given system.

Baris et al (2017) presented a Linear Quadratic Optimal controller design for an inverted pendulum on a cart using Artificial bees' colony (ABC) algorithm[1].

Vinod Kumar E at al (2016) presented a Algebraic Riccati equation based Q and R matrices selection algorithm for optimal LQR applied to tracking control of 3rd order magnetic levitation system[4].

The paper is organised as the section 2 deals with the description and modelling of inverted pendulum on moving cart. Section 3 explains the design of LQR control using optimization algorithm. Section 4 in detail explains the simulation results. Section 5 addresses the conclusion and the ways in which the work can be extended in future.

II. INVERTED PENDULUM SYSTEM

The inverted pendulum is a nonlinear, unstable, under actuated system. The system has two degrees of motion with a single input such systems are difficult to control. The output is the linear motion of the cart and the angular motion of the pendulum. Because of this nature of the system, they are selected for studying various modern control problems. The schematic representation of the inverted pendulum on a moving cart system is shown in Fig. 1

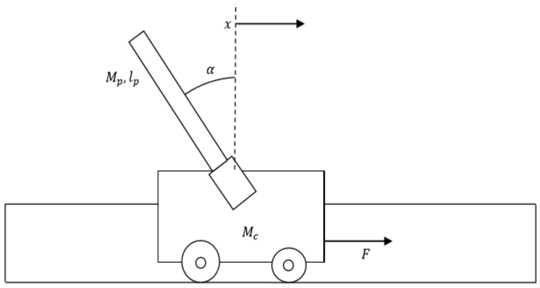

Fig. 1 Cart-Inverted Pendulum System

The cart-inverted pendulum system consists of a pendulum of mass and length attached to the cart of mass and the cart in turn is attached to a motor that drives the cart along the horizontal track by means of gear arrangement. The mass of the cart is given by the sum of the cart mass and the mass of the additional weights that are added to balance the weight of the pendulum attached to the cart. The movement of the cart is constrained only in horizontal direction whereas the pendulum can rotate in the x-y plane[6].

Hence the system can be represented by the two state variables namely, the horizontal displacement of the cart and the angular displacement of the pendulum. The coulomb's frictional force exerted by the cart pinion arrangement and the force on the cart due to pendulum's action are assumed to be negligible for the modelling of the system. The cartesian co-ordinates of the cart-inverted pendulum is represented as shown in Fig 2.

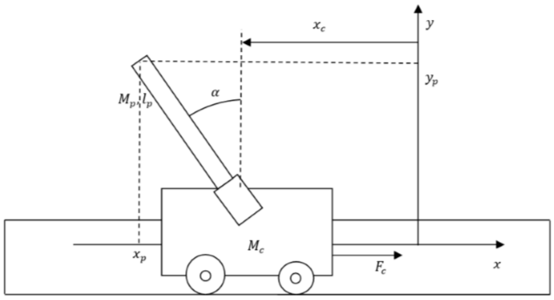

Fig. 2 Cartesian co-ordinates of Cart-Inverted Pendulum System

The global frames are fixed as $x - y$ and the position of the pendulum with respect to the global frame is given by $x_p - y_p$ corresponding to the x and y global reference frame. The mathematical model of the setup shown in Figure 2 is obtained by applying the Euler Lagrangian energy equation.

A. Euler – Lagrangian formulation

The Lagrangian formulation is based on the differentiation of the energy terms with respect to the system's state variables and time [6]. When the complexity of the system increases, the Lagrangian method becomes relatively simpler to use. Lagrangian method is based on the following two generalized equations: one for linear motions and the other for rotational motions. Because of the effectiveness, the Lagrangian method is used for modelling the complex systems which have translational as well as rotational motions. The Lagrangian is defined as

$$L = K - P \qquad (1)$$

Where L is the Lagrangian, K is the total kinetic energy of the system, P is the total potential energy of the system. The equations governing the Lagrangian method is given by

$$F_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} \qquad (2)$$

$$T_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} \qquad (3)$$

Where F_i is the summation of all external forces for a translational motion, T_i is the summation of all external forces for rotational motion, θ_i and x_i are the system variables. Hence in order to get the equations of motion for the system, the energy equations of the system are derived first and then the Lagrangian is differentiated according to Equations (2) and (3).

