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Thermal analysis of Bingham plastic fluid flow between two infinite parallel plates

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ABSTRACT

This work considered a non- Newtonian flow between two parallel plates where one is at restand another in motion. The velocity and temperature is studied by solving the nonlinear coupled equation simultaneously formed by the equivalence of motion, continuity equation and energy equation using Bingham plastic flow model. The temperature is studied for both the cases, one is plates are with same temperature and another is with different. It has been studied analytically to the velocity profile and temperature profile with various parameters and presented in forms of figures.

KEYWORDS: Fluid flow, Parallel plats, Bingham plastic, Velocity, Temperature.

INTRODUCTION

In civil engineering Bingham plastics materials are used extensively in various purpose like hydraulic suspension and heavily loaded machines.The yield pressure is the one of the significant properties of this liquid most significant attribute of Bingham plastics is their yield pressure, which says the cutoff and the point above which stream exists [1].Yeild surface in the sense it is a locus point of the material which separates the two different region with different properties[2].

Hoshyar, H. A., et al.[3] has studied the flow between two parallel plates where upper plate moves up and down and lower one is fixed and with porous surface, Also the study has been done for non- Newtonian fluid.

Kaushik, P et al.[4] investigated the viscoelastic fluid flow between two parallel plates with squeezing motion.

Kumar, Singeetham Pavan et al. [5] has done a theoretical study on Bingham fluid flow between two parallel planes with squeezing motion and analysed the film thickness, pressure distribution and squeezing force with different flow parameters.

Rees, D et al.[6] considered bingham fluid with two unsteady free convection in vertical circular cylinder with porous surface.

Attia, Hazem A., et al.[7] studied the Couette flow with temperature distribution ina infinite parallel plate with porous medium under constant pressure gradient.

Fernández-Galisteo, Daniel et al. [8] has done experimental where analyzation has been done about quasi-isobaric flame propagation between two adiabatic parallel plates.

Siddiqui, A. M., et al.[9] and Szeri, A. Z[10] examined the warmth move stream for Couette stream, plane Poiseuille stream and plane Couette–Poiseuille between two warmed equal plates for the consistent thickness model. Mohammad Mehdi Rashidi [11], examined the incompressible liquid stream between two equal plates with ordinary movement of both the plates.

In the light of the above conversation, it has been seen that a couple of analyst has produced the temperature results with binghum plastic liquid. Thus the principle point of this paper is to examine the warm investigation of Bingham plastic liquid stream between two equal plates by accepting one plate is very still and other one is moving. The speed profile likewise has been analyzed with various stream boundary. A theoretical approach has been made by solving the governing equation analytically up to some extent and then used the numerical methods to get the final solution. All the results are shown in figures and presented well which is well matched with some experimental work.

Mathematical Modeling

2.1 Reynolds had established the governing equations in 1886 for inertia less flow of thin film of Newtonian fluid. However, in many applications, the Newtonian behavior of the lubricant does not exist for a long time, i.e. the linear relationship between shear rate and shear stress collapse in a short period of time.Consider a progression of liquid between two equal plates when one plate is very still and other moving and the liquid follows Bingham model. The hypothetical displaying conditions for the issue viable are [12,13,14]:

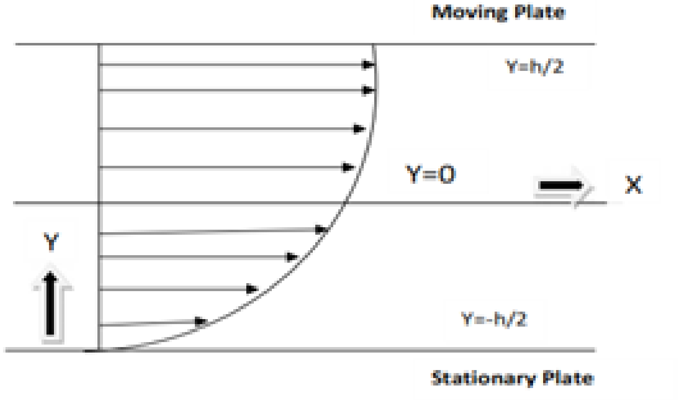

Fig-1: Geometry of the problem

$$\frac{\partial u}{dx}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{dp}{dx}=\frac{d\tau}{dy}, \text{ where } \tau=\tau_0+\mu\frac{du}{dy} \qquad (2)$$

The fluid velocity borderlineenvironmentsfor the system are assumed as $u=0\, and\, u=U\, at\, y=h/2\, and\, y=-h/2\, respectively.$ (3)

Also the velocity in the provinces $-h/2\le y\le 0\, and\, 0\le y\le h/2$ are considered as $\frac{du}{dy}\ge 0\ and\ \frac{du}{dy}\le 0$ respectively. (4)

Now from equation(2) one may get by integrating with respect to $'y'$ as

$$y\frac{dp}{dx}+c=\tau_0+\mu\frac{du}{dy}$$

$$\Rightarrow \frac{du}{dy}=\frac{1}{\mu}\left(y\frac{dp}{dx}-\tau_0+c\right) \qquad (5)$$

Integration of equation(5) with respect to $'y'$, it gives

$$u=\frac{1}{\mu}\left(\left(\frac{y^2}{2}\right)\frac{dp}{dx}-\tau_0 y+cy\right)+d \qquad (6)$$

Use of the above boundary conditions (3), the constants $c\ and\ d$ will be computed as

$$c=\frac{\mu U}{h}+\tau_0 \quad \text{and}\ d=\frac{U}{2}-\frac{h^2}{8\mu}\left(\frac{dp}{dx}\right)$$

Substituting these above $c\ and\ d$ values in equation(6), it gives the velocity distribution of fluid as

$$\frac{u}{U}=\frac{1}{2}+\frac{y}{h}-\frac{h^2}{2\mu U}\frac{dp}{dx}\left(\frac{1}{4}-\frac{y^2}{h^2}\right) \quad \text{for} \quad -\frac{1}{2}\le\frac{y}{h}\le\frac{1}{2} \tag{7}$$

Assuming zero pressure gradients that is $\frac{dp}{dx}=0$ equation (7) reduces to be

$$\frac{u}{U}=\frac{1}{2}+\frac{y}{h} \quad for \quad -\frac{h}{2}\le y\le\frac{h}{2}$$

By using the scheme of non-dimensionalization then equation(7) may be written as

$$\bar{u}=\frac{1}{2}+\bar{y}+\frac{P\left(1-\bar{y}^2\right)}{8} \quad for \ -\frac{1}{2}\le\bar{y}\le\frac{1}{2} \tag{8}$$

The fluid flux can be obtained by integrating the equation of continuity (1) as

$$\int_{-h/2}^{h/2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dy=0$$

$$\frac{\partial}{\partial x}\int_{-h/2}^{h/2}u\ dy\ =c^*, \quad Q=\int_{-\frac{h}{2}}^{\frac{h}{2}}u\ dy\ \ or\ \ Q=\frac{Uh}{2}-\frac{h^3}{12\mu}\left(\frac{dp}{dx}\right)$$

$$\frac{\partial Q}{\partial y}=0\ ,\ then\ c^*=0$$

Average velocity of fluid may be defined as:

$$u_a=\frac{discharge(Q)}{cross-sectional area\ of\ \ fluid\ \ flow\ per\ unit\ length}$$

$$u_a=\frac{Q}{h\times 1}$$

$$u_a=\frac{U}{2}-\frac{h^2}{12\mu}\left(\frac{dp}{dx}\right) \tag{9}$$

Difference equation(5) with respect to 'y'

$$\frac{du}{dy}=\frac{U}{h}+\frac{y}{\mu}\left(\frac{dp}{dx}\right) \tag{10}$$

Temperature Distribution between one plate is moving and other at rest:-

The Energy equation for the above model can be written as

$$k\frac{d^2T}{dy^2}+\tau\frac{du}{dy}=0$$

$$\frac{d^2T}{dy^2}=-\frac{1}{k}\left(\tau_0+\mu\frac{du}{dy}\right)\frac{du}{dy}$$

$$\frac{d^2T}{dy^2}=-\frac{\tau_0}{k}\left(\frac{U}{h}+\frac{y}{\mu}\frac{dp}{dx}\right)-\frac{\mu}{k}\left(\frac{U}{h}+\frac{y}{\mu}\frac{dp}{dx}\right)$$