Now, for the cart-inverted pendulum system, the linear motion is given by cart position x_c and the angular motion is given by pendulum position α. The Euler-Lagrangian for the cart-inverted pendulum system is given by

$$F_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_c} - \frac{\partial L}{\partial x_c} \qquad (4)$$

$$T_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}_i} - \frac{\partial L}{\partial \alpha_i} \qquad (5)$$

Where F_i and T_i are the force applied on the x_c and α co-ordinate respectively. These forces are given by the below expression

$$F_i = F_c(t) - B_{eq}\dot{x}_c \quad (6)$$

$$T_i = -B_p\dot{\alpha}(t) \quad (7)$$

The cart always moves in the horizontal direction and as such, doesn't have any vertical displacement. So the total potential energy is represented by the pendulum's potential energy given by

$$P = M_p g l_p \cos\big(\alpha(t)\big) \quad (8)$$

Kinetic energy gives the measure of total amount of energy in the system due to motion. The total kinetic energy of the cart-inverted pendulum system is given bym

$$K = K_c + K_p \quad (9)$$

Where K_c and K_p are the sum of the translational and rotational kinetic energies of the cart and the pendulum respectively. The translational and the rotational kinetic energy of the cart is given as

$$K_{ct} = \frac{1}{2}M\dot{x}_c^2 \quad (10)$$

$$K_{cr} = \frac{1}{2}\frac{J_m k_g^2 \dot{x}_c^2}{r_{mp}^2} \quad (11)$$

Where J_m is the rotor moment of inertia, k_g is the gear box ratio, r_{mp} is the motor pinion radius. Therefore, the total kinetic energy of the cart can be written as

$$K_c = \frac{1}{2}M_c\dot{x}_c^2 \quad (12)$$

$$M_c = M + \frac{J_m k_g^2}{r_{mp}^2} \quad (13)$$

The total kinetic energy exerted by the pendulum is given by the sum of the translational and the rotational kinetic energy of the pendulum which are given by

$$K_p = K_{pt} + K_{pr} \quad (14)$$

$$K_{pt} = \frac{1}{2}M_p\dot{r}_p^2 \quad (15)$$

$$K_{pr} = \frac{1}{2}I_p\dot{\alpha}^2(t) \quad (16)$$

Where $\dot{r}_p^2 = \dot{x}_p^2 + \dot{y}_p^2$ and from Figure 2 $\dot{x}_p$ and $\dot{y}_p$ can be expressed as

$$\dot{x}_p = \dot{x}_c - l_p\cos\big(\alpha(t)\big)\,\alpha(\dot{t}) \quad (17)$$

$$\dot{y}_p = -l_p\sin\big(\alpha(t)\big)\,\dot{\alpha}(t) \quad (18)$$

Thus, by substituting the above equations, the total kinetic energy of the system is given by

$$K = \frac{1}{2}\left(M_c + M_p\right)\dot{x}_c^2(t) - M_p l_p \cos\left(\alpha(t)\right)\dot{\alpha}(t)\dot{x}_c(t) + \frac{1}{2}\left(I_p + M_p l_p^2\right)\dot{\alpha}^2(t) \quad (19)$$

The Lagrangian is thus expressed as shown below

$$L = \frac{1}{2}\left(M_c + M_p\right)\dot{x}_c^2(t) - M_p l_p \cos\left(\alpha(t)\right)\dot{\alpha}(t)\dot{x}_c(t) + \frac{1}{2}\left(I_p + M_p l_p^2\right)\dot{\alpha}^2(t) - P \quad (20)$$

Upon substituting the obtained Lagrangian (20) in (4) and (5), the nonlinear equations of motion are obtained as

$$F_c(t) = \left(M_c + M_p\right)\ddot{x}_c(t) + B_{eq}\dot{x}_c(t) + M_p l_p \cos\left(\alpha(t)\right)\ddot{\alpha}(t) - M_p l_p \sin\left(\alpha(t)\right)\dot{\alpha}^2(t) \quad (21)$$

And

$$-M_p l_p \cos\left(\alpha(t)\right)\dot{\alpha}(t)\ddot{x}_c(t) + \left(I_p + M_p l_p^2\right)\ddot{\alpha}(t) + B_p\dot{\alpha}(t) - M_p g l_p \sin\left(\alpha(t)\right) = 0 \quad (22)$$

B. Model linearization

The nonlinear model is linearized around the equilibrium point i.e. upright position such that $\sin(\alpha) \cong \alpha$, $\cos(\alpha) \cong 1$. The linearized model is written in the state space form as

$$\dot{X} = AX + BU \quad (23)$$

$$Y = CX \quad (24)$$

Where $X = [x_c\ \alpha\ \dot{x}_c\ \dot{\alpha}]^T$, $U = V$ and $Y = [x_c\ \alpha\ \dot{x}_c\ \dot{\alpha}]^T$

The state space model of the system is thus obtained as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g M_p^2 l_p^2}{(M_p+M_c)I_p + M_c M_p l_p^2} & \frac{-B_{eq}(M_p l_p^2)}{(M_p+M_c)I_p + M_c M_p l_p^2} & \frac{-M_p l_p B_p}{(M_p+M_c)I_p + M_c M_p l_p^2} \\ 0 & \frac{M_p g l_p (M_p+M_c)}{(M_p+M_c)I_p + M_c M_p l_p^2} & \frac{-M_p l_p B_{eq}}{(M_p+M_c)I_p + M_c M_p l_p^2} & \frac{-(M_p+M_c)B_p}{(M_p+M_c)I_p + M_c M_p l_p^2} \end{bmatrix} \quad (25)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{I_p + M_p l_p^2}{(M_p+M_c)I_p + M_c M_p l_p^2} \\ \frac{M_p l_p}{(M_p+M_c)I_p + M_c M_p l_p^2} \end{bmatrix} \quad (26)$$

Linearization is done at the equilibrium point and the state space model of the system is obtained. This model represents the system only within an operating region and whenever the system states exceeds the maximum boundary, the model will not be able to accommodate them. The state and the control input values tend to increase exponentially. Hence, it is important to note the operating region and the conditions that are chosen for testing should fall within the operating rang else the system will become unstable.

Hence care must be taken while selecting the initial conditions for the system. Though the region of operation is small, the state space model addresses the control problem more effectively within the region. The nonlinear equations of motion can be directly used for analysing the system. This could be close to the actual system as these include the nonlinearity in the process equations. Analysis made from such system models will be closer to the actual system.

The cart-pendulum system parameters that are used to obtain the state space model are shown in Table 1 [13]. These system parameters governing the cart-pendulum system are substituted in the equations (25) and (26) to get the state space model.

By substituting the parameters given in Table 1 in Equations (25) and (26), the state space model of the system is obtained as shown below:

$$\begin{bmatrix} \dot{x}_c \\ \dot{\alpha} \\ \ddot{x}_c \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8703 & -4.8987 & -0.0094 \\ 0 & 46.2580 & -21.2169 & -0.5012 \end{bmatrix} \begin{bmatrix} x_c \\ \alpha \\ \dot{x}_c \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.9072 \\ 3.9291 \end{bmatrix} u \quad (27)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \alpha \\ \dot{x}_c \\ \dot{\alpha} \end{bmatrix} \quad (28)$$

Table 1 System Parameters of Cart-inverted pendulum system

Symbol	**Parameter**	**Value**
M_c	Mass of cart	1.0731 K_g
M_p	Pendulum mass	0.127 K_g
l_p	Pendulum length from centre to C.G	0.1778 m
l_p	Pendulum moment of inertia	$1.2 \times 10^{-3} Kgm^2$
g	Acceleration due to gravity	9.81 m/s^2
B_p	Viscous damping co-efficient at pendulum axis	0.0024 Nms/rad
B_{eq}	Viscous damping co-	5.4 Nms/rad

	efficient of motor pinion	

C. Stability of cart-pendulum system

The cart-pendulum system is an unstable system in open loop and this can be verified with the pole zero plots for the obtained state space model. The pole zero plots for the system is shown in Figure 3. As seen from the pole zero map, the poles of the obtained state space model are located at 0, 6.4432, -7.5831 and -4.2600.

This implies that the system inherently is not stable and thus needs a proper controller to be designed that can bring all the closed loop poles to the left half plane making the system stable. The need for proper controller design is thus clearly visible and that the system would be unstable in open loop.