Integrating above equation with respect to ' y '

$$\frac{dT}{dy} = -\frac{\tau_0 U y}{kh} + \frac{\tau_0 P U y^2}{2kh^2} - \frac{\mu U^2 y}{kh^2} - \frac{\mu P^2 U^2 y^3}{3kh^4} + \frac{\mu P U^2 y^2}{kh^3}$$

Again integrating with respect to 'y' where

$$P = \frac{-h^2}{\mu U}\left(\frac{dp}{dx}\right)$$

$$T = -\frac{\tau_0 U y}{2k}\left(\frac{y}{h}\right) + \frac{\tau_0 P U y}{6k}\left(\frac{y}{h}\right)^2 - \frac{\mu U^2}{2k}\left(\frac{y}{h}\right)^2 - \frac{\mu P^2 U^2}{12k}\left(\frac{y}{h}\right)^4 + \frac{\mu P U^2}{3k}\left(\frac{y}{h}\right)^3 + c_1 y + d_1 \quad (9)$$

$\textit{This is for the region: } -\frac{1}{2} \le \frac{y}{h} \le \frac{1}{2}$

Plates are kept at same Temperature :-

Boundary conditions are

$$T = T_L \ \ at \ \ y = -\frac{h}{2}$$

$$T = T_L \ \ at \ \ y = \frac{h}{2}$$

Then we get the constant values c_1 *and* d_1 *are*

$$d_1 = T_L + \frac{\tau_0 U h}{8k} + \frac{\mu U^2}{8k} + \frac{\mu P^2 U^2}{192K}$$

$$c_1 = -\frac{\tau_0 P U}{24k} - \frac{\mu P U^2}{12kh}$$

$$T = T_L + \left[\frac{U}{8k}(\tau_0 h + \mu U) - \frac{UP}{24k}\left(\frac{y}{h}\right)(\tau_0 h + 2\mu U)\right]\left(1 - 4\left(\frac{y}{h}\right)^2\right) + \frac{\mu P^2 U^2}{192k}\left(1 - 16\left(\frac{y}{h}\right)^2\right) \quad (10)$$

$\textit{This is for the region } -\frac{1}{2} \le \frac{y}{h} \le \frac{1}{2};$

$\textit{Take } \bar{y} = \frac{y}{h}; \textit{ Then}$

$$T = T_L + \left[\frac{U}{8k}(\tau_0 h + \mu U) - \frac{PU\bar{y}}{12k}\left(\frac{\tau_0 h}{2} + \mu U\right)\right]\left(1 - 4(\bar{y})^2\right) + \frac{\mu P^2 U^2}{192k}\left(1 - 16(\bar{y})^2\right)$$

$\textit{This is for the region } -\frac{1}{2} \le \bar{y} \le \frac{1}{2};$ (11)

Plates are kept at different Temperature :

Boundary conditions are

$$T = T_L \ \ at \ \ y = -\frac{h}{2}$$

$$T = T_L \ \ at \ \ y = \frac{h}{2}$$

Using the above boundary conditions in equation(9) and also

$c_1 = c_2$, $d_1 = d_2$ *then it gives*

$$d_2 = \left(\frac{T_U + T_L}{2}\right) + \frac{\tau_0 U h}{8k} + \frac{\mu U^2}{8k} + \frac{\mu P^2 U^2}{192k}$$

$$c_2 = \left(\frac{T_U - T_L}{h}\right) - \frac{\tau_0 P U}{24k} - \frac{\mu P U^2}{12kh}$$