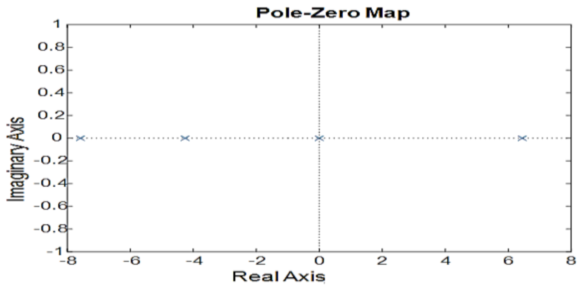

Fig. 3 Pole zero map of the cart-pendulum system model

Hence the design of controller's gains significance for such systems. If the controllers designed, works well for the controlling of the cart-pendulum system, then they can be implemented in a wide variety of real time applications. The controllers designed for such systems find their application in aerospace systems which have much more complex and unstable systems. Also, they can be used to control similar under actuated systems as well.

III. CONTROLLER DESIGN

A. Linear quadratic regulator

The basic idea of linear Quadratic Regulator (LQR) controller is to solve the weighting matrices selection problem. One of the important challenges in the design of LQR for real time applications is the optimal choice of state matrix (Q) and input weighting matrix (R), which play a vital role in determining the performance and optimality of the controller. Commonly, trial and error approach are employed for selecting the weighting matrices, which is not only tedious but also time consuming and results in non-optimal response.

Hence, to choose the elements of Q and R matrices optimally, an optimization algorithm is formulated and applied for minimizing the performance index or the cost function. Moreover, by minimizing a quadratic cost function which consists of two penality matrices (Q and R), LQR yields an optimal response between the control input and speed of response. Hence, the LQR techniques have been successfully applied to a large number of complex systems such as vibration control system, fuel cell system and aircraft[3].

In order to obtain the Q and R matrices optimally, iteration is performed using optimization algorithms. The value of Q and R for which the cost function is minimum is considered to be the optimal values of the matrices Q and R. The block diagram for the LQR controller using various optimization algorithms like artificial bee colony algorithm and particle swarm optimization algorithm is shown in fig 4.

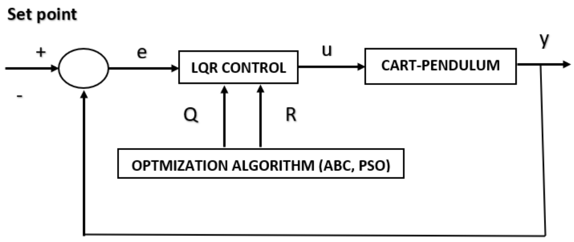

Fig. 4 Block diagram of LQR controller for cart pendulum system

B. Design of LQR control

The LQR control is a powerful technique for designing controllers for complex systems that have stringent performance requirements and it seeks to find the optimal controller that minimizes a given cost function. The cost function is parameterized by two matrices, Q and R which weight the state vector and the system input respectively. LQR method is based on the state-space model and it tries to obtain the optimal control input by solving the algebraic riccatti equation[5].

Consider a linear time invariant system (LTI),

$$\dot{x} = A\,x(t) + B\,u(t) \qquad (29)$$

$$y = C\,x(t) \qquad (30)$$

Where x(t) is the state vector and u(t) is the input vector, determine the matrix

$K = R^{n*m}$ such that the static, full state feedback control law,

$$u = -Kx(t) \qquad (31)$$

$$K = R^{-1}B^{T}P \qquad (32)$$

Satisfies the following criteria,

a) the closed-loop system is asymptotically stable
b) the quadratic performance function and the cost function A

$$J(K) = \frac{1}{2}\int_0^{\infty}[x^T(t)\,Q(t) + u^T(t)\,R\,u(t)]dt \qquad (33)$$

is minimized. Q is a nonnegative definite matrix that penalizes the departure of system states from the equilibrium, and R is a positive definite matrix that penalizes the control input.

The following are the steps to design LQR control where Q and R values are selected by iteration method:

Step:1: Solve the matrix Algebraic Riccati Equation (ARE)

$$-PA - A^T P - Q + PBR^{-1}B^T P = 0 \qquad (34)$$

Step:2: Determine the optimal state $x^*(t)$ from

$$\dot{x}^*(t) = [A - BR^{-1}B^T P]x^*(t) \qquad (35)$$

Step:3: Obtain the optimal control $u^*(t)$ from

$$u^*(t) = -R^{-1}B^T P x^*(t) \qquad (36)$$

Step:4: Obtain the optimal performance index from

$$J^* = \frac{1}{2} {x^*}^T(t) P x(t) \qquad (37)$$

Step:5: Iterate the Q and R values from 0 to n, where n represents the number of iterations to be performed till the performance index or cost function gets minimized.

The weighting matrices Q and R are important components of an LQR optimization process. The composition of Q and R elements has great influences on system performance. The designer need not to worry about the choice of Q and R values as it can be resolved using iteration method[4].

C. Artificial bee's colony algorithm (ABC)

The ABC algorithm is an intelligent search technique used to find solutions to optimization problems. It was proposed by Karaboğa, inspired by the behavior of honey bee swarms that find a food source and share the knowledge of this food source with others. The ABC algorithm divides the artificial bees in a bee colony into three groups as employed bees, onlooker bees, and scout bees. The employed bees search the food sources in the field and share the information of food sources with other bees.

Each employed bee is responsible for only one food source. The onlooker bees wait in the hive and find a food source according to information provided by the employed bees. An employed bee turns into a scout bee after draining the food source and starts to search for new food sources around the hive. The position of a food source means a candidate solution for the corresponding problem. The value of the objective function represented by the nectar amount determines the quality of solution. The algorithm starts with random distribution of employed bees in the search field and production of the initial solutions. For i = 1, 2…, SN (SN is the number of source), each source is a D-dimensional vector. The position of the ith food source in the search space is represented by $X_i = [X_{i1}\, X_{i2} \ldots . X_i]^T$.

Each employed bee searches and produces a modified food source position by the following equation:

$$x'_{ij} = x_{ij} + r_{ij}(x_{ij} - x_{kj}) \qquad (38)$$

where j ∈ 1,2, ⋯, D and k ∈ 1, 2…, SN are randomly chosen indices and k ≠ i. The parameter r_{ij} is also a real random number in the domain [−1,1]. After receiving the food source information, the onlooker bee goes to the food source region at X_i based on probability P_i defined by the following equation

$$P_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \qquad (39)$$

Fitness value fit_i is calculated by using the following equation:

$$fit_i = \begin{cases} \frac{1}{1+f(X_i)} & f(X_i) \geq 0 \\ 1 + |f(X_i| & f(X_i) < 0 \end{cases} \qquad (40)$$

Where $f(X_i)$ is the objective function of source X_i to be minimized. If the new fitness value is better than the previous fitness values, than the bee moves to this new food source. The source information is shared with the onlooker bees after the process is completed by all employed bees and each onlooker bee selects a food source by the probability given above. Each bee searches a better food source until the appropriate solution or maximum iteration number is reached[1].

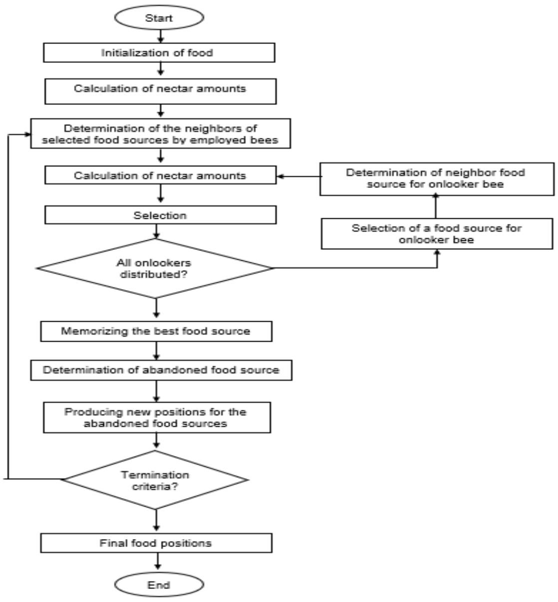

Fig. 5 Flowchart of ABC algorithm

D. Particle swarm optimization (PSO)

The PSO algorithm is a population-based heuristic search technique used to find solutions to optimization problems. It was developed by Kennedy and Eberhart, inspired by the social behavior of bird and fish swarms. The PSO algorithm begins by creating an initial population (swarm) consisting of randomly generated particles within an initialization region. Random position and velocity assigned particles search the optimum solution by navigating in the problem space. The position of each particle corresponds to a candidate solution of the optimization problem represented by objective function f.