Substituting these c_2 and d_2 values in equation(9), it gets

$$T = T_L + \left(\frac{T_U - T_L}{2}\right)\left(1 + 2\left(\frac{y}{h}\right)\right) + \left[\frac{U}{8k}(\tau_0 h + \mu U) - \frac{UP}{24k}\left(\frac{y}{h}\right)(\tau_0 h + 2\mu U)\right]\left(1 - 4\left(\frac{y}{h}\right)^2\right) + \frac{\mu P^2 U^2}{192k}\left(1 - 16\left(\frac{y}{h}\right)^2\right) \quad (12)$$

This is for the region $-\frac{1}{2} \le \frac{y}{h} \le \frac{1}{2}$;

Take $\bar{y} = \frac{y}{h}$ *; then*

$$T = T_L + \left(\frac{T_U - T_L}{2}\right)(1 + 2\bar{y}) + \left[\frac{U}{8k}(\tau_0 h + \mu U) - \frac{UP\bar{y}}{24k}(\tau_0 h + 2\mu U)\right](1 - 4(\bar{y})^2) + \frac{\mu P^2 U^2}{192K}(1 - 16(\bar{y})^2)$$

This is for the region $-\frac{1}{2} \le \bar{y} \le \frac{1}{2}$;

Result and Discussion

The numerical values used for computing the solution for the given system of equation are- $\tau_0 = 0.1,\ y/h = 1,\ \mu = 0.001,\ T_l = 0.4, P = -3, -2, -1, 0, 1, 2, 3$, $a = 0, 0.5, 1, 1.5, 2$

Fig:2, represents the velocity profile which shows that for different values of P, with respect to $\bar{y}$ the fluid velocity increases in the subordinateprovince, and diminutions in the superiorprovince. When P = 0, velocity of fluid *u/U* is linear with*y/h* as shown in the following Fig.2. When P=-1,1 the fluid velocity *u/U* increases throughout between the two plates as *y/h* increases and when P is considered other values, the velocity*u/U* is increasing with*y/h* and then decreases as *y/h* increases as shown in the following Fig.2. When P=-3, -2, -1, 1, 2, 3 the velocity *u/U*is increasing with *y/h* and then decreases as *y/h* increases in Fig.2.

Fig.3 represents the temperature profile and it is considered when both the plates are of same temperature with different P values. When P=0, the fluid temperature $\bar{T}$ increases as $\bar{y}$ increases up to in the middle and then decreases as $\bar{y}$ increases, as shown in Figure.3. This graph is similar to the previous findings when P=0 by Raisinghania and Hermann Schliching[ ]. When P=-1,1, the temperature of the fluid $\bar{T}$ increases with $\bar{y}$ and then decreases as $\bar{y}$ increases. Whereas Fig: 4, shows the varying temperature with respect to P where both the plates are with different temperature. When a=0,the temperature of the fluid $\bar{T}$ is linear in $\bar{y}$. When a=0.5,1,1.5,2 and $-1 \le P \le 1$, the fluid temperature $\bar{T}$ is increasing with $\bar{y}$ up to the lower part of the upper region.Then finally temperature decreases as *y/h* increases. The only difference among them is their magnitudes. matches well with the results of that of Balram Kundu [34].