The fitness of each particle is calculated according to the objective function f and the best position (pbest) ever visited by that particle is determined. After the determination of the pbest value of each particle in the population, the best position ever visited by any particle (qbest) is determined. The velocities and the positions of the particles are calculated by Eqs. (41) and (42), respectively:

$$v_{ij}^{k+1} = w.v_{ij}^{k} + c_1.r_1.\left(pbest_{ij}^{k} - x_{ij}^{k}\right) + c_2.r_2.\left(gbest_{ij}^{k} - x_{ij}^{k}\right) \quad (41)$$

$$x_{ij}^{k+1} = x_{ij}^{k} + v_{ij}^{k+1} \quad (42)$$

In these equations, c_1 and c_2 are the learning factors and w is the inertia weight. The learning factor leads the movement of a particle according to its own experience and the experience of the other particles in the swarm. The inertia weight adjusts the extent of the search area. A small inertia weight enables the local search while a large inertia weight allows the global search. After the update, the fitness of each particle in the new population is recalculated. This process is repeated until the appropriate solution or maximum iteration number is reached.

From every cycle of the optimization algorithms (ABC and PSO), new Q and R matrices are updated and a new feedback gain matrix K is obtained in the following way:

- Solving the algebraic Ricatti equation given in Equation 34 for P where A and B are given above.
- Q and R are updated by the ABC and PSO algorithms.
- Finding the gain matrix using Equation 32.

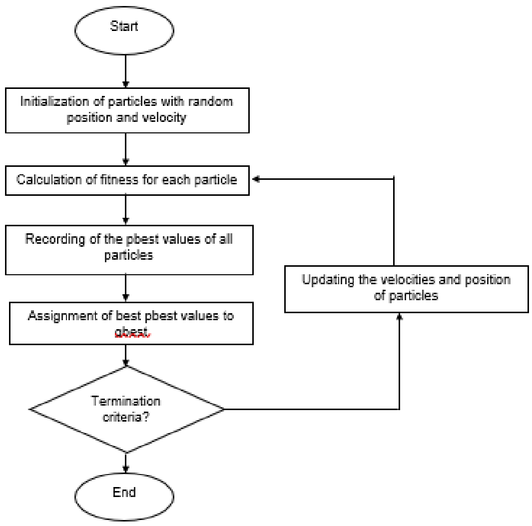

Fig. 6 Flowchart of PSO algorithm

IV.RESULTS AND DISCUSSION

A. Open loop response of the inverted pendulum

The fig 7 shows the open loop response of the inverted pendulum. From response, it is clear that the system is unstable since the output is bounded.

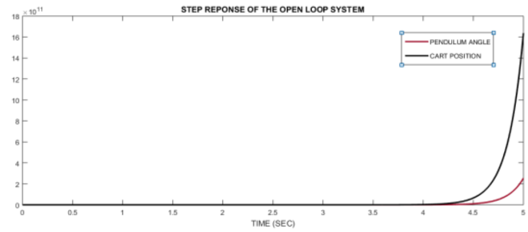

Fig. 7 Step response of open loop system

B. Closed loop response of the inverted pendulum

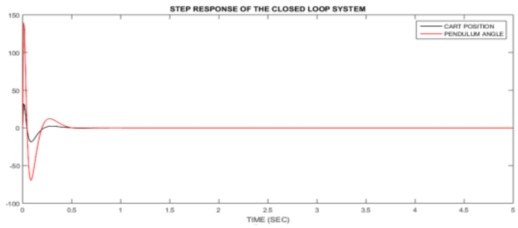

Fig. 8 Step response of the closed loop system

The Fig. 8 shows that the closed loop response of the inverted pendulum system and it can be seen from the figure that it has more overshoot in both pendulum angle and the cart position.

C. Response of the inverted pendulum system using LQR control

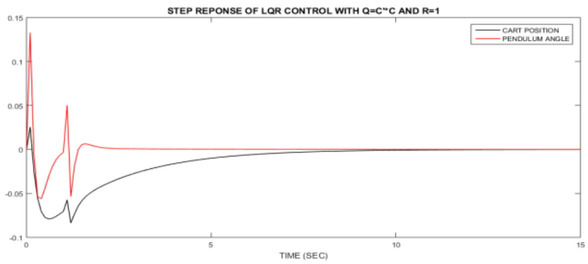

Fig. 9 Step response with LQR control

From the fig. 9 shows that the Q and R values are given (Q=C'*C and R=1). The cart position and pendulum angle settle to a desired value but the settling time and rise time are more.

D. Response of the LQR control using optimization algorithms

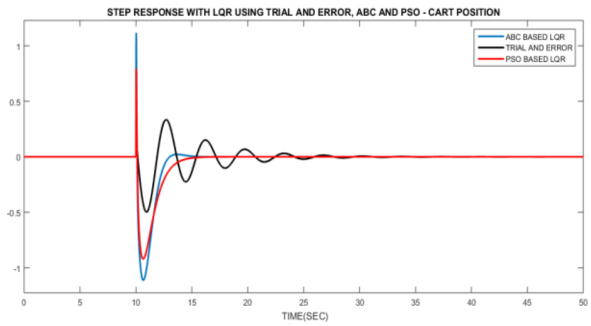

Fig. 10 Step response of LQR control for cart position

The step response of the inverted pendulum with LQR control using optimization algorithm is shown in fig 5 and fig 6 where the Q and R values are obtained by iteration.

Here the overshoot and settling time is less in PSO tuned LQR for cart position and pendulum angle of the inverted pendulum than in the trial and error approach and ABC tuned LQR. The selected weighting matrices for the trial and error approach are shown in Equation 4.1.

$$Q = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 5.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ R=0.0003} \qquad (43)$$

By solving Equation 34 and 32 for the system matrices of the inverted pendulum system, the closed loop optimal control gain is obtained as in Equation 44.

K = [-40.8248 184.9692 -43.7508 18.1548] (44)

The selected weighting matrices for the ABC tuned LQR are shown in Equation 45.

$$Q = diag([625.8682\ 0.0400\ 0.0400\ 0.8782]), \text{ R=1} \qquad (45)$$

The parameters of the ABC algorithm are set in the range [0.1 100], colony size=20 and max cycle=100.

By solving Equation 34 and 32 for the system matrices of the inverted pendulum system, the closed loop optimal control gain is obtained as in Equation 46

K = [-6.7287 -7.6433 24.6789 4.7350] (46)

The selected weighting matrices for the PSO tuned LQR are shown in Equation 47.

$$Q = \mathrm{diag}([64.059\ 0.068\ 265.428\ 1.713]\text{,R=0.3883} \qquad (47)$$

The parameters of the PSO algorithm are population size = 100, Number of Iterations = 100, velocity constant c1=2, velocity constant c2 = 2. By solving Equation 34 and 32 for the system matrices of the inverted pendulum system, the closed loop optimal control gain is obtained as in Equation 48.

K = [-12.8442 114.8787 -36.7504 16.9963] (48)

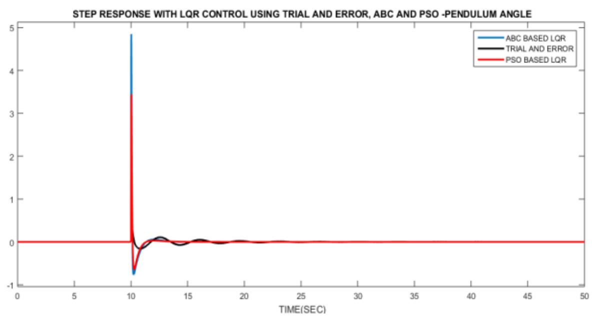

Fig. 11 Step response of LQR control for pendulum angle

	Time response	parameters	Trial and error approach	ABC tuned LQR	PSO tuned LQR
Step response	**Settling time (t_s)**	Cart position Pendulum angle	30 23	17 13	15 12
	Overshoot (%)	Cart position Pendulum angle	0.4 0.5	1.2 4.2	0.6 3.5

Table 2 Comparative analysis – Step response of LQR control using trial and error, ABC tuned LQR and PSO tuned LQR.

V CONCLUSION

In this paper a Linear Quadratic Regulator control using optimization algorithms has been introduced for the inverted pendulum system. The choice of weighting matrices Q and R plays a major role when cost function or performance index is taken into account. Generally, selecting matrices is managed by the trial and error approach and is merely a time-consuming process.

To avoid the conventional trial and error method, an LQR controller is designed using the ABC and PSO algorithms and to determine the weighting matrices to overcome the LQR design difficulties. These methods reduce the time required to select the weighting matrices which is being chosen from user's previous experience.

The value of Q and R is selected for which it results in minimum cost function. The iterations will run for 'n' number of times till the performance index becomes minimum. This project can be extended for discrete time nonlinear systems as a future work. As mentioned earlier, there are many types of optimization algorithms which can be selected based on the requirement or constraints.

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