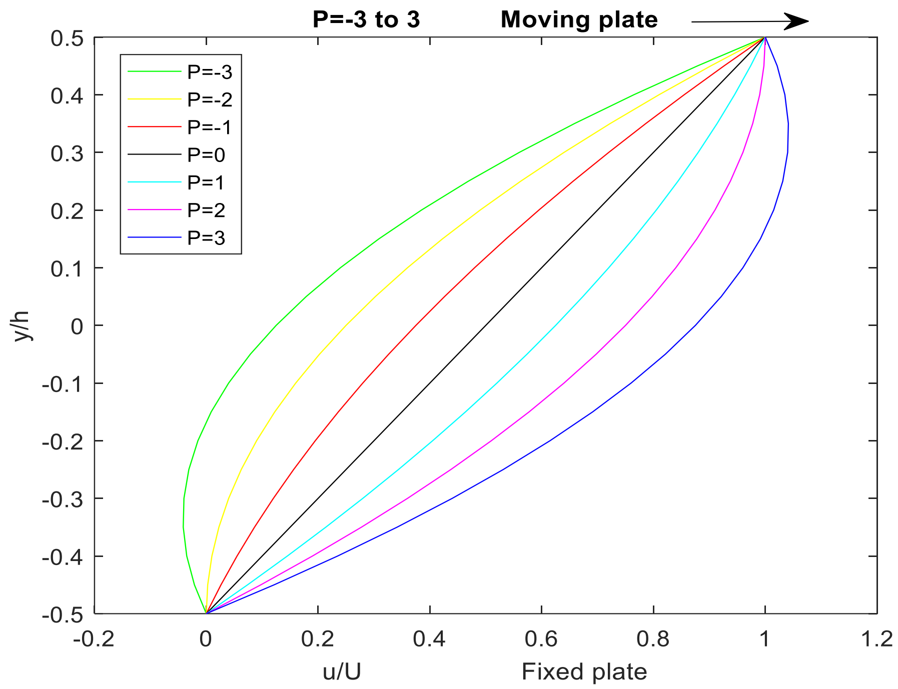

Fig-2 Velocity profile: one plate is moving and another at rest

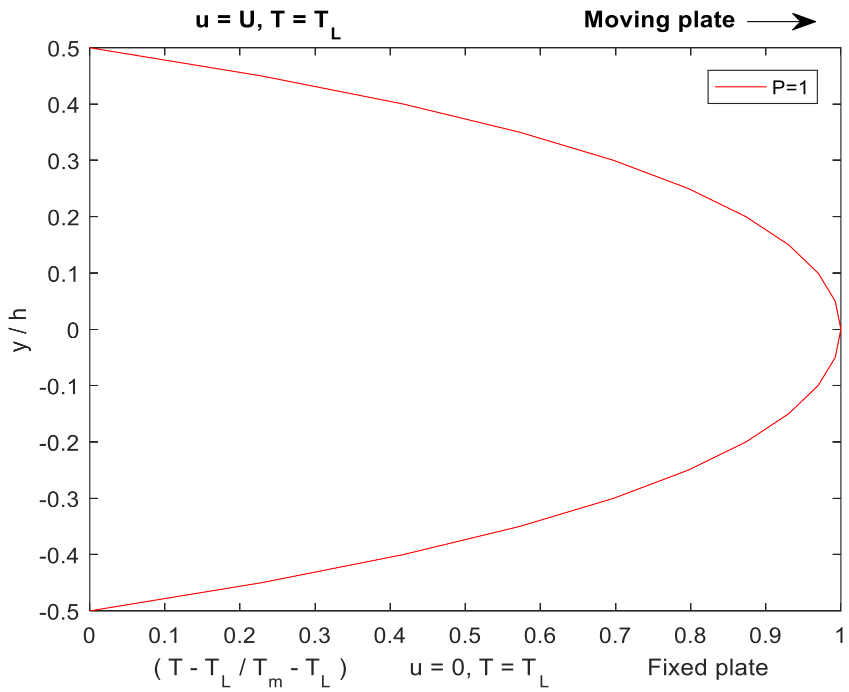

Fig-3 Same Temperature at P= 1

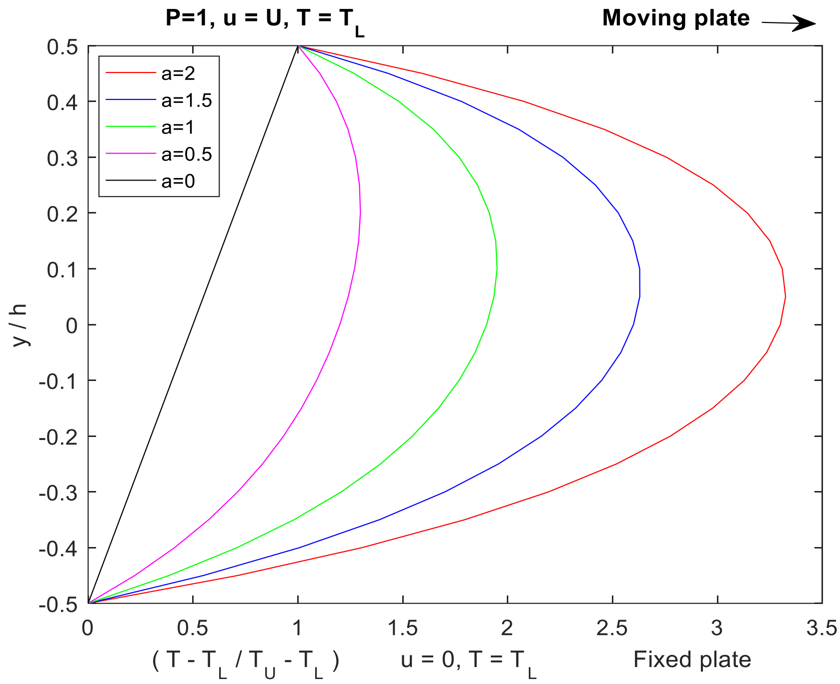

Fig-4 Different temperatures at P= 1

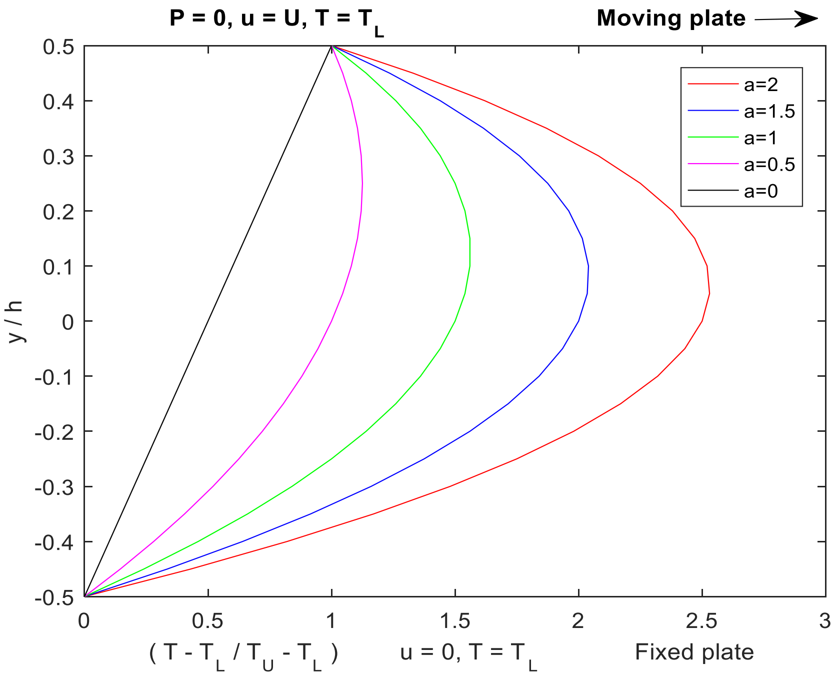

Fig-5 Different temperatures at P= 0

Conclusion

A semi scientific methodology is made to tackle the numerical model where Bingham plastic liquid with consistent state is thought of. The liquid streams between two equal plates where one plate is moving and another is very still. The speed and temperature circulation of the liquid has been researched by thinking about something similar and diverse temperature in both the plates. The speed and temperature dissemination is appeared in charts which acquired by utilizing the MATLAB programming in the wake of settling the condition of movement and energy condition with reasonable limit conditions. Also it has been seen that every one of the outcomes are well concurrence with a portion of the trial work [8].

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